Accreting millisecond X-ray pulsars, from accretion disk to magnetic poles
Promotiecommissie:

Promotor: Prof. dr. M. B. M. van der Klis

Co-Promotor: Dr. R. Wijnands

Overige leden: Prof. dr. W. Hermsen
Prof. dr. S. F. Portegies Zwart
Prof. dr. E. P. J. van den Heuvel
Prof. dr. F. W. M. Verbunt
Prof. dr. R. A. M. J. Wijers
Dr. J. W. T. Hessels

Faculteit der Natuurwetenschappen, Wiskunde en Informatica

The research reported in this thesis was carried out at the Astronomical Institute “Anton Pannekoek”, at the Universiteit van Amsterdam, The Netherlands.
# Contents

1 Introduction .......................................................... 1
   1.1 Theory of neutron stars ........................................ 1
      1.1.1 Neutron star structure .................................. 2
   1.2 Observation of neutron stars .................................. 3
      1.2.1 Radio pulsars .............................................. 5
      1.2.2 Accreting neutron stars .................................. 7
   1.3 Accretion disc ................................................... 9
   1.4 Accreting millisecond X-ray pulsars ......................... 11
      1.4.1 Standard accretion theory ................................ 11
      1.4.2 Periodic variability ...................................... 12
      1.4.3 Quasi-periodic variability .............................. 13
      1.4.4 Thermonuclear X-ray bursts .............................. 13
   1.5 Observations of accreting millisecond X-ray pulsars .... 14
      1.5.1 Rossi X-ray Timing Explorer .............................. 17
      1.5.2 European X-ray Multi-Mirror telescope Newton ...... 19
      1.5.3 Swift ....................................................... 20
   1.6 Observational techniques ..................................... 21
      1.6.1 Periodic timing analysis .................................. 21
      1.6.2 Aperiodic timing analysis ................................. 23
      1.6.3 Spectral Analysis ........................................... 24

2 Coherence of burst oscillations and accretion-powered pulsations in the accreting millisecond pulsar XTE J1814-338 25
   2.1 Introduction ..................................................... 26
   2.2 Timing analysis .................................................. 27
   2.3 Discussion ....................................................... 30
4.4.1 Long-term spin down ........................................ 88
4.4.2 Pulse profile variability ..................................... 92
4.4.3 Motion of the hot spot ...................................... 93
4.4.4 Comparison with previous spin frequency measurements ........................................ 95
4.4.5 Constraints on the magnetic field .......................... 97
4.4.6 Constraints on accretion torques ......................... 98
4.4.7 Discussion of the increasing $P_{\text{orb}}$ .................. 98
4.5 APPENDIX: Improved Optical Position for SAX J1808 ........ 99
4.6 Derivation of Phase Uncertainties .......................... 102

5 The properties of low energy pulsations in SAX J1808.4-3658 103
5.1 Introduction .................................................... 104
5.2 X-ray observation ............................................. 104
5.3 Timing analysis ............................................... 106
5.4 Spectral analysis .............................................. 107
  5.4.1 High-resolution spectroscopy ............................ 109
5.5 Discussion .................................................... 113

6 Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294 115
6.1 Introduction .................................................... 116
6.2 Data reduction and reconstruction of the pulse profiles .................. 117
6.3 Results ........................................................ 119
  6.3.1 Measurement of the spin frequency and its derivative in the presence of timing noise .............. 119
  6.3.2 Relation between timing residuals and X-ray flux ........ 121
  6.3.3 Pulse profiles ........................................... 124
  6.3.4 Short-term $\dot{\nu}$ measurements .......................... 128
6.4 Discussion .................................................... 129
6.5 Conclusions .................................................. 133

7 An alternative interpretation of the timing noise in accreting millisecond pulsars 135
7.1 Introduction .................................................... 136
8 1 Hz flaring in SAX J1808.4–3658: flow instabilities in the propeller stage  
8.1 Introduction  
8.2 X-ray observations and data reduction  
8.2.1 RXTE observations  
8.2.2 Swift-XRT observations  
8.3 Results  
8.3.1 The X-ray lightcurves and the re-flaring state  
8.3.2 The fast decays and the pulse phase drifts  
8.3.3 QPO parameters and flux  
8.3.4 The appearance of the 1 Hz QPO  
8.3.5 Energy dependence of the 1 Hz QPO  
8.3.6 Jumps in 401 Hz pulse phases related to strength of 1 Hz QPO  
8.3.7 Dependence of 401 Hz pulsation on the 1 Hz phase  
8.4 Discussion  
8.4.1 Accretion onto a magnetized neutron star  
8.4.2 Candidate mechanisms: surface instabilities  
8.4.3 Candidate Mechanisms: disk/magnetosphere instabilities  
8.4.4 The mechanism for the 1 Hz QPO  
8.4.5 J1808 and the other AMXPs  
8.5 Conclusions

9 Samenvatting in het Nederlands  
9.1 De accreterende milliseconde röntgenpulsars  
9.2 De inhoud van dit proefschrift

10 List of publications  
10.1 Refereed publications  
10.1.1 Papers submitted to refereed journals  
10.2 Non-refereed publications
This work has the general aim of studying accretion phenomena associated with neutron stars in X-ray binaries. The specific target of this work is a subclass of accreting binaries that show X-ray pulsations with periods in the millisecond range. These pulses are thought to be formed on the neutron star surface and to track its rotation. Several other phenomena are also connected with the accretion flow that produces the X-ray pulsations and are useful to study the disc-magnetosphere interactions, the accretion disc, the thermonuclear X-ray bursts and the accretion torques. In this chapter I provide an introduction to the physics of neutron stars and to the X-ray binaries which harbor these fascinating objects.

1.1 Theory of neutron stars

A neutron star is an extreme object formed in a core-collapse supernova explosion (or in accretion induced collapse of white dwarfs) with a radius of $\sim$ 10 km and a mass of $\sim$ 1–2 M$_\odot$. The matter that composes the innermost part of the neutron star core can reach values significantly exceeding the density of atomic nuclei. It is currently thought that the point of highest density can reach values of up to 5-10 $\rho_0$, where $\rho_0$ is the density of nuclear matter at saturation$^1$ $(2.8 \times 10^{14} \text{g cm}^{-3})$. At such enormous densities the properties of ultradense nuclear matter are very uncertain and the understanding of its equation of state remains a major challenge for both theorists and experimentalists. The equation of state (EoS) is formally the relation between density, pressure and temperature $P(\rho, T)$. However, the term is sometimes used also to indicate the chemical composition and the microphysical model that describes the neutron star cores. Relatively little is currently known about ultradense cold nuclear matter.

---

$^1$Nuclear matter is at saturation density when it is static and the pressure is zero. It is a configuration of minimum energy.
matter, the information we have comes from astrophysical observations more than direct lab experiments.

Neutron stars are also unique laboratories to test theories of gravity in the strong field regime. Their surface gravitational (potential) energy is the highest known among directly observable objects: only black holes have larger gravitational fields close to their event horizons. However, in contrast to black holes, neutron stars have a solid surface which is directly observable by the distant observer. The typical gravitational energy of a particle of mass \( m \) at the surface of a neutron star is \( \sim 0.2mc^2 \), where \( c \) is the speed of light. The gravitational binding energy of a neutron star is, similarly, a significant (\( \sim 20\% \)) fraction of the total neutron star mass, making a neutron star a General relativistic object.

Since their surface gravity is high, gas flowing close to the neutron star surface approaches the speed of light, giving rise to Special relativistic effects. Moreover, neutron stars can rotate at spin periods near a millisecond, with surface velocities as high as several percent of the speed of light.

Finally, neutron stars have magnetic fields which are among the strongest observed in the Universe: typical values range from \( B \sim 10^8 \) Gauss for very old systems, up to the impressive value of \( 10^{15} \) G for young systems, well above the quantum electrodynamic limit of \( B_{QED} = 4.4 \times 10^{13} \) G. If a charged particle flows close enough to a magnetic neutron star, its motion will be completely controlled by the magnetic stresses. Electrons spiraling around such extreme fields emit cyclotron radiation as a flow of X-rays and \( \gamma \)-rays, and when \( B > B_{QED} \) the electron cyclotron energy equals the electron rest mass energy and quantum mechanical effects are dominant, like photon splitting and pair production.

Neutron stars can be therefore considered an extraordinary lab for extreme fundamental physics experiments: ultradense matter, strong gravity and high magnetic fields coexist in a unique environment, not reproducible in Earth based experiments.

1.1.1 Neutron star structure

Our modern view of neutron stars predicts the existence of three main regions: an outer layer of plasma (atmosphere), a thick envelope of atomic nuclei, free electrons, protons and neutrons (crust), and an inner region of ultradense matter whose composition is currently partially unknown (core). Both the crust and the core are also divided in an inner and outer region (see Fig. 1.1).

The atmosphere has a thickness from a few millimeters in systems with a cold surface (\( T \sim 10^5 \) K) up to several ten centimeters for young hot systems (\( T \sim 10^6 \) K). The importance of this layer of plasma is crucial for the
1.2 Observation of neutron stars

The first neutron star observed was Sco X-1 which was also the first extrasolar X-ray source discovered by Riccardo Giacconi and collaborators in 1962 (Giacconi et al. 1962). Sco X-1 was not immediately recognized as a neutron star since its properties were not completely understood at that time and its presence in a binary further complicated the interpretation of experimental data. At that time, it was commonly believed that the supernova explosion generating a neutron star would have disrupted any binary system given the large amount of mass lost in the explosion. It was only in the 1967, with the discovery of the first radio pulsar (PSR B1919+21, Hewish et al. 1968), that the existence of neutron stars was finally proved. In the '70s, with the launch
1. Introduction

![Diagram of a neutron star](image)

**Figure 1.1:** Artist’s impression of a neutron star. (Image from: Dany Page)

of the first X-ray satellite UHURU, it became clear that Sco X-1 belonged to a more general and extended family of X-ray binary sources with a compact accretor stripping gas from a canonical star.

Since then, the number of known neutron stars has increased enormously and neutron stars have been observed in all the wavelength bands from radio up to gamma. We currently know $\sim 2 \times 10^3$ neutron stars, which show a wide variety of behavior, illustrated by the categories of radio pulsars, accreting X-ray pulsars, RRATs, magnetars, isolated neutron stars, compact central objects and so on. From theoretical stellar evolutionary models and population studies we expect the existence of $\sim 10^8 - 10^9$ neutron stars in our Galaxy.

I now briefly review the two categories of neutron stars which are useful to understand the motivation of the present work: radio pulsars and accreting X-ray pulsars.
1.2 Observation of neutron stars

1.2.1 Radio pulsars

Radio pulsars comprise the vast majority of known neutron stars: the present population exceeds \( \sim 1700 \) objects and realistic estimates predict a galactic population of \( \sim 10^5 \) active radio pulsars.

The basic observational property of radio pulsars is their emission of broadband radio noise in the form of a periodic sequence of pulses. The periodicity of the pulse arrival times is thought to track the neutron star rotation. The fastest known rotating pulsar spins at a frequency of 716 Hz (Hessels et al. 2006) while the slowest ones spin at frequencies between 1 and 0.1 Hz.

The radio pulses are produced by a rotating beamed cone that becomes observable when pointing toward the Earth. The mechanism generating radio pulses is still not fully understood although it involves the transformation of the enormous pulsar rotational energy into radiation. However, the idea is now accepted that radio pulses have a tight link with the strong dipolar magnetic field of pulsars.

When observed over long periods of time, all radio pulsars show an increase of their rotational period. When plotting the pulse period and the period derivative in a diagram (the so called P-\( \dot{P} \) diagram, see Fig. 1.2), two groups of pulsars are evident: a large population of slow pulsars that spin down on a fast timescale \( \left( \frac{P}{\dot{P}} \sim 10^{6-7} \text{ yr} \right) \) and a smaller group of fast pulsars (called “millisecond pulsars”) that spin down of very long timescales \( \left( \frac{P}{\dot{P}} \sim 10^{8-9} \text{ yr} \right) \).

If we assume magnetic dipole radiation as the origin of the spin down, it is possible to infer a lower limit to the pulsar magnetic field \( B \propto \left( \frac{P}{\dot{P}} \right)^{1/2} \). Slow radio pulsars have \( B \) fields of \( \sim 10^{12} \) G, while millisecond pulsars have fields of \( \sim 10^8 \) G.

Interestingly, the majority of millisecond pulsars (\( \sim 80\% \)) is found in binaries, in sharp contrast with the \( \sim 1\% \) of slow pulsars in binaries. Finally, in the P-\( \dot{P} \) diagram, there is a large area of long periods and small period derivatives that is not populated. It is thought that when radio pulsars cross the so called “death line” on the P-\( \dot{P} \) diagram, the radio emission mechanism switches off and the pulsar enters the “pulsar graveyard”. The neutron star keeps evolving as an isolated neutron star by emitting thermal radiation from its cooling surface.

All new-born radio pulsars found in supernova remnants have short periods and fast spin down timescales. No millisecond pulsar has been found until now in a young supernova remnant. This and the other sharp differences between millisecond and slow pulsar populations, led to the idea that young pulsars are born with periods between 0.01-10 s, while the fast millisecond pulsars reach
1. Introduction

Figure 1.2: P-\(P\) diagram of isolated (black circles) and binary (open circles) radio pulsars.

Many of these differences can be easily explained in a binary evolutionary scenario that starts with two main sequence stars. The most massive star evolves faster than the companion, and explodes as a type II Supernova, leaving a neutron star remnant. If the binary is not disrupted, the neutron star remains bound to the companion. At some point a phase of accretion can start and the outer layers of the companion are stripped by the gravitational force exerted by the neutron star. The gas and its angular momentum accrete on the neutron star. If the angular momentum brought by the accreted gas is sufficiently high, the neutron star can easily be spun up from \(\sim 10\) s down to millisecond periods during this phase. The accretion also has the effect of moving the neutron star from the graveyard into the bottom-left side of the P-\(P\) diagram: if the companion star is disrupted or the accretion phase terminates, the neutron star is expected to switch on as a millisecond radio
1.2 Observation of neutron stars

This is the so called Recycling Scenario, in which a dead radio pulsar is recycled into a millisecond radio pulsar through accretion in a binary system. However, it is not yet clear why the magnetic field decays by $\sim 4 - 5$ orders of magnitude during the spin up phase. Whether the accretion is responsible for the magnetic field decay is still an open issue. A spontaneous Ohmic decay with time in an isolated neutron star is unfeasible as it requires a timescale longer than the age of the Universe. A significant difference is observed between the $B$ field distribution of isolated radio pulsars and X-ray pulsars in binaries. The idea is now widely accepted that the magnetic field decays more rapidly in binaries, although the exact way in which the field is reduced still has to be identified.

1.2.2 Accreting neutron stars

At some point of the evolution, a star in a binary can transfer part of its matter onto the companion. If one of the stars in the binary increases its radius or the orbital separation decreases, the gravitational force exerted by the other star can pull part of the gas into its Roche lobe. The gas then accretes onto the other star releasing its gravitational potential energy as radiation (see Verbunt 1993 and Tauris & van den Heuvel 2006 for reviews). The release of potential energy is very large if the accreting star is a neutron star or a black hole: if all the energy is converted into radiation, then the expected accretion luminosity is:

$$L_{\text{acc}} = \frac{GM\dot{M}}{R},$$

where $G$ is the universal gravitational constant and $M$, $R$, and $\dot{M}$ are the accretor mass, its radius and the rate of mass accretion. For a neutron star with $M = 1.4 M_\odot$, $R = 10$ km the efficiency of the process is close to $\sim 10\%$ of $mc^2$, where $m$ is the amount of matter accreted.

If the neutron star magnetic field is sufficiently strong to channel a substantial fraction of gas onto the magnetic polar caps, and the rotation and magnetic axes are misaligned, the X-ray radiation produced by the matter infall is modulated at the neutron star spin frequency. The radiation is detected by the distant observer as a sequence of X-ray pulses, whose frequency may be modulated at the binary orbital period due to the Doppler effect.

Depending on how massive the companion star is, accreting neutron stars are found in three different types of X-ray binaries: high, intermediate and low mass X-ray binaries. The classification does not distinguish between X-ray binaries with a neutron star and those with a black hole. Here we will refer only to the properties of the binaries which are known to harbor a neutron star.
High mass X-ray binaries (HMXBs)

The binary is young and the companion star is a massive star (O-B spectral type) with $M > 10 M_{\odot}$. They are found concentrated in the galactic plane and show regular X-ray pulsations, but no thermonuclear bursts. In the majority of these systems, accretion takes places through capture of part of the intense stellar wind emitted by the O-B main sequence or blue supergiant companion. In some rare cases accretion can take place also via Roche lobe overflow (RLOF) and with formation of a small accretion disc (for example Cen X-3, see Bildsten et al. 1997). The neutron star is formed after a supernova explosion of a massive star that does not disrupt the system. Its magnetic field is moderately high ($B \sim 10^{12} \text{G}$) which by interaction with the accreting material leads to spins at frequencies below $\sim 15 \text{ Hz}$. Most HMXBs that contain a neutron star show regular X-ray pulsations, as a result of the channeling of the accreted matter towards their magnetic poles. HMXBs have rather quick evolutionary timescales, since the donor is massive and completes its main sequence phase in less than $10 \text{ Myr}$.

Low mass X-ray binaries (LMXBs)

Low mass X-ray binaries (LMXBs) typically are old systems ($\sim 1 - 10 \text{ Gyr}$) with a companion star whose mass is $\lesssim 1 M_{\odot}$. They are mainly found toward the galactic bulge and in globular clusters, which are mainly composed of old Population II stars. Given the relatively weak stellar wind, LMXBs exhibit detectable X-rays because of RLOF. The gas is transferred through the inner Lagrangian point and carries a relatively high angular momentum. An accretion disc forms, which is thought to be responsible for the quasi-periodic oscillations (QPOs) observed in the X-ray flux (van der Klis 2000).

LMXB orbital periods can go from several days (wide binaries) down to 11 minutes (ultra-compact binaries, Nelson et al. 1986). One of the unsolved issues about binary evolution is how to produce such short orbital periods.

LMXBs show X-ray pulsations only on rare occasions, and the vast majority having never shown pulsations despite large observational surveys. The accreting neutron star is old and has relatively weak magnetic fields of $\sim 10^8 - 10^{11} \text{ G}$. The weak magnetic field allows also the production of Type I X-ray bursts, sudden releases of energy produced by unstable thermonuclear fusion of H and He. Two LMXBs also show the puzzling Type II X-ray bursts (Hoffman et al. 1978), which are X-ray flashes similar to those of Type I, which, however, are produced by accretion instabilities instead of thermonuclear runaways.
Intermediate mass X-ray binaries (IMXBs)

The class of IMXBs comprises all the X-ray binaries whose donor mass lies in the $\sim 1-10 \, M_\odot$ range. From a binary evolutionary point of view, IMXBs are difficult to observe since they are expected to evolve on very fast timescales because of unstable RLOF (Tauris et al. 2000) or because of common envelope evolution (Iben & Livio 1993). This instability emerges when the mass ratio between the neutron star and the donor is close to one, and gives an evolutionary timescale of $\sim 10^3$ yr. This means that the chances to observe such a system during this evolutionary phase are very low, while the detection of wind fed IMXBs is also biased against by the very low luminosities expected by their weak winds.

1.3 Accretion disc

The accretion discs formed in LMXBs are usually studied with the geometrically thin and optically thick disc approximation of Shakura & Syunyaev (1973). In this approximation the scale height, $H$, is much smaller than the radial extension of the disc: $H/R \ll 1$. The gas in the disc rotates at a Keplerian velocity $\Omega_K = \sqrt{GM/R^3}$ and the orbital kinetic energy is transformed into radiation by viscosity, while the angular momentum is transported outward. The viscosity $\nu$ is very uncertain and is parametrized by the so-called “$\alpha$-prescription”:

$$\nu = \alpha c_s H$$  \hspace{1cm} (1.1)

where $c_s$ is the speed of sound and $\alpha$ is a free parameter proportional to the viscosity.

Many LMXBs have unstable accretion discs performing a “limit cycle” oscillation between stable thermal equilibria states: the outburst, with a high viscosity and hot temperatures, and the quiescence with low viscosity and low temperatures. The disc consists of ionized gas in outburst and neutral gas in quiescence. The two states are activated by a thermal-viscous instability that emerges when the gas temperature is larger than the ionization temperature. For a hydrogen disc, the critical temperature is $\sim 6500$K.

For a given radial distance $R$, the disc tracks an $S$-curve in the $\Sigma - T_{\text{eff}}$ plane, where $\Sigma$ is the disc surface density and $T_{\text{eff}}$ is the disc effective temperature (Fig. 1.3). The lower cold branch corresponds to the quiescence, with neutral hydrogen, low viscosities, low temperatures (and thus low disc luminosities) and low surface densities. The upper hot branch is the outburst with high luminosities and an high viscosity ($\alpha \sim 0.1$). In either case the disc is in thermal equilibrium. The middle branch is an unstable thermal equilib-
Fig. 1.3: S-curve of an accretion disc (Image from: Lasota 2001)

The disc moves from outburst to quiescence on a thermal timescale:

\[ t_{th} \sim \frac{R^3 c_s^2}{GM\nu} \]  

(1.2)

While in outburst or quiescence, the disc is in thermal but not in viscous equilibrium and it evolves on a viscous timescale:

\[ t_{visc} \sim \alpha^{-1} \frac{R^2}{Hc_s} \]  

(1.3)

It is clear then why the outburst duration is much shorter than the quiescence: the viscous parameter \( \alpha \) is a factor \( \sim 10 \) smaller in quiescence than in outburst, thus giving a correspondingly longer viscous timescale.

When the disc moves from quiescence to outburst, a heating front propagates through the disc from the inner regions outward or from the outer region inward, depending where the temperature first reaches the hydrogen ionization temperature. As the heating front moves in the disc, an increasingly larger portion of the accretion disc contributes to the total X-ray luminosity. The outburst peak is reached when the whole disc is ionized. A cooling front then propagates from the outer colder regions inward, producing the typical luminosity decay observed in LMXB outbursts. When the whole gas in the disc is recombined, the quiescence starts, and the cycle repeats again.
Accreting millisecond X-ray pulsars

According to the recycling scenario, millisecond radio pulsars are produced by spin up via the transfer of angular momentum through accretion. During this phase (∼ 10 Gyrs), they would be expected to emit X-ray pulsations. Therefore one would expect some accreting neutron stars to be observed as accretion powered X-ray pulsars in the process of spinning up in the millisecond range. After this accreting phase, the companion leaves a remnant (white or brown dwarf) or is destroyed by the X-ray radiation or by the powerful pulsar wind after it turns on as a radio pulsar. A binary or an isolated “rotation powered” millisecond radio pulsar is left at this point of the evolution. Accreting pulsars are therefore expected to be the evolutionary link between the young radio pulsars and the old fast millisecond radio pulsars. This “recycling” model for the origin of millisecond radio pulsars was proposed in 1982 (Alpar et al. 1982, Radhakrishnan & Srinivasan 1982), but it lasted until 1998 before the first accreting millisecond X-ray pulsar was finally discovered.

In 1998, millisecond pulsations from the low mass X-ray binary transient SAX J1808.4-3658 were discovered (Wijnands & van der Klis 1998, see Fig. 1.4). This was the proof of the existence of accreting millisecond X-ray pulsars (AMXPs) and confirmed the recycling scenario. However, the number of known AMXPs is only 10 out of ∼150 non-pulsating LMXBs systems. The reason for this is still open: under certain simplifying assumptions standard accretion theory predicts that all the accreting NSs should pulsate. Moreover, the theory would predict also sub-millisecond periods. However, the fastest observed accreting and radio pulsar spin at 1.7 and 1.4 ms, respectively. There is therefore a large gap of unpopulated periods below 1.4 ms that remains unexplained (Chakrabarty et al. 2003a).

### 1.4.1 Standard accretion theory

Accreting X-ray pulsars are the only objects through which we can directly measure the transfer of angular momentum from the accretion disc onto the neutron star surface in neutron star X-ray binaries.

The inner accretion disc is truncated by the neutron star magnetic field lines at the magnetospheric radius \( r_m = \xi r_A \), where \( r_A \) is the Alfven radius defined as:

\[
    r_A = \left( \frac{\mu^4}{2GM^2} \right)^{1/7}
\]

and \( \xi \sim 1 \) is a dimensionless parameter (Ghosh & Lamb 1979). Here \( \mu \) is the
magnetic moment of the neutron star $B$ field, and for a magnetic dipole it is:

$$\mu = \frac{B_0 R^3}{2}$$  \hspace{1cm} (1.5)

where $B_0$ is the magnetic field at the poles.

The magnetic stress balances the fluid stress when

$$B_p B_\phi r_m^2 \Delta r = \dot{M} \Omega r_m^2$$  \hspace{1cm} (1.6)

with $B_p$ and $B_\phi$ the poloidal and toroidal field component, $\Delta r$ is the magnetospheric-disc boundary layer where the interaction takes place and $\Omega$ is the neutron star angular velocity.

Accretion is inhibited by centrifugal forces if the Keplerian gas velocity at the magnetospheric radius is smaller than the neutron star rotational velocity. This concept is usually expressed by defining the co-rotation radius:

$$r_{co} = \left(\frac{GM}{\Omega^2}\right)^{1/3}$$  \hspace{1cm} (1.7)

which is the radius at which the Keplerian gas velocity equals the neutron star’s rotation rate. If the magnetospheric radius lies inside the co-rotation radius, then the accreting gas applies a positive torque on the neutron star (spin up), while in the opposite case it removes angular momentum slowing down the neutron star rotation (spin down).

### 1.4.2 Periodic variability

Pulsations are observed in AMXPs as modulations of the X-ray flux. Differently from radio pulsars, AMXPs show a broad pulse profile with a large duty cycle and low harmonic content. This is mainly due to the different emission mechanism in X-ray pulsars: a blackbody possibly comptonized by a shock of accreting material, with strong distortions due to gravitational lensing, Doppler shift and aberration (Braje et al. 2000). The source of blackbody emission is the hot spot, which is the region where the magnetic funnel flow impacts the neutron star surface, transforming the kinetic energy into radiation. Since the funnel flow is relativistic, a shock is expected to appear. The thermal radiation emitted by the hot spot can then be Comptonized by the shock and upscattered to higher energies (Poutanen & Gierliński 2003).

Though this picture needs confirmation, it is clear that pulsations are a surface phenomenon and can be used to track the accretion flow directly onto the neutron star surface.
1.4.3 Quasi-periodic variability

Beside the coherent millisecond variability, AMXPs show quasi-periodic variability on timescales of milliseconds up to hundred seconds (Linares et al. 2005, 2007, 2008). Despite considerable efforts in the field, the origin of this fascinating phenomenon is not yet clear. Since accretion occurs through gas revolving in Keplerian orbits in the accretion disc, the fastest quasi-variability observed (∼ 0.1 – 1 ms) is usually associated with the inner regions of the accretion disc (van der Klis 2000). In these regions, the accretion flow has a characteristic timescale close to the dynamical timescale of the system:

\[ \tau_{\text{dyn}} = \sqrt{\frac{R^3}{GM}} \]  

Therefore the fastest quasi-periodic variability is thought to form in regions a few km from the neutron star, and can be useful to probe the motion of matter in the strong gravity field regime.

1.4.4 Thermonuclear X-ray bursts

Accretion is one of the most efficient processes to extract energy from a source. The energy released per baryon accreted is ∼ 200 MeV, while the nuclear energy liberated by hydrogen fusion is ∼ 5 MeV. Moreover both reactions produce blackbody radiation with peak emission in the soft X-rays. Therefore any steady nuclear burning reaction taking place on the neutron star surface is completely swamped by the accretion powered radiation. However, if the nuclear energy is released on a much shorter timescale than the typical accretion timescale, the nuclear powered radiation becomes dominant (see Strohmayer & Bildsten 2006 for a review).

This radiation is indeed observed in many LMXBs and in five AMXPs as thermonuclear runaways, called Type I X-ray bursts: sudden release of nuclear energy observed as enhanced X-ray flux with a rise timescale of ∼ 1 s and a flux decay of ∼ 10 – 1000 s that corresponds to the cooling time of the burnt layer. These timescales are much shorter than any typical outburst length (i.e., viscous/thermal timescale of the accretion disk, lasting weeks to years). The material burned in the Type I bursts can differ depending on the composition of the material accumulated in the neutron star atmosphere from accretion. We currently know three types of bursts: mixed hydrogen/helium burning, pure helium, and carbon bursts. The latter are also known as Superbursts, since the energy released and the length of the burst can be up to 2-3 orders of magnitude larger than in the other cases. An interesting property of Type I X-ray bursts is the possible presence of “burst oscillations”, which are observed
1. Introduction

Table 1.1: Accreting Millisecond X-ray Pulsars

<table>
<thead>
<tr>
<th>Source</th>
<th>$\nu_s$</th>
<th>$P_{orb}$</th>
<th>$f_x$</th>
<th>$M_{c,min}$</th>
<th>Type I Bursts</th>
<th>B-O</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAX J1808.4–3658</td>
<td>401</td>
<td>121</td>
<td>3.8 x 10^{-8}</td>
<td>0.043</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>XTE J1751–305</td>
<td>435</td>
<td>42.4</td>
<td>1.3 x 10^{-6}</td>
<td>0.014</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>XTE J0929–314</td>
<td>185</td>
<td>43.6</td>
<td>2.9 x 10^{-7}</td>
<td>0.0083</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>XTE J1807–294</td>
<td>190</td>
<td>40.1</td>
<td>1.5 x 10^{-7}</td>
<td>0.0066</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>XTE J1814–338</td>
<td>314</td>
<td>257</td>
<td>2.0 x 10^{-3}</td>
<td>0.17</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>IGR J00291+5934</td>
<td>599</td>
<td>147</td>
<td>2.8 x 10^{-5}</td>
<td>0.039</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>HETE J1900.1–2455</td>
<td>377</td>
<td>83.3</td>
<td>2.0 x 10^{-6}</td>
<td>0.016</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Swift J1756.9–2508</td>
<td>182</td>
<td>54.7</td>
<td>1.6 x 10^{-7}</td>
<td>0.007</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Aql X–1…………………</td>
<td>550</td>
<td>1194</td>
<td></td>
<td></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>SAX J1748.9–2021</td>
<td>442</td>
<td>522</td>
<td>4.8 x 10^{-4}</td>
<td>0.1</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

$\nu_s$ is the spin frequency, $P_{orb}$ the orbital period, $f_x$ is the X-ray mass function, $M_{c,min}$ is the minimum companion mass assuming a neutron star mass of 1.4 $M_\odot$. The last two columns identify bursting sources and those that show Burst-Oscillations (B-O).

with a frequency close (within ~ 1 Hz) to the neutron star spin. Only 16 among the 90 bursters show nuclear-powered oscillations and only three of these show also accretion powered pulsations. The reason for this is currently unknown.

The first AMXP to show Type I bursts was SAX J1808.4-3658. The source exhibited burst oscillations, whose frequency drifted above the spin frequency by a few Hz during the burst rise, settling down to an oscillation frequency within 0.1 Hz of the measured accretion-powered pulsations (Chakrabarty et al. 2003a). This behavior is quite anomalous among bursting LMXBs. The typical behavior is a slow increase of the burst oscillation frequency that asymptotically reaches a frequency inferred to be close or equal to the neutron star spin frequency, during the burst decay. The second AMXP to show burst oscillations also has an anomalous behavior, with the oscillation frequency extremely stable and fixed at the neutron star spin frequency (Fig. 1.6).

1.5 Observations of accreting millisecond X-ray pulsars

Ten AMXPs have been observed until now. Eight of them have orbital periods below 4.3 hrs and three are ultra-compact binaries with orbital period $P_{orb}$ ~ 40 min. They all have a low mass companion that in the most extreme case can be as small as 7 Jupiter masses (XTE J1807-294, Kirsch et al. 2004) and they all show a very low long term average mass transfer rate ($\dot{M} \sim 10^{-11} M_\odot$ yr$^{-1}$). Their spin frequencies range from 182 to 599 Hz and three of them show burst oscillations during thermonuclear bursts. In Table 1.1 I summarize their main properties.
Pulsations in SAX~J1808.4-3658 were discovered with Fourier analysis (see § 1.6.2) at a frequency of 401 Hz. From simple considerations on the pulse formation mechanism, Wijnands & van der Klis (1998) estimated a surface $B$ field of $2 - 6 \times 10^{8} \text{ G}$, confirming the expectations of a low magnetic fields for old neutron stars in LMXBs. The pulsation showed a fractional amplitude between 4% and 7% (rms) with little dependence to the photon energies but strong phase lags (Cui et al. 1998).

The pulse frequency had a 2.01 hr modulation at the orbital period of the binary (Fig. 1.5) and a very small mass function (Chakrabarty & Morgan 1998). Bildsten & Chakrabarty (2001) studied the mass function and the long term average mass transfer rate of SAX~J1808.4-3658, and concluded that the companion star was a brown dwarf heated by the neutron star radiation. In more than 10 years of observations, SAX J1808.4–3658 went in outburst six times, and on five occasions high quality data were obtained and the pulsations were studied. I discuss this long baseline timing study of SAX J1808.4-3658 in § 4 and § 5.

Type I X-ray bursts were also observed in this source, and the detection of burst oscillations at $\sim 401 \text{ Hz}$ led to the conclusion that burst oscillations do indeed track the neutron star spin frequency. In June 5, 2003, another AMXP with both Type I bursts and burst oscillations was discovered (XTE J1814-338). The 314 Hz burst oscillation frequency was consistent with the spin frequency measured with accretion powered pulsations and further confirmed the idea that burst oscillations are a good tracer of the spin frequency. I discuss the properties of these oscillations and their relation with the accretion-
1. Introduction

Figure 1.5: Radial velocity curve of SAX J1808.4-3658. The sinusoidal variations are caused by the 2.01 hr orbital period modulation of the pulse frequency. (From Chakrabarty & Morgan 1998).

powered pulsations in § 2.

SAX J1808.4–3658 also showed unexpected variations in its X-ray flux: after the main outburst, the source showed the so-called re-flares, a puzzling phenomenon probably related with instabilities in the accretion disc before the return into quiescence. The most surprising discovery was the observation of a violent flaring superimposed on the re-flares, at a repetition frequency of 1 Hz (Wijnands 2004). This phenomenon is currently poorly understood, and probably related with hydrodynamic instabilities at the disc-magnetospheric interface that develop close to the onset of the propeller stage (Illarionov & Sunyaev 1975). I discuss this phenomenon in detail in § 8.

The study of AMXPs was considered very promising also because of the possibility of measuring spin frequency variations caused by external torques as predicted by standard accretion theory (§ 1.4.1). However, this is one of the most controversial aspects of AMXP research, and there is no general consensus yet on how to measure these torques. The main problem when measuring accretion torques is the presence of unexpected fluctuations of the pulse phase which are not predicted by standard accretion theory. These phase fluctuations are called “timing noise”, and their origin is still unknown. An
1.5 Observations of accreting millisecond X-ray pulsars

Figure 1.6: Type I X-ray bursts and burst oscillations as observed in XTE J1814-338 (From Strohmayer et al. 2003)

initial discussion of the problem is found in § 4 and a detailed test of accretion theory is discussed in § 6 for XTE J1807–294. Finally, a new interpretation of the timing noise is given in § 7.

An interesting recent development has been the discovery of intermittent pulsations in three AMXPs (HETE J1900.1-2455, Aql X-1 and SAX J1748.9-2021). The pulsation appears and disappears sporadically during an outburst (see for example Fig. 1.7 for SAX J1748.9-2021).

All three intermittent sources show also Type I bursts. In one case (Aql X-1) the source showed pulsations for \( \sim 120 \) s out of a total observing time of 1.4 Ms, with a duty cycle of less than 0.01% (Casella et al. 2008). The relation between pulse formation and the presence of Type I X-ray bursts in SAX J1748.9-2021, along with a discussion of the pulse formation timescale and the pulse profile properties is presented in § 3.

1.5.1 Rossi X-ray Timing Explorer

The Rossi X-ray Timing Explorer (RXTE) was launched by a Delta II rocket on December 30, 1995, on a circular orbit with altitude of 580 km, and an orbital period of 96 min. RXTE has remained operational since then, spanning almost 15 years of observations. Three instruments are aboard RXTE: the All Sky-Monitor (ASM), the Proportional Counter Array (PCA) and the High Energy X-ray Timing Experiment (HEXTE). An illustration of the RXTE satellite is shown in Fig. 1.8.

The ASM is composed by three wide-angle proportional counters with a
1. Introduction

Figure 1.7: X-ray lightcurves of three outbursts of the intermittent X-ray pulsar SAX J1748.9–2021 (Altamirano et al. 2008). During the 1998 outburst no pulsations were observed. In 2001 and 2005 the high time resolution observations are marked with grey circles. The pulsations are marked with black squares at the bottom and the occurrence of Type I bursts is marked on top with black squares at the top.

Figure 1.8: The RXTE satellite and its three main instruments: ASM, PCA and HEXTE.

The total collecting area of 90 cm$^2$ and a total energy band of 1.3-12.1 keV. The main property of the ASM is its ability of scanning $\sim 80\%$ of the sky in less than one RXTE orbit. In this way long term monitoring of bright sources is possible. Source state transitions from quiescence to outburst can be readily detected with this instrument.
The PCA is an array of five Proportional Counter Units (PCUs) with a total collecting area of $\sim 6500 \text{cm}^2$ and 256 energy channels sensitive to the 2-60 keV energy band. A collimator provides a field of view with a $\sim 1^\circ$ full width at half maximum (FWHM) that helps in reducing the source confusion in crowded fields. The time of arrival of each photon is registered with a time resolution of up to $2^{-20}$ s. This high temporal resolution, the large collecting area and the broad energy range make the PCA the best available instrument to study millisecond variability in X-ray binaries.

Several changes in the gain of PCUs and the long term use have substantially reduced the capabilities of the instrument. Actually five so-called gain epochs are defined, each of them corresponding to some loss in capabilities. The gain is a measure of the ability of a circuit to increase the output power of the input signal. It is defined as the number of electrons released in the gas of a detector after an X-ray photon (or a high energy particle) is detected. The PCA is the main instrument used in this work, and all these effects have been carefully taken into account in the data reduction and analysis.

The HEXTE instrument consists of two clusters of scintillation detectors with total collecting area of 1600 cm$^2$. Each cluster can “rock” (beamswitch) every 16 s or 128 s along orthogonal directions, to provide background measurements 1.5 or 3.0 degrees away from the source. The HEXTE has a wide energy range of 15-250 keV and a field of view of $1^\circ$ FWHM.

The data collected are pre-processed by on-board system analyzers that prepare the data in different data-modes. The PCA has six different data modes, two Standard modes, and four guest-observer data modes that can be changed and chosen by the observer. In this thesis I use two specific data modes: the GoodXenon mode, with a time resolution of $2^{-20}$ s and 256 energy channels, and the Event mode with a time resolution of $2^{-13}$ s and 64 energy channels.

1.5.2 European X-ray Multi-Mirror telescope Newton

The European X-ray Multi-Mirror (XMM) telescope, named in memory of Isaac Newton, has been launched with an Ariane 504 rocket on December 10, 1999, in a very eccentric orbit that brings the telescope from a distance of 7000 km at the perigee up to a distance of $\sim 100,000$ km in a 42 hr orbit. It consists of 58 mirrors and three telescopes that focus X-ray photons with energies of 0.1-12 keV. The detectors on-board XMM-Newton consist of two X-ray instruments and an optical monitor (OM) used to follow the optical counterparts (down to magnitude 24th) of X-ray sources observed by the other
X-ray instruments. XMM mirrors are most efficient in the energy range from 0.1 to 10 keV, with a maximum around 1.5 keV and a pronounced edge near 2 keV. The design goal was to achieve a collecting area of 1900 cm$^2$ for energies up to 150 eV, 1500 cm$^2$ at 2 keV, 900 cm$^2$ at 7 keV, and 350 cm$^2$ at 10 keV, for each of the three telescopes.

The Reflection Grating Spectrometer (RGS) is mounted behind two of the XMM telescopes and produces high resolution spectra with high resolving power (150 to 800) over a range from 5 to 35 Å [0.33 to 2.5 keV] in the first spectral order. The instrument reaches the highest sensitivity at 15 Å [0.83 keV] (first order) where the effective area is largest. The instrument consists of two Reflection Grating Arrays units (RGA), each mounted on one mirror, two Focal Plane Camera units (RFC), each including a stand-off structure, a radiator and the detector itself, and the electronics.

The European Photon Imaging Camera (EPIC) is the main instrument aboard XMM-Newton and is placed in the main focal plane providing CCD imaging, spectroscopy and timing. The three cameras have independent configurations that can be chosen by the guest-observer for each observation, and comprises an Imaging/Spectral mode and a Timing mode. For spectral studies, the EPIC-pn and the MOS provide broadband spectra with a pn energy resolution at 6.5 keV of $E/dE \sim 50$.

In timing mode, imaging is made only in one dimension, with the other dimension collapsed along the column axis. The read out time is very high, $\sim 2.5$ ms for the MOS 1 & 2 and down to $\sim 30 \mu s$ in Timing mode for the pn. A special mode of $7 \mu s$ (Burst mode) can also be chosen for the pn when observing very bright sources to avoid pile-up. The duty cycle of this mode is low, however, around 3%. The nominal energy band of the EPIC-pn ranges from 0.1 to 15 keV; EPIC-pn provides imaging in a 30 arcmin field of view with a 6 arcsec FWHM.

### 1.5.3 Swift

The Swift Gamma Ray Burst Explorer was launched with a Delta 7320 rocket on November 20, 2004. It is a multi-instrument mission whose main target is the study of Gamma Ray Bursts. The main characteristic of Swift, as its name suggests, is the execution of “swift” pointings of the focusing telescopes in typically $\sim 50$ s. The telescope consists of three instruments: the Burst Alert Telescope (BAT), the X-Ray Telescope (XRT) and the UltraViolet-Optical Telescope (UVOT).

BAT has a sensitivity in the 15-150 keV range and a collecting area of 5200 cm$^2$, and is mainly used in gamma ray burst studies.

The XRT is an X-ray mirror telescope with a CCD Focal Plane Camera As-
assembly (FPCA) used to provide X-ray spectra and refined positions of gamma ray bursts, but is also widely used by the X-ray binary community for pointed short-exposure observations of X-ray sources. Since the primary target of Swift is the study of gamma ray bursts, the pointings may be interrupted, or the observations rescheduled, if a gamma ray burst occurs during the X-ray observations. Swift is therefore mainly used for monitoring or to refine the position of newly discovered X-ray sources.

The FPCA has a useful bandpass of 0.2 to 10 keV and can collect the data in three modes: a PhotoDiode (PD) mode with 0.14 ms time resolution and no spatial information, a Windowed Timing (WT) mode, with one spatial dimension and a full time resolution of 1.8 ms and a Photon Counting (PC) mode with a full spatial resolution and a 2.5 s timing resolution.

The UVOT telescope is a 30 cm modified Ritchey-Chretien UV/optical telescope co-aligned with the X-ray Telescope and provides ultraviolet and optical coverage (170-650 nm) in a $17' \times 17'$ field.

1.6 Observational techniques

As described in § 1.4, the accretion processes taking place in AMXPs are extremely fast and energetic. The dynamical timescale in a region close to the neutron star surface is $\tau_{\text{dyn}} = \sqrt{R^3/GM} \sim 100 \mu s$, where $R$ is a typical neutron star radius of 10 km. The X-ray flux emitted in these regions exhibits a rapid variability that can be studied by collecting data sequentially over time. In this thesis we study time series in both time and frequency domains. The time domain is used to analyze the X-ray pulsations measured in an inertial reference frame. The frequency domain is used to study the aperiodic variability coming from the accretion disc and from the disc-magnetospheric boundary layer.

The power emitted by the inner region of the accretion disc and the neutron star surface is as high as $\sim 10^{38}$ erg s$^{-1}$, partially emitted as thermal radiation. Since the volumes are rather small ($\propto R^3$) the blackbody temperature is as high as $\sim 10^7$ K, which corresponds to peak energies of $\sim 1$ keV. This is why the X-ray band is the most suitable for the study of accretion phenomena in LMXBs.

1.6.1 Periodic timing analysis

Periodic timing analysis has the main target of studying with the highest precision the X-ray pulsar rotation and the orbital evolution of the X-ray binary. The neutron star rotation is known to be extremely stable, due to the
large moment of inertia of the body. Therefore any spin variation has to be studied with extreme care to reveal the small changes of the spin, expected with the transfer of angular momentum during the accretion process. The main requirement of periodic timing analysis is the accounting for variable propagation delays affecting the photon time of arrivals in order to measure the pulsations of the pulsar, and hopefully therefore, its spin.

Pulse arrival times need to be measured in an inertial reference frame to remove the effect of the Earth’s motion around the Solar system barycenter. The most convenient approximation for an inertial reference frame is the Solar system barycenter, where all the times of arrival are referred to. Several corrections need to be applied to each photon arrival time detected and an ephemeris of the Earth’s motion is used, as provided by the Jet Propulsion Laboratory, that specifies the past and future positions of the Sun, Moon, and nine planets in three-dimensional space. The corrections applied take into account the light-travel time of a photon from the X-ray satellite to the Solar system barycenter and all the special and general relativistic corrections due to the changing potential well in which the Earth and the photon move.

Once all the photons are in the correct barycentric time format, the X-ray lightcurve is folded to overlap and sum several hundred thousand pulsations. This step is necessary to increase the signal to noise of a single pulsation. These accreting pulsars rotate at millisecond periods, and the count rate even for the brightest of them never exceeds $\sim 10^3$ ct/s. This means that in one millisecond one expects approximately one photon, which is clearly insufficient to detect the pulse modulation even for the brightest source. Moreover the amplitude of the signal is comparable with the amplitude of the Poissonian noise expected from counting statistics. The main problem therefore is the photon-counting statistics. This problem is usually overcome by averaging stretches of lightcurve much longer than the neutron star spin period, and by rebinning photons into a relatively small number of time bins. This of course reduces the length of the shortest timescales that can be measured, but increases the amplitude of the average pulsation well above the Poissonian noise amplitude.

Once a series of pulse profiles is built, there are two types of X-ray pulse timing studies. In the first type the pulse phase variations due to Doppler shifts are measured to reconstruct the binary orbit. In the second type, the Doppler variations are removed from the pulsations to follow any intrinsic variations in pulse frequency. The pulse phase at time $t$ is described by the Taylor expansion:

$$
\phi(t) = \phi_0 + \nu_0(t - t_0) + \frac{1}{2}\ddot{\nu}_0(t - t_0)^2 + \frac{1}{6}\dddot{\nu}_0(t - t_0)^3 + ... \quad (1.9)
$$
where \( \phi_0 \) is the pulse phase at the reference time \( t_0 \). The difference \( \phi - n \) between the pulse phase \( \phi \) and the closest integer to \( \phi \) is known as residual, in units of cycles. These studies give information about the X-ray binary evolution and the external torques (due for example to the accretion) applied to the neutron star.

Pulse profiles in AMXPs have a sinusoidal shape, sometimes with a strong first overtone detected. When the overtone is detected, the average pulse profile is not stable and its shape changes with time. This is very different from radio pulsars, where the pulse duty cycle is small and the average pulse profile is very stable. Therefore in AMXPs special techniques are used to reduce the effect of pulse shape variability (see for example § 4). Sometimes the pulse profile is split into the fundamental and overtone, and the two phases are measured separately by using standard \( \chi^2 \) fits (see for example § 6).

### 1.6.2 Aperiodic timing analysis

The study of aperiodic variability usually is made in the frequency domain because of the photon-counting statistics problem already discussed in the previous section. Differently from the periodic analysis, where the signal is highly coherent and stable, and can be studied in the time domain by folding the lightcurve, in aperiodic analysis the variability is mainly incoherent and is usually dominated by Poissonian noise. The study of aperiodic variability requires Fourier techniques to average long stretches of data to overcome the counting statistics and measure the amplitude of the variability as a function of frequency.

These techniques (which are also useful to discover periodicities somehow hidden in the data) are used to study modes of variability covering several, usually many, frequency resolution elements. The absolute values of the Fourier amplitudes squared, the so-called power spectrum, diagnose the amplitude of the aperiodic variability.

The maximum Fourier frequency measurable in a power spectrum is set by the sampling time resolution \( t_{\text{samp}} \), and is called Nyquist frequency \( \nu_N = 1/(2t_{\text{samp}}) \). The minimum frequency is given by the length \( T \) of the time series in consideration and is \( \nu_{\text{min}} = 1/T \). For long discrete time series, the computation of a Fourier transform of \( N \) points might be intensive, and requires \( O(N^2) \) operations. Quick efficient numerical methods to make the computations feasible have been developed. One of the most popular ones is the Fast Fourier Transform (FFT) algorithm which computes Fourier transforms in only \( O(N \log N) \) operations. We refer to van der Klis (1989) for a detailed review of discrete Fourier analysis and FFT techniques.
1.6.3 Spectral Analysis

The study of X-ray spectra in this work is based on multi-band photometry and spectral fitting. The photometric approach is useful to determine the approximate broad band spectral shape and uses “hardness ratios” between the photon count rates in two different energy bands. One then plots two colors in a diagram (color-color diagram) or one color and the count rate in one broad energy band, usually called X-ray intensity (hardness-intensity diagram). A specific source moves in these diagrams following specific patterns that give information on the spectral state (hard, soft, intermediate) of the source. For an example of a color-color diagram we refer to § 3.

The broad band spectral fitting uses a different approach since it requires physical models to interpret the X-ray spectral shape. The broad band spectrum can be described by models including several thermal and non-thermal continuum components and a few discrete emission lines (like the fluorescent iron line) produced by X-ray irradiation of heavy elements in the accretion disc.

The high resolution spectra in our work were in particular useful to measure absorption lines and edges due to the presence of interstellar gas along the line of sight between the observer and the X-ray source. We present a detailed broad band and high resolution spectral analysis of SAX J1808.4-3658 in § 5.
Abstract

X-ray timing of the accretion-powered pulsations during the 2003 outburst of the accreting millisecond pulsar XTE J1814-338 reveals variation in the pulse time of arrival residuals. These can be interpreted in several ways, including spin-down or wandering of the fuel impact point around the magnetic pole. In this Letter we show that the burst oscillations of this source are coherent with the persistent pulsations, to the level where they track all of the observed variation in the residuals. Only one burst, which has other unusual properties, shows a significant phase offset. We discuss what might lead to such rigid phase-locking between the modulations in the accretion and thermonuclear burst emission, and consider the implications for spin variation and the burst oscillation mechanism.
2. Coherence of burst oscillations and accretion-powered pulsations in the accreting millisecond pulsar XTE J1814-338

2.1 Introduction

The accreting millisecond X-ray pulsars (AMXPs) are a small class of neutron stars in Low Mass X-ray Binaries that show pulsations in outburst, thought to be caused by magnetic channeling of accreting plasma. Detailed timing studies of these stars reveal diverse behavior that can be interpreted in terms of spin variation or shifts in emission pattern. Both processes may play a role, and the degree to which we can be confident in inferred values of spin-up or spin-down remains a hot topic (Galloway et al. 2002; Burderi et al. 2006, 2007; Papitto et al. 2007, 2008; Hartman et al. 2008a; Riggio et al. 2008).

Some additional way of verifying the timing analysis obtained from the accretion-powered pulsations would be useful, and in this respect two of the AMXPs are particularly valuable. SAX J1808.4-3658 (J1808) and XTE J1814-338 (J1814) also show thermonuclear-powered pulsations, or burst oscillations. These are high frequency variations seen during Type I X-ray bursts, powered by unstable burning of accreted fuel. In these systems the burst oscillation frequency is at (J1814), or very close to (J1808), the spin frequency (Chakrabarty et al. 2003a; Strohmayer et al. 2003, hereafter S03). The frequency is stable in the decaying tails of the bursts, with no sign of the large frequency drifts seen in other burst oscillation sources (Muno et al. 2002a). In J1814 the frequency is also stable during the rising phase of the bursts, making it the most straightforward candidate for burst oscillation timing (Watts et al. 2005, hereafter W05).

J1814’s accretion-powered pulsations show significant pulse time of arrival (TOA) residuals even after correction for orbital Doppler shifts (Papitto et al. 2007, hereafter P07). The cause is still a matter of debate: P07 interpreted the observations in terms of a steady spin-down coupled with some jitter due to wandering of the fuel impact hotspot around the magnetic pole. However there are other possibilities, such as changes in beaming due to the accretion shock, that may also lead to the observed variation. In this respect analysis of the burst oscillations may be simpler: although the process responsible is not yet understood, the thermal spectrum suggests a purely surface mechanism, with the accretion shock contributing little to the observed asymmetry.

Some initial investigation of this issue was carried out by S03, who found that burst oscillations in the first 12 bursts were phase-locked to within 2.5° of the persistent pulsations. Their analysis, however, covered only the first 10 days of the ≈ 50 day outburst; before any of the variation reported by P07 is apparent. The level of coherence between the two types of pulsations is also important in our efforts to understand the burst oscillation mechanism, which remains mysterious (see the reviews by Strohmayer & Bildsten 2006, Galloway...
et al. 2006). The AMXPs are the only sources in which we can quantify the role of the magnetic field. We want to know, for example, whether the burst oscillations couple to the magnetic field or to the fuel stream impact point.

2.2 Timing analysis

J1814 was discovered in 2003 in the Rossi X-ray Timing Explorer (RXTE) Galactic bulge monitoring campaign (Markwardt & Swank 2003), and remained in outburst for nearly 2 months. The pulsar has a spin frequency of 314.4 Hz and resides in a binary with an orbit of 4.3 hours (Markwardt et al. 2003b). During the outburst over 425 ks of high time resolution data were taken with RXTE’s Proportional Counter Array (PCA, Jahoda et al. 2006). A total of 28 X-ray bursts were detected, all with burst oscillations at the spin frequency. Both accretion-powered pulsations and burst oscillations have a strong overtone at twice the spin frequency (S03).

For our timing analysis we use all available pointed observations from the RXTE PCA with Event mode data (time resolution 122 µs, 64 binned energy channels) or Good Xenon mode data (time resolution 1 µs, 256 unbinned energy channels). The latter were rebinned in time to 122 µs time resolution. Data were barycentered using the JPL DE405 ephemeris and a spacecraft ephemeris including fine clock corrections which together provide an absolute timing accuracy of 3-4 µs (Rots et al. 2004), using the source position of Krauss et al. (2005).

In analysing the accretion-powered pulsations we discard the X-ray bursts, removing all data from 50 s before to 200 s after the burst rise, and select only photons in the 2.5-17 keV range to maximize the signal to noise ratio. Pulse profiles are built using the fixed frequency solution of P07 to fold segments of approximately 500 s of data after subtracting the background contamination (using the FTOOL pcabackest). The TOAs were then obtained by cross-correlating the folded profiles with a pure sinusoid whose frequency represents the spin frequency of the neutron star. The same procedure is repeated for the first overtone. The fiducial point used in measuring the TOAs was the peak of the sine wave being cross-correlated. The determination of pulse TOAs and their statistical uncertainties follows the standard radio pulsar technique (Taylor 1993). Fitting a Keplerian orbit plus a constant spin frequency \( \nu \), or spin derivative \( \dot{\nu} \), we obtain solutions consistent with those of P07.

Like P07, we find a strong anti-correlation between flux and the accretion-powered pulse TOA residuals. It is interesting, however, that this anti-correlation becomes weaker when one considers the residuals from an ephemeris that includes a constant \( \dot{\nu} \) term. A rank correlation test between flux and
residuals for a constant frequency model gives a Spearman coefficient of $\rho = -0.71$ (fundamental) and $\rho = -0.90$ (first overtone), with a probability of < 0.01% that the two variables are not anti-correlated. Including spin-down, the magnitudes of the Spearman coefficients fall to $\rho = -0.56$ (probability still < 0.01% that the two variables are not anti-correlated, but larger than the probabilities for the zero spin derivative case) and $\rho = -0.12$ (a probability of 27% that the two variables are not anti-correlated) respectively. As we will argue in Section 2.3, an accretion-rate dependent hot spot location may be able to explain the entire residual record, with no need for spin variation.

We then apply the same timing procedure to the X-ray bursts. As in W05 and Watts & Strohmayer (2006) we use data where the count rate is at least twice the pre-burst level. For the first 27 bursts there is no evidence of frequency variability during the bursts (W05), so we use the entire burst to generate a folded profile. The final burst requires more care, as there is a statistically significant frequency drop in the late stages of the burst rise (Fig.20 of W05). For this burst we use only the first 2 s of the burst rise, before the frequency starts to shift. For each folded burst profile we then compute residuals using the same ephemeris that we used for the accretion-powered pulsations. Again we cross-correlate the folded profile using both a fundamental and an overtone. Figure 2.1 shows the TOA residuals for the accretion-powered pulsations and the burst oscillations.

To check whether the TOAs of the burst oscillations are consistent with having the same temporal dependence as the accretion-powered pulsations we fit a constant frequency model to the two TOA sets separately (excluding the final burst, see below). The fitted frequency is the same within the statistical uncertainties both for the fundamental and the first overtone. We also tried fitting the two TOA sets with a spin frequency plus a frequency derivative. Again the two solutions are consistent within the statistical uncertainties. However P07 already noted that both ephemerides are a poor fit to the accretion-powered pulse TOA residuals, and the same is true for the burst oscillation residuals. They are formally inconsistent with a constant frequency model: for the fundamental this assumption gives a $\chi^2$ of 459 for 23 degrees of freedom (dof). They are also inconsistent with a constant $\dot{\nu}$, as $\chi^2$ in this case is still large, 80 for 22 dof. Similar results are obtained for the first overtone.

It is clear from the Figure that, for all but the final burst, the burst oscillation TOA residuals track the accretion pulsation residuals. To test the phase-lock we computed the phase difference $\Delta \phi_m = \nu \{ \text{TOA}_{\text{bur}} - \text{TOA}_{\text{acc}} \}$, using the accretion pulse TOA residuals from the observation containing each burst. We then fitted a constant. For the fundamental we found a $\chi^2$ of 15.3 (26 dof), with best fit $\Delta \varphi_m = (0.004 \pm 0.002)$ rotational cycles. For the first
2.2 Timing analysis

Figure 2.1: Phase residuals for the accretion-powered pulsations (bars; red) and the burst oscillations (crosses; blue), compared to a model that has a constant frequency and orbital Doppler shifts. The final burst, which has no detectable 1st overtone component, requires special treatment; see text. The units on the right axis are rotational cycles. The top panel shows the outburst light curve.

overtone we found a $\chi^2$ of 11.3 (26 dof), with best fit $\Delta \varphi_m = -(0.001 \pm 0.003)$ rotational cycles. The fact that a constant $\Delta \varphi_m$ is such a good fit confirms that the two sets of pulsations are phase-locked. The small (2$\sigma$) non-zero offset in the fundamental bears comment. Unlike the burst oscillations, the accretion-powered pulsations have soft lags across the 2.5-20 keV band, with higher energy photons arriving earlier in phase (Watts & Strohmayer 2006). Taking this into account, we find that the burst oscillation TOAs are completely coincident with the softer (2.5 - 5 keV) component of the accretion pulsations, thought to originate from stellar surface (Section 2.3). The only exception to the phase-locking is the final burst, with $\Delta \varphi_m = (0.2 \pm 0.04)$ rotational cycles.

When calculating burst oscillation residuals some care is required, since if accretion continues during the burst, there might still be a contribution from the accretion-powered pulsations. The resulting bias in the burst oscillation phase can be calculated easily by considering the profile that results from the addition of two offset sinusoidal profiles. Standard trigonometric identities
yield a relation between $\Delta \varphi_m$ (the measured offset between burst oscillation phase and accretion-powered pulse phase) and $\Delta \varphi_b$ (the bias, i.e. the offset between measured and true burst oscillation phase caused by residual accretion):

$$\tan \Delta \varphi_m = \frac{\sin(\Delta \varphi_m - \Delta \varphi_b)}{[N_{\text{acc}} r_{\text{acc}} / N_{\text{bur}} r_{\text{bur}}] + \cos(\Delta \varphi_m - \Delta \varphi_b)}.$$  \hspace{1cm} (2.1)

where $N_{\text{acc}}$ and $N_{\text{bur}}$ are the number of accretion and burst photons in the folded profile, $r_{\text{acc}}$ and $r_{\text{bur}}$ being the associated fractional amplitudes. The quantity $\Delta \varphi_m$ was measured earlier: $\Delta \varphi_m \lesssim 0.01$ cycles at the 99% confidence level. Using the values of $N_{\text{acc}}$, $N_{\text{bur}}$, $r_{\text{acc}}$ and $r_{\text{bur}}$ from Table 1 of W05, Equation (2.1) gives a 99% confidence upper limit on the bias introduced by any residual accretion pulsation of $10^{-3}$ cycles. This is sufficiently small that it does not affect our analysis. Similar conclusions can be reached for the overtone. Note that in computing these limits we assume that the accretion flow parameters ($N_{\text{acc}}, r_{\text{acc}}, \Delta \varphi_m$) are unchanged during a burst. If accretion is inhibited during a burst, due to radiation pressure, the bias will be lower.

2.3 Discussion

J1814 has unusual burst oscillation properties compared to other sources: they occur in hydrogen-rich bursts, have negligible frequency and amplitude variation, and have a soft spectrum (S03, W05, Watts & Strohmayer 2006). Our analysis has shown that the burst oscillations are also phase-locked to the accretion-powered pulsations (to within 3° at 99% confidence) despite the substantial phase-wander exhibited by the latter over the course of the outburst.

In fact the burst oscillations are not only phase-locked but also coincident, having the same phase as the soft (2.5-5 keV), lagging, part of the accretion-powered pulsations (although they are also at the 2σ level coincident with the entire 2.5-20 keV accretion-powered pulse). Detailed modeling of the accretion pulsations has yet to be done for J1814, but modeling for other AMXPs with similar pulse properties suggests that the soft pulsed component comes from a hot spot on the stellar surface, with the hard component originating in the accretion funnel (Poutanen & Gierliński 2003; Gierliński & Poutanen 2005; Falanga & Titarchuk 2007).

We first consider what the exceptional degree of phase-locking implies about the cause of the variation in the TOA residuals. There are several parts of the system whose variation might affect both types of pulsation: surface rotation, the accretion funnel/disk, the magnetic field, and the fuel deposition footprint.
2.3 Discussion

Case 1: Genuine changes in the spin rate of the stellar surface. Our result would be consistent with a model where all of the variation is due to spin changes, both sets of pulsations being locked to the surface. However this requires alternating spin-up and spin-down with $|\dot{\nu}| \sim 10^{-12}$ Hz/s. Even if the crust were decoupled from the fluid core, this is high compared to what is achievable from the expected accretion or gravitational wave torques (Andersson et al. 2005; Bildsten 1998a). Fitting a constant spin-down term $\dot{\nu} \approx 6 \times 10^{-14}$ Hz/s, as argued by P07, does improve the quality of the fits somewhat. However spin derivatives $|\dot{\nu}| \sim 10^{-12}$ Hz/s would still be required on shorter timescales to explain the remaining excursions. Our results would also be consistent with precession, but modeling by Chung et al. (2008) suggests that precession is unlikely in this source.

Case 2: Changes in beaming/scattering by the accretion funnel or disk. The accretion shock in the funnel is thought to contribute to the pulsed emission of the accretion-powered pulsations, leaving a signature of hard emission in the spectrum. If the funnel were to have a similar effect on the much stronger burst emission it would have to do this without leaving any trace in the spectrum (Strohmayer et al. 2003; Krauss et al. 2005). This does not seem feasible. Our result also casts doubt on the accretion disk being the source of the soft lagging component of the accretion-powered pulsations (one of the possibilities considered by Falanga & Titarchuk 2007), since it is hard to understand why the burst oscillations (a surface process) would track a component generated in the surrounding environs.

Case 3: Wander of the magnetic pole, or changes in field geometry. Motion of the magnetic pole would affect location of the accretion hot spot. If the magnetic field also determines the location of the nuclear burning hot spot through modulation of ignition or emission, this could also explain our result. However, the observed variability would require localized burial or amplification of the poloidal field component on timescales of order a day. The accretion rate in the peak of the outburst is at most a few percent of the Eddington rate (Galloway et al. 2004). Current modeling suggests that this is insufficient to cause burial of the polar field on the required timescales (Brown & Bildsten 1998; Cumming et al. 2001; Payne & Melatos 2004). There is also no obvious mechanism for field amplification: material arriving via a funnel flow will have almost no angular momentum differential compared to the stellar surface. The heating associated with accretion could bring a buried field to the surface (Cumming et al. 2001), but as previously stated burial is unlikely at such low accretion rates.

Case 4: Wander of the fuel deposition point around the magnetic pole. Simulations of funneled accretion have shown that the fuel deposition point can
Coherence of burst oscillations and accretion-powered pulsations in the accreting millisecond pulsar XTE J1814-338 exhibit phase excursions around a fixed magnetic pole as accretion rate fluctuates, particularly for small misalignment angles between the magnetic pole and the spin axis (Romanova et al. 2003, 2004; Lamb et al. 2008b)\(^1\). Such a model might neatly account for the correlation between residuals and flux without requiring any non-zero \(\dot{\nu}\), since the stable position for the fuel impact point will vary in azimuth depending on accretion rate. The fact that the anti-correlation between flux and residuals is stronger when we set \(\dot{\nu} = 0\) supports this idea.

If the most plausible explanation for the variability in the two sets of pulsations is the last, what physical mechanisms might lead to phase-locking between the fuel impact point - which moves with accretion rate relative to field geometry on timescales of a few days - and the nuclear burning hot spots?

One possibility is some degree of magnetic confinement, leading to accumulation of fuel at the accretion hot spot. Material deposited near the polar cap will be prevented from spreading until the over-pressure is sufficient to distort the field lines (Brown & Bildsten 1998), even if the impact point is not precisely on the polar cap. To ensure that the magnetic propeller effect does not disrupt accretion, the magnetic field of J1814 must be \(\lesssim 10^9\) G (Psaltis & Chakrabarty 1999; Rappaport et al. 2004). This is consistent with the upper limit on spin-down inferred from pulse timing (P07). At this upper limit helium could be confined until a column depth \(\sim 10^6\) g cm\(^{-2}\), but this is still well before the material reaches the bursting layer at column depth \(\sim 10^8\) g cm\(^{-2}\) (Bildsten & Cumming 1998). Hydrogen will spread even more easily. Based on these estimates we conclude that fuel confinement is not effective, although it has been advanced as a possible explanation for the short burst recurrence times (Galloway et al. 2004).

The other factor distinguishing the fuel impact point is its temperature, which is higher than the rest of the star. The magnitude of the temperature differential has not been determined observationally, but it could certainly affect burst emission. One possibility is that the higher temperatures modify the local composition (via steady burning between bursts, for example). The higher starting temperature and/or composition could in principle modify the flux from the burst once it starts; perhaps driving more energetic reactions or enhancing convection. Whether this effect would be large enough to explain

---

\(^1\)The pulse profile modeling for this source which has been attempted has suggested large misalignment angles (Bhattacharyya et al. 2005; Leahy et al. 2009). If this were the case, the observed phase variability would require the fuel impact point to migrate back and forth by several km over the course of the outburst - an uncomfortably large amount. It seems more likely that some of the assumptions in these models, particularly the use of single temperature circular hotspots (known to be problematic, see Watts & Strohmayer 2006), need to be revisited.
the high fractional amplitudes is, however, not clear.

An alternative is the effect that a higher local temperature would have on ignition conditions. Previous studies that concluded that ignition would occur predominantly near the equator (Spitkovsky et al. 2002; Cooper & Narayan 2007) did not consider the effect of non-uniformities in temperature. A small increase in temperature can have a large effect on the column depth required for ignition, with the hotter area requiring a lower column depth (Bildsten & Cumming 1998). Material at the fuel impact point could therefore reach ignition conditions well ahead of the rest of the star. In this scenario the burning front might stall before spreading across the whole star, depending on the rate of heat transfer across the burning front. This would result in a brightness asymmetry centered on the fuel impact point. Premature ignition at the fuel deposition point, followed by stalling, could explain several observational features: the rather small black body radius of the bursts (Galloway et al. 2006), the shorter than expected burst recurrence times (Galloway et al. 2004), and the burst shapes, which suggest off-equatorial ignition (Maurer & Watts 2008). Note that off-equatorial ignition alone is not sufficient to explain the presence of an asymmetry in the burst tail: something else, such as stalling, is required.

Whatever the mechanism, it must fail once the accretion rate drops (or perhaps when the burst is more energetic), since the oscillations in the final burst are offset. The fuel impact footprint is by this time thought to be smaller, since the fractional amplitude of the accretion-powered pulsations is rising (W05). In addition the accreted fuel has more time to spread and equilibrate between bursts. Both factors will act to reduce the temperature differential between the fuel stream impact point and the rest of the star, which would make ignition at the fuel impact point less likely. The oscillation properties of this final burst (substantially lower fractional amplitude, frequency drift) are very different to the rest of the sample, and it is quite plausible that a different burst oscillation mechanism operates in this final burst.
Abstract

We present a phase-coherent timing analysis of the intermittent accreting millisecond pulsar SAX J1748.9–2021. A new timing solution for the pulsar spin period and the Keplerian binary orbital parameters was achieved by phase connecting all episodes of intermittent pulsations visible during the 2001 outburst. We investigate the pulse profile shapes, their energy dependence and the possible influence of Type I X-ray bursts on the time of arrival and fractional amplitude of the pulsations. We find that the timing solution of SAX J1748.9–2021 shows an erratic behavior when selecting different subsets of data, that is related to substantial timing noise in the timing post-fit residuals. The pulse profiles are very sinusoidal and their fractional amplitude increases linearly with energy and no second harmonic is detected. The reason why this pulsar is intermittent is still unknown but we can rule out a one-to-one correspondence between Type I X-ray bursts and the appearance of the pulsations.
3. Phase-coherent timing of the accreting millisecond pulsar SAX J1748.9-2021

3.1 Introduction

Recently it has been shown (Kaaret et al. 2006, Galloway et al. 2007, Casella et al. 2008, Gavriil et al. 2007, Altamirano et al. 2008) that intermittent pulsations can be observed during the transient outbursts of some accreting millisecond pulsars (AMPs). It was emphasized (Casella et al. 2008, Gavriil et al. 2007, Altamirano et al. 2008) that a new class of pulsators is emerging with a variety of phenomenology i.e., pulsations lasting only for the first two months of the outburst (HETE J1900.1-2455, Kaaret et al. 2006, Galloway et al. 2007), switching on and off throughout the observations (SAX J1748.9-2021, Altamirano et al. 2008, and HETE J1900.1-2455 Galloway et al. 2007), or even appearing for only 150 s over 1.3 Ms of observations (Aql X-1, Casella et al. 2008). This is different from the behavior of other known AMPs, where the pulsations persist throughout the outburst until they drop below the sensitivity level of the instrumentation.

Galloway et al. (2007) and Altamirano et al. (2008) pointed out the clear, albeit non-trivial, connection between the appearance of pulsations in HETE J1900.1-2455 and SAX J1748.9-2021 and the occurrence of thermonuclear bursts (Type I X-ray bursts). Galloway et al. (2007) demonstrated that increases of pulse amplitude in HETE J1900.0-2455 sometimes but not always follow the occurrence of Type I bursts nor do all bursts trigger an increase in pulse amplitude. Altamirano et al. (2008) showed that in SAX J1748.9–2021 the pulsations can re-appear after an absence of pulsations of a few orbital periods or even a few hundred seconds. All the pulsing episodes occur close in time to the detection of a Type I X-ray burst, but again not all the Type I bursts are followed by pulsations so it is unclear if the pulsations are indeed triggered by Type I X-ray bursts.

The pulse energy dependence and the spectral state in which pulsations are detected also differ between these objects. As shown by Cui et al. (1998), Galloway et al. (2005), Gierliński & Poutanen (2005) and Galloway et al. (2007), the fractional amplitude of the pulsations in AMPs monotonically decreases with increasing energies between 2 and \( \approx 20 \text{keV} \). However in IGR J00291+5934, Falanga et al. (2005b) found a slight decrease in the pulse amplitude between 5 and 8 keV followed by an increase up to energies of 100 keV. For the single pulse episode of Aql X-1 the pulsation amplitude increased with energy between 2 and \( \approx 20 \text{keV} \) (Casella et al. 2008). Interestingly in Aql X-1 the pulsations appeared during the soft state, contrary to all the other known AMPs which pulsate solely in their hard state (although many of these objects have not been observed in a soft state). Therefore it is interesting to investigate these issues for SAX J1748.9–2021 as well.
### Table 3.1: Observations analyzed for each outburst

<table>
<thead>
<tr>
<th>Outburst (year)</th>
<th>Start (MJD)</th>
<th>End (MJD)</th>
<th>Time (ks)</th>
<th>Observation IDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>51051.3</td>
<td>51051.6</td>
<td>14.8</td>
<td>30425-01-01-00</td>
</tr>
<tr>
<td>2001</td>
<td>52138.8</td>
<td>52198.4</td>
<td>138.0</td>
<td>60035-<em><strong>-</strong></em></td>
</tr>
<tr>
<td>2005</td>
<td>53523.0</td>
<td>53581.4</td>
<td>82.2</td>
<td>91050-<em><strong>-</strong></em></td>
</tr>
</tbody>
</table>

In order to study these various aspects of SAX J1748.9−2021 an improved timing solution is necessary. The spin frequency of the neutron star and the orbital parameters of SAX J1748.9−2021 were measured by Altamirano et al. (2008) using a frequency-domain technique that measures the Doppler shift of the pulsar spin period due to the orbital motion of the neutron star around the center of mass of the binary. The source was found to be spinning at a frequency of 442.361 Hz in a binary with an orbit of 8.7 hrs and a donor companion mass of \( \sim 0.1 - 1 M_\odot \). However, the timing solution obtained with this technique was not sufficiently precise to phase connect between epochs of visible pulsations. The intermittency of the pulsations creates many gaps between measurable pulse arrival times, complicating the use of standard techniques to phase connect the times of arrival (TOAs). The current work presents a follow-up study to the discovery paper of intermittent pulsations in SAX J1748.9−2021 (Altamirano et al. 2008) and provides for the first time a timing solution obtained by phase connecting the pulsations observed during the 2001 outburst. In §2 we present the observations and in §3 we explain the technique employed to phase connect the pulsations. In §4 we show our results and in §5 we discuss them. In §6 we outline our conclusions.

### 3.2 X-ray observations and data reduction

We reduced all the pointed observations from the RXTE Proportional Counter Array (PCA, Jahoda et al. 2006) that cover the 1998, 2001 and 2005 outbursts of SAX J1748.9−2021. The PCA instrument provides an array of five proportional counter units (PCUs) with a collecting area of 1200 cm\(^2\) per unit operating in the 2-60 keV range and a field of view of \( \approx 1^\circ \). The number of active PCUs during the observations varied between two and five. We used all the data available in the Event mode with a time resolution of 122 \( \mu \)s and 64 binned energy channels and in the Good Xenon mode with a time resolution of 1 \( \mu \)s and 256 unbinned channels; the latter were re-binned in time to a resolution of 122 \( \mu \)s. All the observations are listed in Table 1.

For obtaining the pulse timing solution we selected only photons with energies between \( \approx 5 \) and 24 keV to avoid the strong background at higher energies.
and to avoid the region below $\approx 5\text{keV}$ where the pulsed fraction is below $\approx 1\%$. The use of a wider energy band decreases the signal to noise ratio of the pulsations. We used only the good time intervals excluding Earth occultations, passages of the satellite through the South Atlantic Anomaly and intervals of unstable pointing. We barycentered our data with the tool \textit{faxbary} using the JPL DE-405 ephemeris along with a spacecraft ephemeris including fine clock corrections that together provide an absolute timing accuracy of $\approx 3.4\ \mu\text{s}$ (Rots et al. 2004). The best available source position comes from \textit{Chandra} observations (in’t Zand et al. 2001 and Pooley et al. 2002, see Table 2). The background was subtracted using the FTOOL \textit{pcabackest}.

### 3.3 The timing technique

The intermittent nature of SAX J1748.9–2021 requires special care when trying to obtain a full phase connected solution (i.e., a solution that accounts for all the spin cycles of the pulsar between periods of visible pulsations). With pulsations detected in only $\approx 12\%$ of the on-source data of the 2001 outburst, and only a few hundred seconds of observed pulsations during the 2005 outburst, the solution of Altamirano et al. (2008) does not have the precision required to directly phase connect the pulses. Their spin frequency uncertainty of $\sigma_s = 10^{-3}\text{Hz}$ implies a phase uncertainty of half a spin cycle after only 500 s, while the gaps between pulse episodes in the 2001 outburst were as long as 2 days. Moreover, the low pulsed fraction ($\lesssim 2.5\%$ in the 5-24 keV band) limits the number of TOAs with high signal to noise ratio.

Since the solution of Altamirano et al. (2008) lacks the precision required to phase connect the TOAs, we had to find an improved initial set of ephemeris to be used as a starting point for the phase-connection. To obtain a better initial timing model we selected all the chunks of data where the pulsations could clearly be detected in the average power density spectrum (see Altamirano et al. 2008 for a description of the technique). The total amount of time intervals with visible pulsations were $M=11$ with a time length $500\text{s} \lesssim t_{\text{obs},i} \lesssim 3000\text{s}$ in the $i$-th chunk, with $i=1,...,M$. We call the Modified Julian Date (MJD) where the pulsations appear in the $i$-th chunk as $\text{MJD}_{\text{start},i}$ while the final MJD where the pulsation disappears is $\text{MJD}_{\text{end},i} = \text{MJD}_{\text{start},i} + t_{\text{obs},i}$. We then measured the spin period $P_{s,i}$, the spin period derivative $P_{s,i}$ and, when required, the spin period second derivative $\dot{P}_{s,i}$ to align the phases of the pulsations (folded in a profile of 20 bins) for each chunk of data, i.e., to keep any residual phase drift to less than one phase bin (0.05 cycles) over the full length $t_{\text{obs},i}$. These three spin parameters are a mere local measure of the spin variation as a consequence of, primarily, the orbital Doppler shifts and cannot
be used to predict pulse phases outside the i-th chunk of data; they provide a ‘local phase-connection’ of the pulses within the i-th chunk. Using this set of $3 \times M$ parameters we generated a series of spin periods $P_s(t)$ in each i-th chunk of data with $t \in [\text{MJD}_{\text{start},i}, \text{MJD}_{\text{end},i}]$. The predicted spin period of the pulsar $P_s$ at time $t$ is given by the equation:

$$P_s(t) = P_{s,i} + \dot{P}_{s,i} t + \frac{1}{2} \ddot{P}_{s,i} t^2$$  \hspace{1cm} (3.1)

with $i = 1 \ldots M$. We then fitted a sinusoid representing a circular Keplerian orbit to all the predicted spin periods $P_s(t)$ and obtained an improved set of orbital and spin parameters.

With this new solution we folded 300 s intervals of data to create a new series of pulse profiles with higher signal to noise ratio. We then fitted a sinusoid plus a constant to these folded profiles and selected only those profiles with a S/N (defined as the ratio between the pulse amplitude and its $1\sigma$ statistical error) larger than 3.4. With a value of $S/N \geq 3.4$ we expect less than one false detection when considering the total number of folded profiles. The TOAs were then obtained by cross-correlating these significant folded profiles with a pure sinusoid. The determination of pulse TOAs and their statistical uncertainties closely resembles the standard radio pulsar technique (see for example Taylor 1993).

If the pulsar is isolated and the measured TOAs are error-free, the pulse phase $\phi$ at time $t$ can be expressed as a polynomial expansion:

$$\phi_P(t) = \phi_0 + \nu_0(t - t_0) + \frac{1}{2} \dot{\nu}_0(t - t_0)^2 + ...$$  \hspace{1cm} (3.2)

where the subscript “0” refers to the epoch time $t_0$. In an AMP, the quantities $\nu_0$ and $\dot{\nu}_0$ are the spin frequency of the pulsar and the spin frequency derivative (related to the spin torque), respectively. We note that the detection of a spin up/down in AMPs is still an open issue, since many physical processes can mimic a spin up/down (see for example Hartman et al. 2008a, but see also Burderi et al. 2006, Papitto et al. 2007, Riggio et al. 2008 for a different point of view). Indeed in the real situation the pulse phase of an AMP can be expressed as:

$$\phi(t) = \phi_P(t) + \phi_O(t) + \phi_M(t) + \phi_N(t)$$  \hspace{1cm} (3.3)

with the subscripts P, O, M, and N referring respectively to the polynomial expansion in eq. 3.2 (truncated at the second order), the orbital and the measurement-error components and any intrinsic timing noise of unknown origin that might be included in the data. For example the timing noise is
observed in young isolated radio pulsars (e.g., Groth 1975a, Cordes & Helfand 1980) or has been detected as red noise in SAX J1808.4-3658 (Hartman et al. 2008a) or in high mass X-ray binaries (Bildsten et al. 1997). The term $\phi_O(t)$ takes into account the phase variation introduced by the orbital motion of the pulsar around the companion. We expect that the error-measurement component $\phi_M(t)$ is given by a set of independent values and is normally distributed with an amplitude that can be predicted from counting statistics (e.g., Taylor 1993). Fitting the TOAs with a constant frequency model and a zero-eccentricity orbit we phase connected the 2001 outburst pulsations in a few iterations, re-folding the profiles with the improved solution and fitting the TOAs using the software package TEMPO2 (Hobbs et al. 2006). No significant improvement is obtained fitting for a spin derivative and an eccentricity (using the ELL1 model in TEMPO2). We did not fit for the orbital period in TEMPO2, because our initial fit, described earlier in this section, included data from the 2005 outburst and thus provided more precision on this parameter. A consistency check was made folding the pulsations of the 2005 outburst with the two different solutions. With the phase-connected solution of the 2001 outburst we were not able to properly fold the pulsations in the 2005 outburst, due to the relatively large error in the fitted orbital period, whereas with the hybrid solution (pulse spin frequency $\nu$, epoch of ascending node passage $T_{asc}$, and projected semi-major axis $a_x \sin i$ from the TEMPO2 fit to 2001 only; $P_{orb}$ kept from the sinusoidal fit to $P_{s}(t)$ for all the data) we were able to fold the pulsations and obtained a $\approx 8\sigma$ pulse detection with approximately 500 s of the 2005 outburst data.

3.4 Results

3.4.1 Timing solution

With the new timing solution we were able to search with higher sensitivity for additional pulsation episodes throughout the 1998, 2001 and 2005 outbursts. We folded chunks of data with a length between approximately 300s (to minimize the pulse smearing due to short timescale timing noise) and 1 hr (to increase the S/N when the pulsations are weak). Two new pulsation episodes of length $\approx 300$s each, were detected on MJD 52180.7 and MJD 52190.5, about eleven days and one day before the first earlier detection respectively, with a significance of $\approx 3.5\sigma$. We refitted a constant frequency model and a zero eccentricity orbit to the new ensemble of TOAs and obtained new timing residuals and a new timing solution. The post-fit residuals are all less than $\approx 0.2$ spin cycles, but their rms amounts to $164\mu s$, well in excess of the value
3.4 Results

Figure 3.1: Post-fit timing residuals of the 5-24 keV TOAs of the 2001 outburst. A negative/positive value of the residuals means that a pulsation is leading/lagging with respect to the timing model. All the pulse profiles used have $S/N > 3$. Strong timing noise appears as a large scatter within each group of points and misalignment between groups.

expected from counting statistics of $\simeq 70 \mu s$; the reduced $\chi^2$ is 9 (for 86 degrees of freedom, see the timing residuals in Figure 3.1). Trying to fit higher frequency derivatives or an eccentricity does not significantly improve the fit. At first glance, the residual variance, described by the timing noise term $\phi_N$ in eq 3.2, has two components: a long-term and a short-term component to the timing noise $\phi_N$. The short term timing noise acts on a timescale of a few hundred seconds and creates the large scatter observed within each group of points in Fig. 3.1. The long term timing noise acts on a timescale of a few days and can be recognized by a misalignment of the TOA residuals between groups, where with a simple white noise component we would expect within each group a distribution of TOAs spread around zero. The short term and the long term timing noise, which could of course well be part of the same process, are together responsible for the bad $\chi^2$ of the fit and the large rms of the residuals.
A histogram of the timing residuals shows a non-Gaussian distribution with an extended lower tail. A superposition of two Gaussians distribution can fit this. The first Gaussian is composed of TOAs that are on average earlier than predicted by the timing solution. The second Gaussian comprises TOAs that are on average later than predicted. The separation between the two mean values of the Gaussians is a few hundred microseconds. We found that the lagging TOAs correspond to systematically higher S/N pulsations than the leading ones. Since pulsations with higher S/N usually have high fractional amplitude, we analyzed the fractional amplitude dependence of the residuals. As can be seen in Fig. 3.2 above a fractional amplitude of \( \approx 1.8\% \) (the “upper group”), the TOAs are on average 103 \( \mu \text{s} \) later than predicted by the timing solution; the remaining TOAs (the “lower group”) are 42 \( \mu \text{s} \) early. The lower group TOAs are not related in a one-to-one relation with the fractional amplitude (Fig. 3.2), meaning that probably some kind of noise process is affecting the lower group TOAs in addition to the effect the varying amplitude has on the TOAs.

We fitted separate constant frequency and zero eccentricity models to just the upper and lower groups of TOAs and again checked the post-fit residuals (see Fig. 3.3). The reduced \( \chi^2 \) values of the two solutions are still large (\( \approx 4.4 \) and \( \approx 9.6 \) for the upper and lower solution, respectively). The maximum residual amplitude is about 200 \( \mu \text{s} \) and 450 \( \mu \text{s} \) in the upper and lower solution, respectively. Long term timing noise is still apparent, contributing more to the lower TOAs group than to the upper one. While the fitted orbital parameters \( (T_{\text{asc}} \text{ and } a_x \sin i) \) are in agreement to within 1\( \sigma \), the two solutions deviate in pulse frequency by \( 1.2 \times 10^{-7} \text{ Hz} \), about 4\( \sigma \).

We attribute this difference to the unmodeled timing noise component \( \phi_N \), which is ignored in the estimate of the parameter errors. Selecting groups of TOAs with fractional amplitude thresholds different from 1.8\% always gives different timing solutions, with deviations larger than 3\( \sigma \) sometimes also in \( T_{\text{asc}} \) and \( a_x \sin i \).

To investigate if this indicates a systematic connection between timing noise excursions and fractional amplitude or is just a consequence of selecting subsets of data out of a record affected by systematic noise, we selected subsets of data using different criteria: we split the data in chunks with TOAs earlier and later than a fixed MJD or we selected only TOAs with S/N larger than 3.4. In each case the fitted solutions deviated by several standard deviations, indicating that the differences are due to random selection of data segments out of a record affected by correlated noise (the timing noise). To take this effect into account in reporting the timing solution we increased our statistical errors by a constant factor such that the reduced \( \chi^2 \) of the timing solution fit was
3.4 Results

Figure 3.2: Dependence of the residuals on fractional amplitude of the pulsations. The vertical dashed line divides the plot in the upper (right side of the panel) and lower group (left side) of TOAs at a fractional amplitude of 1.8%.

Close to unity and refitted the parameters. The parameters and the errors for the global solution, calculated in this way, are reported in Table 2. They provide an improvement of three and five orders of magnitude with respect to the Keplerian and spin frequency parameter uncertainties, respectively, when compared with the solution reported in Altamirano et al. (2008).

To check whether the timing noise is energy dependent, we selected three energy bands: a soft band (5-13 keV), an intermediate band (11-17 keV) and a hard one (13-24 keV). Then we repeated the procedure, fitting all the significant TOAs with a constant spin frequency and circular Keplerian orbit, without increasing the statistical errors on the TOAs.

The two solutions of the soft and intermediate bands were in agreement to within 1σ for ν, T_asc and a_x sin i, while the hard band solution deviated by more than 3σ for the spin frequency. The reduced χ^2 of the sinusoidal fit to the pulse profiles in the highest energy band was systematically larger than in the other two bands. Fitting a second and a third harmonic did not significantly
Figure 3.3: Timing residuals of the upper (right panel) and lower (left panel) group. All the TOAs have at least a S/N larger than 3.4 and refer to the 5-24 keV energy band. The short timescale timing noise affects more strongly the lower group than the upper one, which is reflected in the bad $\chi^2$ of the fit (4.4 and 9.6 for the upper and lower group respectively). The new TOA around MJD 52181 belongs to the lower group. The two plots have the same scale on the residuals-axis to put in evidence the large scatter of the post-fit residuals in the lower group with respect to the upper one.

improve the fit according to an F-test.

The bad $\chi^2$ can be ascribed to a non-Poissonian noise process that dominates at high energies or to an intrinsic non-sinusoidal shape of the pulse profiles. Since the hard energy band has a count rate approximately 10 times smaller than the soft band, the influence of this on the global timing solution calculated for the energy band 5-24 keV is negligible. We refolded all the profiles and calculated another solution for the 5-17 keV energy band to avoid the region of high energies and found an agreement to within 1$\sigma$ with the solution reported in Table 2 and with the solution obtained for the 5-13 keV and for the 11-17 keV energy bands. This also shows that the timing noise is substantially independent from the energy band selected for energies below 17 keV.

We checked for the possible existence of characteristic frequencies in a power density spectrum (PDS) of the timing residuals using a Lomb-Scargle technique. No peaks above three sigma were found in either the PDS of the upper and lower group TOAs or the PDS of the global solution combining these two groups. The post-fit timing residuals are also not correlated with the reduced $\chi^2$ of each fitted pulse profile. We also checked the influence of very large events (VLE) due to energetic particles in the detector on the TOA residuals but no correlation was found. There is also no link between the source flux and the magnitude of the timing residuals and between the timing residuals and the orbital phase.
Table 3.2: Timing parameters for SAX J1748.9−2021

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right Ascension (α) (J2000)</td>
<td>17°48′52″.163</td>
</tr>
<tr>
<td>Declination (δ) (J2000)</td>
<td>−20°21′32″.40</td>
</tr>
<tr>
<td>Orbital period, $P_{orb}$ (hr)</td>
<td>8.76525(3) hr</td>
</tr>
<tr>
<td>Projected semi major axis, $a_x \sin i$ (ligh-ms)</td>
<td>387.60(4)</td>
</tr>
<tr>
<td>Epoch of ascending node passage $T_{asc}$ (MJD, TDB)</td>
<td>52191.50710(4)</td>
</tr>
<tr>
<td>Eccentricity, $e$ (95% confidence upper limit)</td>
<td>$&lt; 2.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Spin frequency $\nu_0$ (Hz)</td>
<td>442.36108118(5)</td>
</tr>
<tr>
<td>Reference Epoch (MJD)</td>
<td>52190.0</td>
</tr>
</tbody>
</table>

Note. — All the uncertainties quoted correspond to 1σ confidence level (i.e., $\Delta \chi^2 = 1$).

3.4.2 Pulse shape variations

To look for possible pulse profile changes, we fitted all significant folded pulse profiles with a fundamental and a second harmonic sinusoid (plus a constant level). No significant second harmonic detections ($> 3\sigma$) were made in the 5-24 keV energy band with upper limits of 0.9%, 0.5% and 0.4% amplitude at 98% confidence for the 1998, 2001 and 2005 pulsations episodes respectively. We repeated the procedure for different energy bands, obtaining similar results.

The pulse profiles of SAX J1748.9−2021 are therefore extremely sinusoidal with the fundamental amplitude at least $\approx 11$ times that of the second harmonic both in the soft and hard energy bands. Thus there are no detectable pulse profile shape variations nor the possibility to detect sudden changes of the relative phase between the fundamental and the second harmonic such as observed in SAX J1808.4-3658 (Burderi et al. 2006, Hartman et al. 2008a) and XTE J1814-338 (Papitto et al. 2007).

3.4.3 Pulse energy dependence and time lags

We analyzed the energy dependence of the pulse profiles selecting ten energy bands and measuring the strength of the pulsations. We folded all the observations where we detect significant pulsations. The pulse fraction is as small as $\approx 0.5\%$ between 2.5 and 4 keV and increases up to $\approx 4.0\%$ above 17 keV following a linear trend. Counting statistics prevents the measurement of pulsations above 24 keV. Fitting a linear relation to the points in Fig. 3.4 gives a slope of $(0.17 \pm 0.01) \% \text{keV}^{-1}$. 

45
The pulses of SAX J1748.9−2021 exhibit a similar energy dependence to Aql X-1 (Casella et al. 2008), which is opposite to that measured in three other AMPs (SAX J1808.4-3658 Cui et al. 1998, XTE J1751-305 Gierliński & Poutanen 2005, XTE J0929-314 Galloway et al. 2007), where the amplitude is stronger at low energies and decreases at high energies. In another source however (IGR J00291+5934) Falanga et al. (2005b) pointed out an increase of the fractional amplitude with energy which seems to resemble what has been observed for SAX J1748.9−2021 and Aql X-1. In IGR J00291+5934 a slight decrease in the fractional amplitude between 5 and 8 keV is followed by a slight increase up to 100 keV. However between 6 and 24 keV the slight increase is also consistent with a constant when considering the 1σ error bars. In SAX J1748.9−2021 and Aql X-1 the increase in fractional amplitude with energy is much steeper.

We did not detect any significant time lag between soft and hard photons, with an upper limit of 250µs at the 3σ level, using a coherent analysis between the 2.5 and 24 keV bands (see fig. 3.4). If a time lag with a magnitude smaller than our limit exists, this might explain the dependence of TOA on pulse amplitude if for example low amplitude pulses are dominated by soft photons whereas the high amplitude pulses are dominated by hard photons or vice versa. To check this possibility we re-folded all the low and high amplitude pulses selecting a low energy (5-13 keV) and an high energy band (13-24 keV). The folded profiles do not show any strong energy dependence, with the low and high energy bands equally contributing to the low and high amplitude profiles. Therefore energy dependent lags are unlikely to be the origin of the TOA dependence on amplitude.

### 3.4.4 Pulse dependence on flux

In both the 2001 and 2005 outbursts the pulsations disappear at both low and high X-ray flux. In Fig. 3.5 we show the light curve of the 2001 outburst, with pulsating episodes indicated with filled circles. The pulses appear in a broad flux band but not below ≈ 150 and not above ≈ 240 Ct/s/PCU. It is also intriguing that pulsations sporadically disappear on a timescale of a few hundred seconds even when the flux is in the band where we see pulsations. The few non-pulsating episodes at high and low flux could be random, however some non-pulsing episodes are observed at the same fluxes where we observe pulsations at different times. This indicates the absence of a flux threshold above or below which the pulse formation mechanism is at work.
3.4 Results

Figure 3.4: Time lags (left panel) and energy dependence of the fractional amplitudes of the pulsations (right panel). The plots stop at 24 keV after which the counts strongly drop below the detection level. The plots refer to all the observations where we detected pulsations. **Left:** no significant time lags are measured, with all the points being consistent with a zero lag at the three sigma level with respect to the first energy band, chosen here to be our reference time. **Right:** the fractional amplitude starts with a very low value of \( \approx 0.5\% \) around 2 keV and linearly increases up to 4\% at around 15 keV.

3.4.5 Type I X-ray bursts

The possibility that the pulsations are triggered by Type I X-ray bursts can be tested by measuring the evolution of the fractional amplitude of the pulsations in time. Our time resolution corresponds to the time interval selected to fold the data, i.e., \( \approx 300s \). As can be seen in Fig. 3.6, there are cases in which the fractional amplitude increases after a Type I burst (e.g., MJD 52191.74, 52192.25, 52192.31, 52196.24, 52198.2, 52198.36, 53534.46) on timescales of a few hundred seconds, while after other Type I bursts the fractional amplitude does not change significantly (MJD 52190.38, 52190.47, 52193.32, 52193.40, 52193.45, 52195.3, 52195.5) or increased prior to the Type I burst (MJD 52195.62). In four cases pulsations are present without any occurrence of a Type I burst (MJD 52181,52190.30, 52191.65, 52198.27) and in one case the fractional amplitude decreases after the burst (MJD 52192.40). Clearly there is no single response of the pulsation characteristics to the occurrence of a burst. There is also no clear link between the strength of a Type I burst and the increase of the pulsation amplitudes as is evident from Fig. 3.6. The maximum fractional amplitude (3.5\%) is reached during the 2005 outburst, after a Type I X-ray burst. However, after \( \approx 500s \) the pulsations were not seen anymore.
Figure 3.5: Light curve of the 2001 outburst from the first pulsating episode until the end of the outburst. The time resolution of the light curve is 300 s, and each point corresponds to a folded light curve. When a pulsation is detected a filled circle is plotted. The dotted lines are limits of highest and lowest flux where the pulses appear. Note that even between the dashed lines, there are non-pulsating episodes. These are often followed by pulsations on a timescale of a few hundred seconds.

3.4.6 Color–color diagram

We use the 16 s time resolution Standard 2 mode data to calculate X-ray colors. Hard and soft colors are defined respectively as the 9.7–16.0 keV / 6.0–9.7 keV and the 3.5–6.0 keV / 2.0–3.5 keV count rate ratio. The energy-channel conversion is done using the pca_e2c_e05v02 table provided by the RXTE Team. Type I X-ray bursts were removed, background subtracted, and dead-time corrections made. In order to correct for the gain changes as well as the differences in effective area between the PCUs themselves, we normalized our colors by the corresponding Crab Nebula color values (see Kuulkers et al. 1997; van Straaten et al. 2003, see table 2 in Altamirano et al. 2008 for average colors of the Crab Nebula per PCU) that are closest in time but in the same RXTE gain epoch, i.e., with the same high voltage setting of the PCUs Jahoda
et al. (2006).

The PCA observations sample the source behavior during three different outbursts (see Fig 1 in Altamirano et al. 2008). In Figure 3.7 we show the color–color diagram for all the observations of the three outbursts. Grey circles mark the 16 second averaged colors while black crosses and triangles mark the average color of each of the 8 observations from which pulsations were detected. As it can be seen, pulsations only appear in soft state of the source (banana state). During the observations of the 1998 outburst, the source was always in the so called Island/Extreme island state while during the 2001 and 2005 outburst the source was observed in both island and banana state.

### 3.4.7 Radio pulse search

We also used the new phase-coherent timing solution presented here to search for possible radio pulsations from SAX J1748.9–2021. For this, we used 2-GHz archival radio data from Green Bank Telescope (GBT) observations of the 6 known radio millisecond pulsars (MSPs) in the globular cluster (GC) NGC 6440 (see Freire et al. 2007 for a description of these data). The known radio pulsars in NGC 6440 have spin periods from 3.8–288.6 ms and dispersion measures (DMs) between 219–227 pc cm$^{-3}$. With a spin period of 2.3 ms, SAX J1748.9–2021 is thus the fastest-spinning pulsar known in NGC 6440, where the average spin period of the radio pulsars (11.3 ms when the 288-ms pulsar B1745–20 is excluded) is relatively long compared to other GCs.

We searched 11 data sets taken between 2006 December and 2007 March. These data sets have total integration times between 0.5–5 hr and a combined total time of 20.3 hr (73 ks). The data were dedispersed into 10 time series with trial DMs between 219–228 pc cm$^{-3}$. To check if pulsations were present, these time series were then folded with the timing solution presented in this paper. Because potential radio pulsations may also be transient, we folded not only the full data sets, but also overlapping chunks of 1/4, 1/10, and 1/50 of the individual data set (thus probing timescales between 100 s and hours). Furthermore, because these data were taken 6 years after the data that was used to construct the timing solution, we folded the data using both the exact period prediction from the timing solution as well as allowing for a small search around this value.

No obvious pulsations were detected in this analysis, where the reduced $\chi^2$ of the integrated pulse profile was used to judge if a given fold was worthy of further investigation. This is perhaps not surprising as radio pulsations have, as of yet, never been detected from an LMXB or AMP. Nonetheless, we plan to continue these searches on the large amount of available radio data which we have not yet searched with this technique.
Figure 3.6: Time dependence of the fractional amplitude and the TOA residuals in the 5-24 keV band during the 2001 outburst. Each diagram displays the residuals (upper panel) and their fractional amplitude (lower panel) versus time. All the points correspond to pulses with a S/N $\geq 3.4$. Type I bursts are also plotted in both panels in arbitrary units but scaled by a common constant factor.
Figure 3.7: Color-color diagram for the 1998, 2001 and 2005 outbursts (values are normalized by the Crab Nebula, see Section 3.4.6). Gray circles mark the 16 seconds average color of all available data. Black crosses (2001 outburst) and triangles (2005 outburst) mark the average color per observation in which we find pulsations.
3.5 Discussion

We have presented a new timing solution for SAX J1748.9–2021 obtained by phase connecting the 2001 outburst pulsations. We discovered the presence of timing noise on short (hundred seconds) and long (few days) timescales. We cannot exclude the presence of timing noise on different timescales, since the short and the long timescales found are also the two timescales the TOAs probe. The pulse profiles of SAX J1748.9–2021 keep their sinusoidal shape below 17 keV throughout the outburst, but do experience considerable shifts relative to the co-rotating reference frame both apparently randomly as a function of time and systematically with amplitude. Above 17 keV the pulse profiles show deviations from a sinusoidal shape that cannot be modeled adding a 2nd harmonic. The fit needs a very high number of harmonics to satisfactorily account the shape of the pulsation. This is probably due to the effect of some underlying unknown non-Poissonian noise process that produces several sharp spikes in the pulse profiles. The lack of a detectable second harmonic prevents us from studying shape variations of the pulse profiles such as was done for SAX J1808.4–3658 (Burderi et al. 2006; Hartman et al. 2008a) where sudden changes between the phase of the fundamental and the second harmonic were clearly linked with the outburst phase.

Another interesting aspect is the energy dependence of the pulse profiles, which can be a test of current pulse formation theories. In one model of AMPs, Poutanen & Gierliński (2003) explain the pulsations by a modulation of Comptonized radiation whose seeds photons come from blackbody radiation. The thermal emission is given by the hot spot and/or emitting column produced by the in-falling material that follows the magnetic dipole field lines of the neutron star plasma rotating with the surface of the neutron star. Part of the blackbody photons can be scattered to higher energies by a slab of shocked plasma that forms a comptonizing region above the hot spot (Basko & Sunyaev 1976).

In three AMPs (Cui et al. 1998; Gierliński & Poutanen 2005; Poutanen & Gierliński 2003; Watts & Strohmayer 2006; Galloway et al. 2007) the fractional pulse amplitudes decreases toward high energies with the soft photons always lagging the hard ones. However in SAX J1748.9–2021 the energy dependence is opposite, with the fractional pulse amplitudes increasing toward higher energies and no detectable lags (although the large upper limit of 250 µs does not rule out the presence of time lags with magnitude similar to those detected in other AMPs). Moreover, all the pulsating episodes happen when the source is in the soft state (although not all soft states show pulsations). This is similar to Aql X-1 (see Casella et al. 2008) where the only pulsating episode occurred
during a soft state, and the pulsed fraction also increased with energy. Remarkably all the other sources which show persistent pulsations have hard colors typical of the extremely island state.

Both the energy dependence and the presence of the pulsations during the soft state strongly suggest a pulse formation pattern for these two intermittent sources which is different from that of the other known AMPs and the intermittent pulsar HETE J1900.1-2455. As discussed in Muno et al. 2002b, Muno et al. 2003, a hot spot region emitting as a blackbody with a temperature contrast with respect to the neutron star surface produces pulsations with an increasing fractional amplitude with energy in the observer rest frame. The exact variation of the fractional amplitude with energy however has a complex dependence on several free parameters as the mass and radius of the neutron star and the number, size, position and temperature of the hot spot and viewing angle of the observer. The observed slope of 0.2% keV$^{-1}$ is consistent with this scenario, i.e., a pure blackbody emission from a hot spot with a temperature contrast and a weak comptonization (see Falanga & Titarchuk 2007, Muno et al. 2002b, Muno et al. 2003). However it is not possible to exclude a strong comptonization given the unknown initial slope of the fractional amplitude.

The pulse shapes above 17 keV are non-sinusoidal and are apparently affected by some non-Poissonian noise process or can be partially produced by an emission mechanism different from the one responsible of the formation of the soft pulses (see for example Poutanen & Gierliński (2003) for an explanation of how the soft pulses can form). We found that the TOAs of the pulsations are independent of the energy band below 17 keV but selection on the fractional amplitude of the pulsations strongly affects the TOAs: high amplitude pulses arrive later. However, this does not affect the timing solution beyond what is expected from fitting other data selections: apparently the TOAs are affected by correlated timing noise. If we take into account the timing noise ($\phi_N$) in estimating the parameter errors, then the timing solutions are consistent to within 2$\sigma$. In the following we examine several possibilities for the physical process producing the $\phi_N$ term.

### 3.5.1 Influence of Type I X-ray bursts on the TOAs

The occurrence of Type I X-ray bursts in coincidence with the appearance of the pulsations suggests a possible intriguing relation between the two phenomena. However as we have seen in §4.4, there is not a strict link between the appearance of pulsations and Type I burst episodes. Only after $\approx$ 30% of the Type I bursts the pulsations appeared or increased their fractional amplitude. The appearance of pulsations seems more related with a period of global surface activity during which both the pulsations and the Type I bursts
occur. This is different from what has been observed in HETE J1900.1-2455 (Galloway et al. 2007) where the Type I X-ray bursts were followed by an increase of the pulse amplitudes that were exponentially decaying with time. We note that during the pulse episodes at MJD 52191.7, 52193.40, 52193.45 and 52195.5 the TOA residuals just before and after a Type I burst are shifted by 300-500 $\mu$s suggesting a possible good candidate for the large timing noise observed. However in two other episodes no substantial shift is observed in the timing residuals (see Fig. 3.6). We also note that there is no relation between the Type I burst peak flux and the magnitude of the shifts in the residuals.

### 3.5.2 Other possibilities

The timing noise and its larger amplitude in the weak pulses might be related with some noise process that becomes effective only when the pulsations have a low fractional amplitude. Kulkarni & Romanova (2008) have shown that above certain critical mass transfer rates an unstable regime of accretion can set in, giving rise to low-$m$ modes in the accreting plasma which produce an irregular light curve. The appearance of these modes inhibits magnetic channeling and hence dilutes the coherent variability of the pulsations. Such a model can explain pulse intermittency by positing that we see the pulses only when the accretion flow is stable. The predominance of timing noise in the low fractional amplitude group could be explained if with the onset of unstable accretion the $m$-modes gradually set in and affect the pulsations, lowering their fractional amplitude until they are undetectable. However the lack of a correlation between pulse fractional amplitude and X-ray flux is not predicted by the model.

Another interesting aspect are the apparent large jumps that we observe in the phase residuals on a timescale of a few hundred seconds. Similar jumps have been observed for example in SAX J1808.4-3658 during the 2002 and 2005 outbursts (Burderi et al. 2006; Hartman et al. 2008a. In that case however the phases jump by approximately 0.2 spin cycles on a timescale of a few days, while here we observe phase jumps of about 0.4 cycles on a timescale of hundred seconds. Other AMPs where phase jumps were observed (XTE J1814-338 Papitto et al. 2007, XTE J1807-294 Riggio et al. 2008) have shown timescales of a few days similar to SAX J1808.4-3658. This is a further clue that the kind of noise we are observing in SAX J1748.9--2021 is somewhat different from what has been previously observed.
3.6 Conclusions

We have shown that it is possible to phase connect the intermittent pulsations seen in SAX J1748.9−2021 and we have found a coherent timing solution for the spin period of the neutron star and for the Keplerian orbital parameters of the binary.

We found strong correlated timing noise in the post-fit residuals and we discovered that this noise is strongest in low fractional amplitude pulses and is not related with the orbital phase. Higher-amplitude (≥1.8%) pulsations arrive systematically later than lower-amplitude (≤1.8%) ones, by on average 145μs. The pulsations of SAX J1748.9−2021 are sinusoidal in the 5-17 keV band, with a fractional amplitude linearly increasing in the energy range considered. The pulsations appear when the source is in the soft state, similarly to what has been previously found in the intermittent pulsar Aql X-1. The origin of the intermittency is still unknown, but we can rule out a one-to-one correspondence between Type I X-ray bursts and the appearance of pulsations.
The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658


Abstract

We present a 7 yr timing study of the 2.5 ms X-ray pulsar SAX J1748, an X-ray transient with a recurrence time of \( \approx 2 \) yr, using data from the Rossi X-ray Timing Explorer covering 4 transient outbursts (1998–2005). We verify that the 401 Hz pulsation traces the spin frequency fundamental and not a harmonic. Substantial pulse shape variability, both stochastic and systematic, was observed during each outburst. Analysis of the systematic pulse shape changes suggests that, as an outburst dims, the X-ray “hot spot” on the pulsar surface drifts longitudinally and a second hot spot may appear. The overall pulse shape variability limits the ability to measure spin frequency evolution within a given X-ray outburst (and calls previous \( \dot{\nu} \) measurements of this source into question), with typical upper limits of \( |\dot{\nu}| \lesssim 2.5 \times 10^{-14} \) Hz s\(^{-1}\) (2\( \sigma \)). However, combining data from all the outbursts shows with high (6\( \sigma \)) significance that the pulsar is undergoing long-term spin down at a rate \( \dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16} \) Hz s\(^{-1}\), with most of the spin evolution occurring during X-ray quiescence. We discuss the possible contributions of magnetic pro-
peller torques, magnetic dipole radiation, and gravitational radiation to the measured spin down, setting an upper limit of $B < 1.5 \times 10^8$ G for the pulsar’s surface dipole magnetic field and $Q/I < 5 \times 10^{-9}$ for the fractional mass quadrupole moment. We also measured an orbital period derivative of $\dot{P}_{\text{orb}} = (3.5 \pm 0.2) \times 10^{-12}$ s s$^{-1}$. This surprising large $\dot{P}_{\text{orb}}$ is reminiscent of the large and quasi-cyclic orbital period variation observed in the so-called “black widow” millisecond radio pulsars. This further strengthens previous speculation that SAX J1748 may turn on as a radio pulsar during quiescence. In an appendix we derive an improved (0.15 arcsec) source position from optical data.
4.1 Introduction

The growing class of accretion-powered millisecond X-ray pulsars discovered by the \textit{Rossi X-Ray Timing Explorer} (\textit{RXTE}) has verified the hypothesis that old millisecond pulsars obtained their rapid spins through sustained accretion in X-ray binaries. These objects provide a versatile laboratory. The X-ray pulse shapes arising from the magnetically channeled accretion flow can constrain the compactness (and hence the equation of state) of the neutron star. Tracking the arrival times of these X-ray pulses allows us to measure the pulsar spin evolution, which directly probes magnetic disk accretion torque theory in a particularly interesting regime (Psaltis & Chakrabarty 1999) and also allows exploration of torques arising from other mechanisms such as gravitational wave emission (Bildsten & Cumming 1998). There have been several reports of significant spin evolution in accreting millisecond pulsars, some with implied torques that are difficult to reconcile with standard accretion torque theory (Markwardt et al. 2003a; Morgan et al. 2003; Burderi et al. 2006, 2007). However, a variety of effects (including limited data spans, pulse shape variability, and non-Gaussian noise sources) can complicate the interpretation of these measurements. In this paper, we address these difficulties using a comprehensive analysis of the most extensive data set.

Of the eight accretion-powered millisecond pulsars currently known, the first one remains the best-studied example. The X-ray transient SAX J1808 was discovered during an outburst in 1996 by the \textit{BeppoSAX} Wide Field Cameras (in ’t Zand et al. 1998). Timing analysis of \textit{RXTE} data from a second outburst in 1998 revealed the presence of a 401 Hz (2.5 ms) accreting pulsar in a 2 hr binary (Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998). The source is a recurrent X-ray transient, with subsequent \(\approx1\) month long X-ray outbursts detected in 2000, 2002, and 2005; it is the only known accreting millisecond pulsar for which pulsations have been detected during multiple outbursts. Faint quiescent X-ray emission has also been observed between outbursts, although no pulsations were detected (Stella et al. 2000; Campana et al. 2002; Heinke et al. 2007). A source distance of 3.4–3.6 kpc is estimated from X-ray burst properties (in ’t Zand et al. 2001; Galloway et al. 2006). The pulsar is a weakly magnetized neutron star \(([1–10] \times 10^8 \text{ G at the surface; Psaltis & Chakrabarty 1999})\) while the mass donor is likely an extremely low-mass \((\approx0.05 \, M_\odot)\) brown dwarf (Bildsten & Chakrabarty 2001). SAX J1808 is the only source known to exhibit all three of the rapid X-ray variability phenomena associated with neutron stars in LMXBs: accretion-powered millisecond pulsations (Wijnands & van der Klis 1998), millisecond oscillations during thermonuclear X-ray bursts (Chakrabarty et al. 2003b), and kilohertz
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

quasi-periodic oscillations (Wijnands et al. 2003). After submitting this paper, we learned of an independent analysis by di Salvo et al. (2008) reporting an increasing orbital period (see §4.3.7).

An optical counterpart has been detected both during outburst (Roche et al. 1998; Giles et al. 1999) and quiescence (Homer et al. 2001). The relatively high optical luminosity during X-ray quiescence has led to speculation that the neutron star may be an active radio pulsar during these intervals (Burderi et al. 2003; Campana et al. 2004), although radio pulsations have not been detected (Burgay et al. 2003). Transient unpulsed radio emission (Gaensler et al. 1999; Rupen et al. 2005) and an infrared excess (Wang et al. 2001; Greenhill et al. 2006), both attributed to synchrotron radiation in an outflow, have been reported during X-ray outbursts.

In this paper, we describe our application of phase-connected timing solutions for each outburst to study the spin history and pulse profile variability of SAX J1808, providing the first look at the evolution of an accretion-powered X-ray pulsar. In section 4.2, we outline our analysis methods, noting the difficulties raised by pulse profile noise and describing a new technique to obtain a minimum-variance estimate of the spin phase in the presence of such noise. In section 4.3, we present the results of this analysis. In particular, we observe that the source is spinning down between outbursts, the binary orbital period is increasing, and the pulse profiles change in a characteristic manner as the outbursts progress. Finally, in section 4.4, we discuss the implications of these results to the properties of the neutron star and accretion geometry.

4.2 X-ray Observations and Data Analysis

4.2.1 RXTE data reduction

The RXTE Proportional Counter Array (PCA; Jahoda et al. 1996) has repeatedly observed SAX J1808, primarily during outburst. These observations total 307 separate pointings and an exposure time of 1,371 ks from 1998 through 2005. The PCA comprises five identical gas-filled proportional counter units (PCUs) sensitive to X-rays between 2.5 and 60 keV. Each PCU has an effective area of 1200 cm$^2$. It is uncommon for all five PCUs to be active: some are periodically disabled to decrease their rates of electrical breakdown (Jahoda et al. 2006). The average number of active PCUs has declined as the RXTE ages, and most observations during the 2002 and 2005 outbursts of SAX J1808 only include two or three.

All but three of the observations of SAX J1808 were taken with the E_125US_64M_0_1S mode, which records the arrival of each photon with a
4.2 X-ray Observations and Data Analysis

Table 4.1: Observations analyzed for each outburst

<table>
<thead>
<tr>
<th>Date</th>
<th>Data range (MJD)</th>
<th>Time (ks)</th>
<th>Avg. No. of PCUs</th>
<th>Observation IDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 Apr</td>
<td>50914.8 – 50939.6</td>
<td>178.1</td>
<td>4.67</td>
<td>30411-01-*</td>
</tr>
<tr>
<td>2000 Feb</td>
<td>51564.1 – 51601.9</td>
<td>126.8</td>
<td>3.74</td>
<td>40035-01-01-00 – 40035-01-04-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>40035-05-02-00 – 40035-05-18-00</td>
</tr>
<tr>
<td>2002 Oct</td>
<td>52562.1 – 52602.8</td>
<td>714.5</td>
<td>3.25</td>
<td>70080-01-**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70080-03-05-00 – 70080-03-24-00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70080-03-25-01, 70518-01-**</td>
</tr>
<tr>
<td>2005 Jun</td>
<td>53523.0 – 53581.4</td>
<td>284.3</td>
<td>2.84</td>
<td>91056-01-01-01 – 91056-01-04-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>91418-01-01-00 – 91418-01-07-00</td>
</tr>
</tbody>
</table>

Note. — The ranges of observation IDs given here are for numerically sorted IDs, which do not always reflect temporal order.

time resolution of 122 $\mu$s and 64 energy channels covering the full range of the detectors. The other observations were rebinned to be compatible with the 122 $\mu$s resolution data; using higher time resolutions provides no benefit. We shifted the photon arrival times to the Earth’s geocenter\(^1\) using our improved optical position of R.A. = 18\(^{h}\)08\(^{m}\)27\(^{s}\).62, Decl. = −36\(^{\circ}\)58\(^{\prime}\)43\(^{\prime\prime}\)3 (equinox J2000), with an uncertainty of 0\(^{\prime\prime}\)15. (Please refer to Appendix 4.5 for details on this improved position.) We then applied the RXTE fine clock correction, which provides absolute time measurements with errors of less than 3.4 $\mu$s (99% confidence; Jahoda et al. 2006). Finally, we filtered the data to remove Earth occultations, intervals of unstable pointing, and thermonuclear X-ray bursts. For three observations at the start and end of the 1998 outburst, we relaxed our requirement of stable pointing and included raster scanning data to extend our baseline for measuring the frequency evolution during this outburst. These observations provided additional valid phase measurements, but they were not used to calculate fractional amplitudes since the contribution of the source and background varied as the RXTE panned across the source. Table 4.1 lists all the observations that we included in our analysis.

We consistently used an energy cut of roughly 2–15 keV for our timing analysis. While the source is readily detectable in the PCA at higher energies, the background dominates above 15 keV, especially in the dimmer tails of

\(^1\)We shifted the photons to the geocenter rather than the solar system barycenter since the TEMPO pulse timing program, which we used to fit phase models to the arrival times, was designed for radio timing and thus expects photon arrival times at some point on the Earth. TEMPO itself performed the barycentric corrections using the quoted position.
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

the outbursts. Excluding these high-energy counts optimized the detection of pulsations when the source was dim, providing a longer baseline for our timing analysis.

4.2.2 Pulse timing analysis with tempo

The core of this analysis resembles the work long-done for radio pulsars and slowly rotating X-ray pulsars. We first folded intervals of data according to a phase timing model to obtain pulse profiles. We next compared the profile from each interval to a template profile in order to calculate the offset between the observed and the predicted pulse times of arrival (TOAs). We then improved the initial phase model by fitting it to these TOA residuals.

We used the TEMPO pulsar timing program\(^2\), version 11.005, to calculate pulse arrival times from a phase model and to improve a phase model by fitting it to arrival time residuals. TEMPO reads in a list of TOAs and a set of parameters describing the pulsar timing model. It then adjusts the model to minimize the timing residuals between the predicted and observed arrival times. The output files include a revised timing model, a covariance matrix for the fit parameters, and a list of the timing residuals. TEMPO also includes a predictive mode, which takes a timing model and generates a series of polynomial expansions that give the model’s pulse arrival times during a specified time interval. TEMPO has been a standard tool of the radio pulsar community for decades and is well-tested at the microsecond-level accuracies with which we are measuring TOAs.

Our timing models fit for the following parameters: the pulsar spin frequency and (if necessary) the first-order frequency derivative; the times and magnitudes of any instantaneous changes in the frequency; and the orbital parameters. Our models supplied, but did not fit, the position of the source from Appendix 4.5. Because we fit the outbursts separately, the \(\approx 1\) month of data that each provides was not sufficient to improve the source position: the position of SAX J1808 (in particular, its right ascension) is degenerate with the frequency and frequency derivative on such timescales.

To parametrize the orbit of SAX J1808, we used TEMPO’s ELL1 binary model, which employs the Laplace parameters \(e \sin \omega\) and \(e \cos \omega\), where \(e\) is the eccentricity and \(\omega\) the longitude of periastron passage. This parametrization avoids the degeneracy of \(\omega\) in low-eccentricity systems (Deeter et al. 1981). For most of the fits, we held \(e = 0\) and solely fit the projected semimajor axis \(a_x \sin i\), the orbital period \(P_{\text{orb}}\), and the time of ascending node\(^3\) \(T_{\text{asc}}\). As a

\(^2\)http://www.atnf.csiro.au/research/pulsar/tempo/

\(^3\)Past pulsar timing of SAX J1808 uses the \(T_{90}\) fiducial, marking a time at which the
test of this assumption, we also repeated the fits allowing \( e \) to vary. It was always consistent with zero.

This analysis depends critically on the accurate calculation and processing of the TOAs. To verify our results, they were independently calculated using two entirely separate data pathways. One corrected the \( RXTE \) count data to the geocenter and used \textsc{tempo} to barycenter the TOAs, as described in the previous section; the other used the FTOOL\(^4\) \texttt{faxbary} to barycenter the count data. Independent codes were then used to divide the count data into 512 s intervals, fold it according to a phase model, and measure the pulse times of arrival. Finally, we used both \textsc{tempo} and its replacement, \textsc{tempo2} (Hobbs et al. 2006), to process the TOAs and refine the timing models. In all cases, the agreement between the final timing models was good.

4.2.3 TOA calculation in the presence of profile noise

Special care must be taken when measuring the pulse TOAs for rapidly rotating accretion-powered pulsars. In these systems, the pulse profiles exhibit variability on timescales of \( \sim 10 \) hr and longer that is well in excess of the Poisson noise expected from counting statistics. In this section, we develop a procedure to obtain a minimum-variance estimate of the timing residuals in the presence of such noise.

We use the term “noise” with respect to spin timing analysis simply to mean phase variability of one or more harmonics that does not seem to be due to underlying spin frequency changes. While some of this profile variability may in fact be quite ordered — distinctive pulse shape changes that occur in every outburst, for instance — we cannot model all of them and thus consider the unmodeled profile variability as “noise” from the phase-timing perspective. In this section, we attempt to minimize the impact of such variability on the accuracy of our pulse arrival times by favorably weighting data from less-noisy harmonics; in §4.2.4 we describe a Monte Carlo technique to estimate its impact on the timing model parameters.

To calculate the TOAs, we divided the timing data into 512 s intervals and determined one pulse arrival time per interval. We chose this length because it provides sufficient counts to make accurate measurements in the dim tails of the outbursts, while it still is short enough to sample within the 7249 s orbital period. This is necessary to improve the binary model and resolve any additional short-timescale variability.

For each outburst, we used \textsc{tempo}’s predictive mode to generate a series of

\[^{4}\text{http://heasarc.gsfc.nasa.gov/ftools/}\]


\[^{63}\]
polynomial expansions predicting the times of pulse arrivals at the geocenter. These ephemerides are based on the revised optical position, our best-known orbital parameters, and a simple, constant-frequency spin model. Using the expansions, we calculated the expected phase for each photon arrival time. For each 512 s interval, we then divided the phases into $n$ phase bins in order to create folded pulse profiles.

We then decomposed the profiles into their Fourier components. For a given folded profile, let $x_j$ designate the number of photons in the $j$th phase bin, and $N_{\text{ph}} = \sum_{j=1}^{n} x_j$ is the total number of photons. The complex amplitude of the $k$th harmonic is then

$$a_k = \sum_{j=1}^{n} x_j \exp\left(2\pi i j k / n\right).$$  \hspace{1cm} (4.1)

Throughout this paper, we number the harmonics such that the $k$th harmonic is $k$ times the frequency of the 401 Hz fundamental. Since our analysis for the most part handles the phases and amplitudes of harmonics separately, we define these quantities explicitly as follows:

$$A_k \exp\left[2\pi i k (\phi_k + \Delta \phi_k)\right] = 2a_k.$$ \hspace{1cm} (4.2)

Here we are interested in the amplitude\(^5\) $A_k$ and the phase residual $\Delta \phi_k$, which we measure relative to a fixed phase offset $\phi_k$. We include these offsets because we are principally interested in measuring the phase deviations from a fixed template profile, which we obtain by transforming the overall folded pulse profile from an outburst:\(^6\)

$$A'_k \exp\left(2\pi i k \phi_k\right) = 2 \sum_{j=1}^{n} x'_j \exp\left(2\pi i j k / n\right).$$ \hspace{1cm} (4.3)

Here $x'_j$ and $N'_{\text{ph}}$ give the phase bin counts and total counts for the template pulse profile.

Note that we define the phases such that shifting a fixed pulse profile by some phase $\Delta \phi$ produces the same shift in the phase of each of its harmonic: $\Delta \phi_k = \Delta \phi$. Hence the unique phases for each $\Delta \phi_k$ range from 0 to $1/k$.

---

\(^5\)We define $A_k$ such that it is the actual amplitude, in photons, of the observed pulsations. We must therefore include both the positive and negative frequency components (which are equal for real signals), introducing the factor of 2 on the right-hand side of eqs. (4.2) and (4.3).

\(^6\)In the presence of sudden pulse profile changes, we may use multiple templates during a single outburst, thus using different values of $\phi_k$ on either side of the change. Further description is at the end of this section.
Positive phase residuals corresponding to time lags: $\Delta \phi_k > 0$ indicates that the $k$th harmonic arrived later than predicted by the model.

The uncertainty in the phase residuals $\Delta \phi_k$ due to Poisson noise (derived in Appendix 4.6) are

$$\sigma_k = \sqrt{\frac{2N_{ph}}{2\pi k A_k}}.$$ (4.4)

For our analysis, we rejected phase measurements with uncertainties greater than 0.1 ms (i.e., 0.04 cycles). Generally, this cut only removed points in the tails of the outbursts, where the flux was low.

The measured fractional rms amplitudes are

$$r_k = \frac{A_k}{\sqrt{2 (N_{ph} - B)}},$$ (4.5)

where $B$ is the approximate number of background events within our energy range and time interval, estimated using the FTOOL pcabackest.7 The $r_k$ add in quadrature: the total rms fractional amplitude for a pulse profile described with $m$ harmonics is $r = \left(\sum_{k=1}^{m} r_k^2 \right)^{1/2}$. Uncertainties on the fractional amplitudes are computed using the method described by Groth (1975b) and Vaughan et al. (1994), which accounts for the addition of noise to the complex amplitude of the signal. The probability that the detection of a harmonic is due solely to Poisson noise is $\exp(-P_k)$, where $P_k = \frac{1}{4} A_k^2 / N_{ph}$ is the unit-normalized power for the $k$th harmonic. For a fuller review of Fourier techniques in X-ray timing, we defer to van der Klis (1989).

Since each harmonic provides an independent measurement of the phase residual, we can combine them to provide the overall phase residual for the sample pulse. We obtain the optimal estimator by weighting each measurement according to its variance:

$$\Delta \phi = \sum_{k=1}^{m} w_k \Delta \phi_k / \sum_{k=1}^{m} w_k ;$$ (4.6)

$$w_k = \frac{1}{\sigma_k^2} = \frac{k^2 A_k^2}{N_{ph}}.$$ (4.7)

Thus far, this analytical method closely parallels the work long done on spin-powered pulsars (e.g., Taylor 1993, Appendix A).

However, there are some essential differences that must be taken into account when dealing with accretion-powered pulsars. Unlike spin-powered pulsars, which usually show one or more sharp, asymmetric pulses per cycle, there

7http://heasarc.gsfc.nasa.gov/docs/xte/recipes/pcabackest.html
is little harmonic content in the pulsations of SAX J1808 beyond \( k = 2 \), so we truncate the series there. Additionally, while the individual pulses of radio pulsars show appreciable variability from one period to the next, their integrated profiles are very stable (Manchester & Taylor 1977). In such cases, one expects that the pulse fractions of sample pulses are similar to the template \( (A_k/N_{ph} \approx A_k'/N_{ph}') \) and that the harmonics reflect a common phase residual \( (\Delta \phi_k \approx \Delta \phi) \) that traces the rotational phase of the star. Indeed, the standard template-matching analysis is predicated on these assumptions. Furthermore, any variability is assumed to be due to Poisson noise, which is of equal magnitude at all timescales (i.e., it is white noise). In contrast, the accretion-powered pulsars show substantial pulse profile variability. Beyond the usual Poisson noise \( (\sigma_k) \), three additional issues complicate the usual approach of template matching: long-timescale correlations (i.e., red noise) in the observed pulse fractions, with each harmonic’s \( A_k \) varying independently; red noise in the phase offsets \( \Delta \phi_k \); and sudden pulse profile changes, in which the phase offset between the two measured harmonics changes drastically on the timescale of the observations.

In their timing analysis of 283 s accretion-powered pulsar Vela X-1, Boynton et al. (1984) partially address the issue of intrinsic pulse profile noise in the harmonics. In the most general case, the amplitude of the variability in the phase residuals \( \Delta \phi_k \) is different for each harmonic \( k \). This is the case for both Vela X-1 and SAX J1808. They correct for the harmonic dependence of these fluctuations by scaling the phase residuals of each harmonic by constants chosen such that the phase residuals all have the same amplitude of variability (Boynton & Deeter 1985b). Thus the influence of particularly noisy harmonics was diminished, and they were able to measure with much greater accuracy the underlying spin of Vela X-1.

Our approach was similar. For each outburst, we measured the total rms amplitude of the phase residuals for each harmonic with respect to a best-fit constant-frequency model. These residuals will represent the combined effect of the Poisson noise and any intrinsic profile noise:

\[
\sigma_{k,\text{rms}}^2 = \langle \sigma_k^2 \rangle + \sigma_{k,\text{int}}^2.
\] (4.8)

We calculated the Poisson contribution \( \langle \sigma_k^2 \rangle \) as a weighted mean\(^8\) of the results from equation (4.4), giving us a value for \( \sigma_{k,\text{int}}^2 \). We then incorporate this

\(^8\)To calculate the mean of the variances \( \sigma_k^2 \), we weight each according to equation (4.7). Labeling each uncertainty as \( \sigma_{kl} \) for intervals \( l = 1, \ldots, N_{\text{int}} \), we have \( \langle \sigma_k^2 \rangle = \left( \sum_{l=1}^{N_{\text{int}}} \sigma_{kl}^{-2} / N_{\text{int}} \right)^{-1} \). This weighting scheme prevents large variances during the tails of the outbursts from skewing the results.
additional uncertainty into our weighting to determine $\Delta \phi$:

$$w_k = \frac{1}{\sigma_k^2 + \sigma_{k,\text{int}}^2}.$$  \hspace{1cm} (4.9)

$\sigma_k^2$ changes from one TOA measurement to the next due to the variability of the pulse fraction and count rate; there is no assumption that these are constant, as there is in the case of standard template fitting. $\sigma_{k,\text{int}}^2$ is a constant measured independently for each harmonic of each outburst. The result is a minimum-variance estimator for $\Delta \phi_k$. For instance, if the 802 Hz second harmonic has smaller intrinsic fluctuations than the fundamental, then our method presumes that it better reflects the spin of the NS and will weight it more strongly.

Sudden pulse profile changes are somewhat simpler to deal with. We use a different template pulse profile (and hence different measurements on either side of the change of $\phi_k$, defined in eq. [4.3]). We only modeled one such sudden profile change in this way: at the end of the main body of the 2002 outburst (around MJD 52576), the fundamental phase $\phi_1$ experienced a shift while the second harmonic, $\phi_2$, remained constant. The stability of $\phi_2$ allowed us to phase connect across the feature, as Burderi et al. (2006) also noted.

Our distinction between sudden profile changes and pulse profile noise is admittedly somewhat arbitrary. We make it solely in the interest of best estimating the rotational phase of the star — we are not claiming to model some underlying difference in physical processes. In 2002, the phase residuals on either side of the modeled pulse profile change were quite stable, albeit with different values of $\phi_1$. This stability in both harmonics makes it a good candidate for such treatment. In contrast, the phase residuals of both harmonics during the 2005 outburst show greater amplitude fluctuations at nearly all timescales. While its residuals and the 2002 residuals follow a similar pattern at the end of the main body of the outburst (the phases of the second harmonic remain roughly constant, while the phases of the fundamental drop appreciably), the phase of the fundamental continues to fluctuate wildly after this event rather than settling down on a “new” template profile. Therefore we elect to attribute these profile changes to intrinsic noise and weight the relatively stable second harmonic more strongly.

However, when the phases of both harmonics are continuously changing, the ability to define pulse arrival times breaks down, and the data are of little use for determining the spin of the star. For SAX J1808, the pulse profile is changing throughout the rises and peaks of the outbursts, so we excluded these data from our measurements of the spin frequency. We did include these data when calculating the orbital parameters. Since the timescale for these pulse profile changes ($\gtrsim 10$ hr) is many times the orbital period, they tend to average out and have little impact on these measurements.
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

The resulting phase residuals give the best estimator for the offset between the measured and predicted pulse arrival times. By adding these offsets to the phases predicted by the TEMPO ephemerides, we arrived at more accurate pulse arrival times for each interval.

4.2.4 Parameter fitting and uncertainty estimation

After measuring the pulse times of arrival, we input them into TEMPO to re-fit the timing solution. In order to interpret the resulting models, we must understand the nature of the noise in the TOAs and how it affects the model parameters. The harmonic weighting system described above makes the optimal choice to mitigate the phase variability due to a particularly noisy harmonic, but the TOA residuals are still of substantially greater magnitude than would be expected from Poisson noise alone. This leaves us with the task of estimating the fit uncertainties in the presence of such noise. These uncertainties are crucial to our construction of timing models, as they are needed to estimate the significance of fit components, such as frequency derivatives and instantaneous frequency changes.

We noted in the previous section that we treat the TOA residuals as noise, despite some of their variability arising from pulse shape changes that recur in every outburst. This is not a bad approximation: because we fit our models separately for each outburst, correlations in the pulse profile variability between outbursts are not relevant. Furthermore, the power spectra of the TOA residuals (see Fig. 4.4 in §4.3.3) resemble the power-law noise spectra typically observed in actual red noise processes, so treating it as such is reasonable.

We initially used the simplest possible timing model when fitting the TOAs of each outburst in TEMPO: a circular orbit and a constant frequency. (Note that we fit independent models for each outburst. The uncertainties were too large to phase connect between outbursts.) When this simple model proved insufficient to account for the phase residuals, we introduced a nonzero $\dot{\nu}$ and instantaneous frequency changes, as needed. However, there is a danger of overfitting the data. It is important to recognize that some of the features in the residuals are probably pulse profile variability rather than spin evolution. We took great care in our attempts to distinguish between the $\dot{\nu}$ measurements and the artifacts of intrinsic timing noise.

The colored nature of the timing noise in both harmonics is the primary difficulty in the interpretation of the parameter fits. TEMPO assumes that the TOA uncertainties one gives it are white and approximately Gaussian, as is the case of pure Poisson noise. As a result, it systematically underestimates the uncertainties in the fitted parameters in the presence of timing noise. Red timing noise is particularly problematic, because it dominates on the long
timescales on which $\nu$ and $\dot{\nu}$ measurements depend.

Instead of adopting this white noise assumption of TEMPO, we estimated confidence intervals for $\nu$ and $\dot{\nu}$ using Monte Carlo simulations of the timing residuals of each outburst. After using TEMPO to obtain the best fit for a timing model, we calculated the power spectrum $P(f)$ of the timing residuals that TEMPO output. This spectrum is a convolution of the true noise spectrum and the sampling function; most notably, there is excess power around 1 d, an artifact of RXTE observations often being scheduled approximately a day apart, and at the RXTE orbital period of 96 min due to Earth occultations. We applied a low-pass filter to remove these peaks in an attempt to approximate the underlying noise spectrum, $P'(f)$:

$$P'(f) = P(f) \times \left[(1 - A) \exp(-f^2\tau_c^2) + A\right]. \quad (4.10)$$

$\tau_c$ gives the time scale for the low-pass cutoff. $A$ gives the fraction of high-frequency noise to let through, reproducing the short-timescale scatter (principally but not entirely Poisson) that we observed within each observation. Typical values were 3 d and 10–20%.

We then created thousands of sets of artificial phase residuals with the noise properties of the filtered spectrum, $P'(f)$. To reproduce the sampling irregularities, we removed all points at times absent in the original data. The parameters of the low-pass filter were tuned such that the mean power spectrum of the resulting Monte Carlo residuals was as close as possible to the original power spectrum, $P(f)$. For each set of residuals, we measured the frequency of the best linear fit (or, if our TEMPO model fit for $\dot{\nu}$, the frequency derivative of the best quadratic fit). The standard deviations of these measurements provided uncertainty estimates for the respective parameter, this time more accurately accounting for the noise spectrum.

We relied solely on TEMPO to calculate the uncertainties in the binary orbit parameters. While the intrinsic pulse profile noise spectrum is colored on the timescale of days to weeks, the phase residuals are approximately white on timescales equal to and shorter than the 2 hr orbital period. The amplitude of the short-timescale variability is roughly 1.5 times what one would expect from counting noise alone, so we scaled our Poisson-derived phase uncertainties accordingly when estimating the uncertainties of the orbital parameter fits. This rescaling makes TEMPO’s uncertainty estimates for the orbital parameters reasonably accurate. One important consistency check of this simple approach worked nicely: the 1998, 2002, and 2005 measurements of $P_{\text{orb}}$ and $a_\alpha \sin i$, two parameters that should be the same for each outburst at our level of accuracy, were indeed found to be constant, with reduced $\chi^2$ statistics close to unity.

In deriving new binary and spin parameters, the new values sometimes differed considerably from the parameters with which we initially folded the
The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658.

Figure 4.1: The light curves, phase residuals, and fractional amplitudes for all four outbursts. The top panel shows the fractional amplitudes of the fundamental and second harmonic. Positive phases indicate pulse arrivals later than predicted by the phase model. The error bars reflect the statistical errors only, as outlined in equation (4.4). The black points indicate the times of observations. The black lines indicate the best-fit constant-frequency model for each outburst. The top panel shows the fractional amplitudes of the fundamental and second harmonic. The strips along the top of the graphs indicate the times of observations.
4.3 Results

The results of our pulse timing solutions are shown in Figure 4.1, which compares the light curves, phase residuals, and fractional amplitudes for each outburst. Inspecting the best-fit frequency lines in the phase residual plots, it is clear that a constant pulse profile attached to a constant-frequency rotator does not adequately describe the observed residuals. We consider five sources of phase residuals relative to a best-fit constant-frequency model: Poisson timing noise, intrinsic pulse profile noise, sudden and well-defined pulse profile changes, additional spin frequency derivatives, and instantaneous frequency changes in the underlying rotation of the star. In this section, we will consider all these possible contributions to the residuals and their relationships with each other and the other properties of each outburst.

4.3.1 Light curves of the outbursts

The light curves of each outburst are quite similar in shape. We divide them into four stages: the rise, which was only definitively captured in 2005 and took ≈5 d; the short-lived peak at a 2–25 keV flux of $(1.9 \pm 2.6) \times 10^{-9}$ erg cm$^{-2}$ s$^{-1}$, equal to a luminosity of $(4.7 \pm 6.4) \times 10^{35}$ erg s$^{-1}$ using the distance of 3.5 kpc and bolometric correction of $L_{\text{bol}}/L_{2-25\text{ keV}} = 2.12$ derived by Galloway et al. (2006); a slow decay in luminosity, lasting 10–15 d, until the source reaches approximately $8 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ ($= 2.0 \times 10^{35}$ erg s$^{-1}$); and a sudden drop followed by low-luminosity flaring as the outburst flickers out, with the timescale between flares on the order of 5 d. Figure 4.2 shows a cartoon of a typical outburst from SAX J1808, with each of these stages labeled.

---

The orbit introduces a periodic frequency modulation with amplitude $\Delta \nu = \nu_0 \cdot 2\pi a_x \sin i/c_{\text{orb}} > 1/512$, an inaccurate orbital ephemeris can significantly reduce detection strength. In contrast, the spin frequency is remarkably stable through all the observations, so there was no need to recalculate TOAs upon the relatively minor revisions to the spin model.
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

Figure 4.2: The anatomy of a typical outburst from SAX J1808. The features of the light curve and their fluxes and timescales are similar to those observed during the 1998, 2002, and 2005 outbursts. Bolometric luminosities assume a distance of 3.5 kpc and a correction of $L_{\text{bol}}/L_{2-25 \text{ keV}} = 2.12$ (Galloway et al. 2006).

The RXTE first collected high-resolution timing data from SAX J1808 during the 1998 April outburst. Data from the RXTE All Sky Monitor (ASM) show that the peak luminosity occurred approximately three days before the first PCA observation. (See Fig. 2 of Galloway et al. 2006 for a comparison of the ASM and PCA light curves; note that its 1998 plot does not include two raster scans analyzed here.) Unfortunately, PCA observations stopped shortly after the body of the outburst and do not sample the tail.

SAX J1808 was discovered to be again in outburst when it emerged from behind the Sun in 2000 January. Coverage of the outburst was limited and included only the outburst tail. Wijnands et al. (2001) comment on the erratic nature of the flaring during the tail. ASM data indicate that the peak occurred 2 weeks prior to the first PCA observation and that we are observing the later, dimmer stage of the flaring tail. Comparison of the PCA data with the 2002 and 2005 outbursts suggests likewise.

The 2002 outburst, detected in mid-October and observed for the next two months, was the brightest, had the best PCA coverage, and included the
4.3 Results

detection of four extremely bright thermonuclear X-ray bursts during its peak. Its light curve was very similar in shape to the 1998 outburst.

In 2005 June, SAX J1808 was again in outburst. This time, the detection preceded the peak by a few days, providing a full sampling of the light curve. This outburst was somewhat dimmer, with a peak luminosity of only 70% of the 2002 peak and a correspondingly shorter slow-decay stage. The subsequent rapid decay and flaring tail look quite similar to the other outbursts.

4.3.2 Characteristic pulse profile changes

Just as the light curves of each outburst were quite similar, the evolution of the pulse profile during each outburst was remarkably consistent. Figure 4.3 illustrates the full range of pulse profiles that we observed from SAX J1808. In many instances, the similarity of the pulse profiles between outbursts is quite striking. In this section, we describe how these profiles change throughout the outbursts.

We observed the outburst rise, labeled as profile 1 in Figure 4.3, exclusively during the 2005 outburst. The profiles are smooth and asymmetric, with a slow rise followed by a more rapid drop-off after the peak. There is no sign of a second peak.

We observed the outburst maxima during 2002 and 2005. The similarity of the pulse profile evolution between the two outbursts is remarkable. During the first half of the maxima (labeled as profile set 2), the profiles show a secondary bump lagging the main pulse. Compared to the burst rise, the fractional amplitude has decreased somewhat. During the second half of the maximum, the pulse becomes broader, subsuming the lagging secondary bump. (See profile set 3.) This change appears to be gradual: in both outbursts, a mid-peak observation exhibited an intermediate pulse profile.

Profile set 4 shows the pulse profiles during the slow decay stages of the outbursts. The 1998 and 2002 profiles are quite similar: the pulses are somewhat asymmetric, rising more steeply than they fall. In both outbursts, this pulse profile is very stable during the approximately 10 d of the decay in luminosity. During the 2005 outburst, this asymmetry is more pronounced, and the profile varies between observations. Initially, the pulse exhibited a small lagging bump (profile 4A), quite similar to the pulse profile during the first half of the outburst maximum. The relative size of that bump varied substantially, in some observations appearing as a small secondary peak (profile 4B). Over the course of the decline, the source switched back and forth between a double-peaked and single-peaked profile as indicated in the figure. A given state would typically be seen for two or three observations (1–2 d) before switching to the other. Profile set 5 covers the rapid drop in flux at
the end of the outbursts. During 1998, the pulse profile was quite stable and did not appreciably change during this drop, although its fractional amplitude increased somewhat. In contrast, the 2002 and 2005 outbursts show a major pulse profile shift concurrent with the drop in luminosity. Prior to the drop, the pulses in set 4 show a quick rise and a slower fall. After the drop, the asymmetry of the 2002 and 2005 profiles reverses: profiles 5B show a slow rise and a quick drop. In terms of harmonic components, these changes represent a shift in the phase of the fundamental by approximately 0.15 cycles as it went from leading the second harmonic to lagging behind it. The phase of the harmonic did not change. In both outbursts, observations during the ≈2 d of rapid luminosity decline reveal an intermediate stage in which the main pulse is momentarily symmetric (profiles 5A). During this transition, small but significant secondary pulses are present.

During the flaring tail of the outburst (profile set 6), the pulse profile again showed substantial variability. In 2002, the profile repeatedly switched between an asymmetric pulse (profile 6A, identical to the pulse profile at the end of the rapid dimming stage) and a double-peaked profile (profile 6B). The double peaked pulse profile occurs principally (but not exclusively) at the end of the flares, as their luminosity declines. These pulse profile changes are almost entirely the result of changing fractional amplitudes of the harmonic components; the phase offset between the fundamental and second harmonic remains for the most part constant. A notable exception occurs during the decay of the first flare at around MJD 52582. At this time the phase of the fundamental jumped by ≈0.2 cycles, indicating a sudden lag of this amount behind its previous arrival time. By the next observation, less than two hours later, the phase residual of the fundamental returned to its previous value.

The tail of the 2005 outburst is more chaotic. The fractional amplitudes and phases both exhibit strong red noise, producing a pulse profile that is sometimes asymmetric with a slow rise and quick fall (6A); at other times asymmetric with a quick rise and slow fall (6C; not shown, but basically just the reverse of profile 6A); and in one instance clearly double-peaked (6B). The observations were sparse and generally short, so it was impossible to better characterize the evolution of these pulse profile fluctuations. The flaring tail of the 2000 outburst was quite similar, with a highly variable pulse profile that included double-peaked profiles and asymmetric single pulses of both orientations. We did not include it because the observations were few and sparse.
4.3 Results

Figure 4.3: A comprehensive view of the 2–15 keV pulse profiles observed from SAX J1808. Each pulse profile was calculated by folding the observations within the indicated time intervals using the best-fit constant-frequency model of each outburst, so any movement of the peaks reflects the phase offsets from the constant frequency. The profiles are background-subtracted, normalized such that the phase bins have a mean value of unity, and plotted on 0.80–1.20. Thus the plotted profiles accurately show the change in fractional amplitude during the outburst. The profiles are numbered according their position within the outburst: 1 indicates the burst rise; 2, the beginning of the outburst maximum; 3, the end of the maximum; 4, the slow decay stage; 5, the steep luminosity drop marking the end of the main outburst; and 6, the flaring tail. During some parts of the burst, two pulse profiles are present, with the source switching between them. In these cases, we show both profiles and label the regions of the light curve in which they occurred accordingly. The solid black line shows the fluxes from the PCA observations; the grey boxes show the fluxes from the ASM daily averages.
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

Table 4.2: Noise properties of the outbursts

<table>
<thead>
<tr>
<th>Date</th>
<th>Fundamental</th>
<th>Second Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\langle \sigma_1^2 \rangle^{1/2}$</td>
<td>$\sigma_{1,\text{int}}$</td>
</tr>
<tr>
<td>1998 Apr</td>
<td>0.007</td>
<td>0.014</td>
</tr>
<tr>
<td>2000 Feb</td>
<td>0.023</td>
<td>0.052</td>
</tr>
<tr>
<td>2002 Oct</td>
<td>0.012</td>
<td>0.016</td>
</tr>
<tr>
<td>2005 Jun</td>
<td>0.013</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Note. — All phases are in cycles (i.e., fractions of the 2.5 ms spin period). $\langle \sigma_k^2 \rangle^{1/2}$ gives the mean contribution of Poisson noise; $\sigma_{k,\text{int}}$ is the amplitude of pulse profile variability in excess of the Poisson noise; and $\gamma_{\text{PLN}}$ is the slope of the power law best fit to the spectrum of $\sigma_{k,\text{int}}$. We did not attempt to estimate power law noise slopes for the 2000 outburst because of its low-quality data.

4.3.3 Noise properties of the timing residuals

To measure the spin phase of SAX J1808 using the formalism developed in §4.2.3, we must characterize the variability of the harmonic components. This variability encompasses both the pulse profile changes discussed in the previous section as well as any noise in the spin phase of the star.

In our analysis of the phase residuals, we took into account the rms amplitude of the intrinsic pulse profile noise in each harmonic, $\sigma_{k,\text{int}}^2$, defined in equation (4.8). A casual glance at the phase residuals of Figure 4.1 reveals that the magnitudes of these fluctuations vary substantially between outbursts. Table 4.2 summarizes these amplitudes for each outburst and compares them to the mean amplitudes of their Poisson noise, $\langle \sigma_k^2 \rangle^{1/2}$. These values are then used in equation (4.9). For instance, in the 2002 outburst the fundamental is more heavily weighted in measuring the spin phase than the second harmonic, while in 2005 the opposite is true.

The scatter of the phase residuals between observations is generally greater than the scatter within an observation, suggesting that the pulse profile noise is red. Power spectra of the phase residuals, shown in Figure 4.4, confirm this. We estimated these power spectra using Fourier transforms of the residuals from equally spaced 512 s bins. Here we have not attempted to deconvolve the uneven sampling periodicities at 1 d and 96 min due to the RXTE observation schedule and orbit. There are no peaks at the 2 hr binary orbital period, indicating that the pulse profile is independent of orbital phase.

The resulting noise powers are around 2 decades higher at long periods
4.3 Results

Figure 4.4: Power spectra of the phase residuals for the fundamental (black squares) and the second harmonic (grey circles) relative to the best-fit constant-frequency models. (The 2002 model also includes a phase shift in the fundamental to account for the profile change at MJD 52577.) The dashed lines show the power level due to counting statistics, a white-noise contribution proportional to $\langle \sigma_k^2 \rangle$. The data points show the powers $P_k(f)$, from which we have subtracted the contribution of counting statistics. These powers are normalized such that $\int_{10^{-3} \text{ Hz}}^{10^{-7} \text{ Hz}} P_k(f) \, df = \langle \sigma_{k, \text{int}}^2 \rangle$, as defined in equation (4.8). The vertical dotted lines show the relevant time scales for the spectra: the 96 min and $\approx 1$ d periodicities of the RXTE observations, and the 121 min SAX J1808 orbital period.

($\approx 3$ d or longer) than at short periods for 1998, and even more for 2005. The 2002 outburst spectra exhibit less profile noise at long timescales, but still are somewhat red. Poisson statistics produce an uncolored lower limit.
on noise. This white noise dominates at timescales shorter than the orbital period, except in the case of the particularly noisy fundamental of 2005. The spectra of the intrinsic profile noise (i.e., the spectra after subtracting off the Poisson contribution) roughly followed a power law noise spectrum, which we parametrized as $P_k(f) \propto f^{-\gamma_{\text{PLN}}}$. The best-fit values of $\gamma_{\text{PLN}}$, listed in Table 4.2, varied from roughly 0.4 to 1.

### 4.3.4 Fractional amplitudes of the harmonics

In our time-domain discussion of the pulse profiles, an apparent trend is the tendency of the pulses to become narrower, more asymmetric, or doubly peaked — generally speaking, to become less sinusoidal — as the outburst’s flux decreases. In the frequency domain, the relation is striking: the fractional amplitude of the 802 Hz second harmonic, $r_2$, strongly anticorrelates with the

**Figure 4.5:** The fractional amplitude of the second harmonic scales with flux according to a power law of slope $-0.50 \pm 0.01$, shown by the dashed line. Each point gives the mean amplitude and flux for a single observation. The scatter is commensurate with the mean uncertainty in fractional amplitude, which is shown by the error cross in the upper right.
background-subtracted 2–25 keV flux, $f_x$, as shown in Figure 4.5. This power-law dependency has a slope of $-0.50 \pm 0.01$. The agreement with the data is excellent for such a simple model, giving a reduced $\chi^2$ statistic of $\chi^2_{\nu} = 1.15$ with 1816 degrees of freedom. It spans two and a half decades in luminosity and includes every detected harmonic amplitude from all four outbursts.

In terms of the pulse profile, the second harmonic contributes in two ways. If its peak is 45° out of phase with the peak of the fundamental, it will produce an asymmetric pulse profile (e.g., profile 6A in Fig. 4.3). If it is in phase, a narrower primary pulse with a small second peak will result (as in profile 6B). If the components are 90° out of phase, the profile will be profile 6B flipped, but we never observed such a configuration.

To further understand the influence of flux on the pulse profile, we decomposed the second harmonic’s fractional amplitude into its asymmetric and double-peaked components,

$$r_{2,\text{asym}} = r_2 |\sin 4\pi \psi|$$
$$r_{2,\text{dp}} = r_2 |\cos 4\pi \psi|,$$ (4.11)

where $\psi$ is the phase offset between the peaks of the two harmonics: $\psi = (\phi_2 + \Delta \phi_2) - (\phi_1 + \Delta \phi_1)$. The resulting plots have substantially more scatter than Figure 4.3 due to the uncertainty of $\psi$, which is considerable, particularly at low fluxes. However, they both roughly conform to the $r_2 \propto f_x^{-1/2}$ power law. We conclude that the decrease in flux increases the asymmetry of the pulses and the presence of secondary pulses in approximately equal measure.

In contrast, the fractional amplitude of the fundamental behaves unpredictably. During the slow-decay stage of 1998, it is unvarying and strong, at a constant 5.5% rms. During this stage of 2002, it is weaker (4%) and somewhat variable; during 2005, it is weaker still and erratically changing by up to a full percent between observations. Its behavior is more consistent in the tail. In all outbursts, the fractional amplitude of the fundamental varies widely, usually (but not always) having its maxima around the peaks of the flares and its minima during the fading portion of the flares.

For the most part, a pulse profile model only including the fundamental and second harmonic adequately describes the folded profiles. However, folding long stretches of data does sometimes result in the detection of a third harmonic with fractional amplitudes ranging up to $\approx 0.25\%$ rms. We do not reliably detect any higher harmonics.

### 4.3.5 Upper limits on the subharmonics

With some assumptions, we can strongly constrain the presence of subharmonics and half-integral harmonics. The most straightforward approach is to
Table 4.3. Upper limits on subharmonics and half-integral harmonics

<table>
<thead>
<tr>
<th>Harmonic Factor</th>
<th>Upper limita (%) rms</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>100.2</td>
<td>0.017</td>
<td>0.019</td>
<td>0.52</td>
</tr>
<tr>
<td>1/2</td>
<td>200.5</td>
<td>0.022</td>
<td>0.024</td>
<td>0.45</td>
</tr>
<tr>
<td>3/2</td>
<td>601.5</td>
<td>0.018</td>
<td>0.021</td>
<td>0.43</td>
</tr>
<tr>
<td>5/2</td>
<td>1002.4</td>
<td>0.026</td>
<td>0.024</td>
<td>0.42</td>
</tr>
</tbody>
</table>

a These background-corrected upper limits are quoted at the 95% confidence level. These limits result from combining all the observations (column A), combining only bright observations (B), and not combining any observations (C). See the text for more details.

b Frequencies listed here are approximate. The upper limits were obtained using exact multiples of the best-fit constant-$\nu$ models.

fold all the observations using multiples of the best-fit frequency models from each outburst. The amplitude of the resulting profile will give an upper limit. The resulting 95% confidence upper limits are listed in column A of Table 4.3. However, this approach is only statistically valid if the uncorrected fractional amplitude (i.e., the fractional amplitude relative to the source counts and the background) is constant. Clearly this assumption is false. Aside from the varying proportion of source photons, the background-subtracted fractional amplitudes of the fundamental and the second harmonic fluctuate throughout the outburst, spanning nearly an order of magnitude in the tails of the burst. There is no reason to believe that a subharmonic would not fluctuate similarly.

Column B of Table 4.3 takes the more moderate approach of only folding together observations during which the 2–25 keV flux exceeds the value of $5 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$, thereby only including the main body of the outbursts. Background photons are thus a much smaller contribution, and the fractional amplitudes of the observed two harmonics were relatively stable during these times. Nevertheless, we still are folding enough photons to obtain very stringent upper limits: in the case of the 200 Hz subharmonic, we get a 95% confidence upper limit of 0.024% rms. We feel that these numbers are our most reliable, not making unreasonable assumptions about the fractional
amplitude fluctuations.

For completeness, we also include the most conservative upper limits, which make no assumptions whatsoever about the fractional amplitudes of the subharmonics. For instance, it would be possible in principle for the subharmonic to be present only during a single observation and zero-amplitude everywhere else. To constrain the resulting upper limits at least somewhat, we again only used observations during which the source was brighter than $5 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ and that had at least $10^6$ counts. These single-observation limits are tabulated in column C.

The stringent upper limits of column B provide the best evidence yet that the spin frequency of the star is indeed 401 Hz. If the star was spinning at 200.5 Hz, with two antipodal hot spots each emitting pulses to produce the observed frequency, a 200.5 Hz subharmonic would almost certainly be present.

### 4.3.6 Spin frequency measurements and constraints

We initially performed the simplest possible fits to the phase residuals of each outburst: constant-frequency models. We did not include the data at the very beginning of the 2002 and 2005 outbursts, where pulse profile changes during the rise and peak obscure any variations in the phase. We also excluded the residuals during 2002’s mid-outburst pulse profile change, but included the residuals of the fundamental on both sides of the shift by using different profile templates before and after it.

The resulting frequency measurements are shown in Figure 4.6 and summarized in Table 4.4. These data clearly indicate that the source is spinning down. The probability that the actual spin frequency is constant or increasing is less than $10^{-9}$ given the uncertainty estimates. These uncertainties do assume that our optical position is exact, but the position error is excluded because its effects are highly correlated; for instance, the 1998 and 2002 outbursts are six months apart on the calendar, so a position offset would produce equal and opposite frequency displacements for the measurements from these outbursts. There is no position that would provide a statistically feasible constant or increasing frequency.

The linear fit through the measured frequencies is not particularly good: its $\chi^2$ statistic is 9.7 with 2 degrees of freedom, yielding a probability of about 1% that the frequencies are drawn from a linear progression. Once again, changing the source position does not significantly change the result or improve the fit, and changes in the position by more than the $1\sigma$ uncertainty along the ecliptic substantially worsen the linear fit. To estimate the uncertainties of the linear slope in light of this poor fit, we rescaled the measurement errors.
Figure 4.6: Constant-frequency measurements of the SAX J1808 outbursts, showing the spin down of the star. The frequencies are relative to $\nu_0 = 400.97521000$ Hz. The error bars are estimated using Monte Carlo simulations of phase residuals with the same noise properties as the actual outburst; they do not account for the uncertainty in the source position. The $\times$'s mark what the frequencies would be if the fit source position differed from the actual position by 2$\sigma$ along the ecliptic plane in the direction of increasing RA. The same position error in the decreasing RA direction would move the frequency points by an equal amount in the opposite sense.

such that reduced $\chi^2$ statistic would be unity. The resulting first-order spin derivative is $\dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16}$ Hz s$^{-1}$. The large 1$\sigma$ uncertainty reflects the uncertainty in the slope of the frequency change, not in the observation that the source is spinning down. The probability that the frequency is not decreasing is less than $10^{-9}$, as mentioned above, a confidence of better than 6$\sigma$.

Fitting second-order frequency models established that $\dot{\nu}$ is consistent with zero during all the outbursts. These measurements are particularly sensitive to pulse profile variations, so care must be taken to not overfit such features. We again exclude the initial observations of the 1998, 2002, and 2005 outbursts, because the pulse profile changes would induce large non-zero $\dot{\nu}$ measurements that most likely do not reflect the spin of the underlying neutron star. Using
Table 4.4. Best-fit constant frequencies, and their $\dot{\nu}$ upper limits

<table>
<thead>
<tr>
<th>Data included (MJD)</th>
<th>$\nu - \nu_0$ a ($\mu$Hz)</th>
<th>$\dot{\nu}$ b ($10^{-14}$ Hz s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 Apr 50914.8 – 50936.9</td>
<td>0.371 ± 0.018 (−7.5, 7.3)</td>
<td></td>
</tr>
<tr>
<td>2000 Feb 51564.0 – 51601.9</td>
<td>0.254 ± 0.012 (−1.1, 1.4)</td>
<td></td>
</tr>
<tr>
<td>2002 Oct 52565.0 – 52602.8 c</td>
<td>0.221 ± 0.006 (−1.3, 2.5)</td>
<td></td>
</tr>
<tr>
<td>2005 Jun 53529.6 – 53581.5</td>
<td>0.195 ± 0.016 (−0.5, 2.4)</td>
<td></td>
</tr>
</tbody>
</table>

a The frequencies are relative to $\nu_0 = 400.975210$ Hz.
b 95% confidence intervals from the Monte Carlo simulations.
c Excluding MJD 52575.7–52577.7.

TEMPO to find the best-fit $\dot{\nu}$’s and applying Monte Carlos to estimate their uncertainties, we arrived at the 95% confidence intervals of Table 4.4. Excluding the 1998 outburst, which had the shortest span of timing data and thus the most poorly constrained $\dot{\nu}$, these 95% confidence upper limits were all of order $|\dot{\nu}| \lesssim 2.5 \times 10^{-14}$ Hz s$^{-1}$.

The uncertainties in the measurement of the frequency preclude phase connection between outbursts. During the 920 d gap between the 2002 and 2005 outbursts, the 6 nHz frequency uncertainty from 2002 would accumulate to a phase uncertainty of 0.5 cycles; the $2 \times 10^{-16}$ Hz s$^{-1}$ uncertainty in the long-term spin down would contribute 0.6 cycles. Worse, these estimates are best-case scenarios, since they assume that the spin down is constant.

During the 1998 and 2002 outbursts, we observed an abrupt change in the slope of the phase residuals at the end of the main outburst. We modeled these apparent instantaneous changes of frequency by including frequency glitches in our TEMPO fits. (While the TEMPO glitch models are useful in describing the data, we do not believe that we observed actual sudden changes in the spin frequency of the star, a point discussed in detail in §4.4.3.) Figure 4.7 shows the phase residuals of these outbursts and their best-fit glitch models. These models only employ an instantaneous change in frequency; including a phase jump or introducing a $\dot{\nu}$ after the events did not significantly improve the fits.\(^\dagger\)

\(^\dagger\)During the 2002 outburst, the absence of a phase jump refers only to the second harmonic, which we believe is a better tracer of the neutron star spin during this period of time (in agreement with the conclusions of Burderi et al. 2006).
Figure 4.7: Comparison of the 1998 and 2002 glitch-like events. The phase plots show the phase residuals relative to a constant-frequency model for the fundamental (black points) and the harmonic (grey points), binned such that there is one point per observation. The black lines indicate the best timing models fit by TEMPO. The middle plot shows the 1998 and 2002 light curves for comparison. The 1998 light curve has been vertically offset by 1 erg cm$^{-2}$ s$^{-1}$ for clarity. The data are displayed such that the apparent changes in frequency are aligned. Notice that this alignment also has the effect of closely matching up the light curves.

den drop in flux that marks the transition from the slow-decay stage to the flaring tail stage. At the same time, the fractional amplitudes of the fundamental and harmonic increase, and, in the case of 2002, the pulse profile change occurs. (This pulse profile change, discussed earlier in §4.3.2, is apparent in Fig. 4.7 as the rapid advance of the fundamental phase.) If we view the phase residuals with respect to the pre-transition frequencies, as is the case in Figure 4.7, the residuals following the transition skew upward, indicating
progressively increasing lags. This effect is more pronounced in 1998, but its coverage is far better in 2002. If we were to interpret these changes in slope as abrupt spin frequency changes, they would represent drops of 0.21 µHz and 0.03 µHz for 1998 and 2002, respectively. (Again, we consider this scenario unlikely; see §4.4.3.) If we instead interpret them as the motion of a radiating spot, the drift rates would be 6.5° d\(^{-1}\) and 1.0° d\(^{-1}\), retrograde. The total observed shifts between the start of the flaring tail and the loss of the signal are substantial: 0.15 cycles (54°) in 1998 and 0.06 cycles (22°) in 2002. The data are not good enough to distinguish whether these drifts are continuous. For instance, it is possible that the hot spot made a retrograde jump every time there was a flare.

We did not observe the main body of the 2000 outburst, so we cannot measure whether the apparent frequency decreased when it entered the flaring tail stage. But if it did, and if the decrease in the apparent frequency was of similar magnitude to that observed in 1998 and 2002, then including the main body of the 2000 outburst would raise the overall frequency of the outburst somewhat. This correction might put it in line with the other frequency measurements in Figure 4.6, reducing the large \(\chi^2\) statistic of the constant-\(\dot{\nu}\) fit. Therefore we cannot conclude that the change in the observed frequency from one outburst to the next is incompatible with a linear progression.

During the 2005 outburst, the substantial pulse profile noise during the tail prevented us from measuring a change in apparent frequency. The uncertainty in the measurement of the frequency during the tail was 0.03 µHz, as estimated using Monte Carlo simulations of the profile noise, and the phase residuals jumped by as much as 0.1 cycles from one observation to the next. If there was a smaller drift, as seen during the 2002 outburst, we would not necessarily detect it.

### 4.3.7 Evolution of the binary orbit

We fit the orbital parameters separately for each outburst. Table 4.5 lists the results. As expected, the values of \(a_x \sin i\) and \(P_{\text{orb}}\) were consistent among the outbursts. The fit parameters \(e \sin \omega\) and \(e \cos \omega\) were consistent with zero. We used them to improve significantly on previous upper limits on the eccentricity.

The measured time of ascending node advanced with each outburst, relative to the times expected if the period was constant. Figure 4.8 shows these \(T_{\text{asc}}\) residuals. A quadratic provides a good fit (\(\chi^2 = 1.01\) with a single degree of freedom), yielding a constant orbital period derivative of \(\dot{P}_{\text{orb}} = (3.5 \pm 0.2) \times 10^{-12} \text{ s s}^{-1}\) and a significance of 15.6 \(\sigma\). In an independent analysis of the same data, di Salvo et al. (2008) report a consistent value for \(\dot{P}_{\text{orb}}\). They derive a smaller uncertainty and larger \(\chi^2\), most likely reflecting an underestimate of
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

Figure 4.8: Measurement of an orbital period derivative. The points show the observed times of ascending node, relative to the expected times for a constant period. The $T_{\text{asc}}$ of each outburst comes progressively later, indicating a period derivative of $(3.5 \pm 0.2) \times 10^{-12}$ s s$^{-1}$.

Table 4.6 summarizes all the parameters for the pulse timing of SAX J1808.

4.4 Discussion

Our analysis of multiple outbursts from SAX J1808 allows us to greatly improve our understanding of the behavior of this low-mass X-ray binary. By comparing the observed frequency from each outburst, we can see the long-term spin down, which is too small to be detectable from a single outburst. Comparison of the pulse profiles from each outburst lead us to conclude that we are seeing characteristic, repeated profile changes as the outbursts progress, rather than a purely random noise process. Finally, fitting of the orbital parameters over the seven years of observation provides a greatly improved orbital ephemeris.
4.4 Discussion

**Table 4.5:** Binary parameter measurements from each outburst

<table>
<thead>
<tr>
<th>Outburst</th>
<th>$P_{\text{orb}}$ (s)</th>
<th>$a_\times \sin i$ (light-ms)</th>
<th>$T_{\text{asc}}$ (MJD, TDB)</th>
<th>$e \sin \omega$ (10$^{-6}$)</th>
<th>$e \cos \omega$ (10$^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 Apr</td>
<td>7249.1565(18)</td>
<td>62.8080(46)</td>
<td>50921.7584194(12)</td>
<td>$-60 \pm 64$</td>
<td>$-86 \pm 64$</td>
</tr>
<tr>
<td>2000 Feb</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2002 Oct</td>
<td>7249.156(6)</td>
<td>62.8147(31)</td>
<td>52570.0186514(9)</td>
<td>8 $\pm$ 57</td>
<td>41 $\pm$ 57</td>
</tr>
<tr>
<td>2005 Jun</td>
<td>7249.1547(24)</td>
<td>62.8283(109)</td>
<td>53524.9944192(32)</td>
<td>$-173 \pm 83$</td>
<td>$53 \pm 83$</td>
</tr>
</tbody>
</table>

Note. — We excluded the 2000 outburst when calculating everything but $T_{\text{asc}}$ because its data were noisy and sparse.

**Table 4.6:** Combined timing parameters for SAX J1808

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital period, $P_{\text{orb}}$ (s) $^a$</td>
<td>7249.1566961(14)</td>
</tr>
<tr>
<td>Orbital period derivative, $\dot{P}_{\text{orb}}$ (10$^{-12}$ s s$^{-1}$)</td>
<td>3.48(23)</td>
</tr>
<tr>
<td>Projected semimajor axis, $a_\times \sin i$ (light-ms)</td>
<td>62.8132(24)</td>
</tr>
<tr>
<td>Time of ascending node, $T_{\text{asc}}$ (MJD, TDB)</td>
<td>52499.9602477(10)</td>
</tr>
<tr>
<td>Eccentricity, $e$ (95% confidence upper limit)</td>
<td>$&lt; 1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Spin frequency, $\nu$ (Hz) $^a$</td>
<td>400.975210240(11)</td>
</tr>
<tr>
<td>Spin frequency derivative, $\dot{\nu}$ (10$^{-16}$ Hz s$^{-1}$)</td>
<td>$-5.6(2.0)$</td>
</tr>
</tbody>
</table>

$^aP_{\text{orb}}$ and $\nu$ are specified for the time $T_{\text{asc}}$.  

87
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

4.4.1 Long-term spin down

By observing the mean spin frequency of each outburst, we found that SAX J1808 is spinning down at a rate of \( \dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16} \) Hz s\(^{-1}\). This spin down results in a loss of rotational energy at a rate of \( \dot{E} = 4\pi^2 I \dot{\nu} \dot{\nu} = 9 \times 10^{33} \) erg s\(^{-1}\), assuming a canonical value of \( I = 10^{45} \) g cm\(^2\) for the neutron star (NS) moment of inertia.

Most of this spin down occurs during X-ray quiescence; accretion torques during the outbursts play a minimal role. Over the seven years of our observations, the mean outburst frequency decreases by \( \nu_{2005} - \nu_{1998} = -0.18 \pm 0.02 \) \( \mu \)Hz. Let us suppose that this frequency change happens only during the X-ray outbursts (which have a duty cycle of \( \lesssim 5\% \)). Since the outburst light curves are quite similar, it is reasonable to presume that each would contribute roughly the same frequency shift, thus splitting this frequency change into three equal steps. If the spin down is due to a constant \( \dot{\nu}_{\text{outburst}} \) that acts during the \( \approx 20 \) d of each outburst,\(^{11}\) then

\[
\dot{\nu}_{\text{outburst}} \approx \frac{-0.18 \mu \text{Hz}}{3 \times 20 \text{ d}} = -3.5 \times 10^{-14} \text{ Hz s}^{-1}. \quad (4.12)
\]

By contrast, we were able to set stringent (95% confidence) upper limits of \( |\dot{\nu}| \lesssim 2.5 \times 10^{-14} \) Hz s\(^{-1}\) during the outbursts (Table 4.4). We conclude that the spin down is dominated by torques exerted during X-ray quiescence.

We will thus consider three possible sources of torque during quiescence: magnetic dipole radiation, the expulsion of matter by the magnetic field (i.e., the propeller effect), and gravitational radiation. In general, we assume that all three mechanisms contribute additively to the observed spin down of SAX J1808,

\[
N_{\text{obs}} = N_{\text{dipole}} + N_{\text{prop}} + N_{\text{gr}}. \quad (4.13)
\]

We discuss each below.

**Magnetic dipole torque**

A spinning dipolar magnetic field will produce a significant spin down during quiescence for the \( 10^8 \) G field strengths expected for a millisecond pulsar. Relativistic force-free MHD models of pulsar magnetospheres by Spitkovsky (2006) give a torque of \( N_{\text{dipole}} = -\mu^2(2\pi \nu/c)^3(1 + \sin^2 \alpha) \), where \( \mu \) is the magnetic dipole moment and \( \alpha \) is the angle between the magnetic and rotational

\(^{11}\)In reality, \( \dot{\nu} \) would almost certainly not be constant during as the accretion rate varies, but for argument’s sake we make the most conservative assumptions possible. A varying \( \dot{\nu} \) would require that it be sometimes greater than the value from equation (4.12), making it even less plausible that it would escape detection.
poles. Pulse profile modeling of the 1998 outburst by Poutanen & Gierliński (2003) suggests that the magnetic hot spot is not far from the rotational pole, separated by an angle of 5–20°. While other effects might also contribute to the spin down, the rotating magnetic field will always be present and provides an upper limit on the dipole moment:

$$\mu < 0.77 \times 10^{26} \left(1 + \sin^2 \alpha\right)^{-1/2}$$

$$\times \left(\frac{I}{10^{45} \text{ g cm}^2}\right)^{1/2} \left(\frac{\nu}{401 \text{ Hz}}\right)^{-3/2}$$

$$\times \left(\frac{-\dot{\nu}}{5.6 \times 10^{-16} \text{ Hz s}^{-1}}\right)^{1/2} \text{ G cm}^3. \quad (4.14)$$

For $\alpha = 15^\circ$, this upper limit on the dipole is $0.75 \times 10^{26} \text{ G cm}^3$, yielding a field strength of roughly $B = 1.5 \times 10^8 \text{ G}$ at the magnetic poles.\textsuperscript{12} We emphasize that this upper limit on the magnetic field is for a purely dipolar field. The presence of higher-order multipoles would require a stronger field at the NS surface to produce the observed $\dot{\nu}$. This field estimate is consistent with the limits implied by accretion physics (see §4.4.5).

If magnetic dipole torque is a significant contributor to the spin down of SAX J1808, then the source may behave like a rotation-powered pulsar during quiescence, producing radio pulsations and a particle wind. The heating of the companion by a particle wind has been invoked as an explanation of why the companion is significantly brighter than expected in the optical. Burderi et al. (2003) predicted a dipole moment of $\mu = 5 \times 10^{26} \text{ G cm}^3$ based on the optical observations, somewhat higher than our approximate upper limit on $\mu$, but most likely within the uncertainties of the model. A similar analysis by Campana et al. (2004) found the needed irradiation luminosity to be $L_x = (4^{+3}_{-1}) \times 10^{33} \text{ erg s}^{-1}$, compatible with the observed $\dot{E} = 9 \times 10^{33} \text{ erg s}^{-1}$ loss of rotational energy. No radio emission has been detected during quiescence. The upper limits of 0.5 mJy (Gaensler et al. 1999; Burgay et al. 2003) are not particularly constraining.

The X-ray luminosities of isolated millisecond pulsars, for which magnetic dipole radiation is the primary spin-down mechanism, shows a strong correlation with their rates of rotational energy loss. From the tables compiled in Zavlin (2006) and Cameron et al. (2007), the 5–10 keV X-ray luminosity goes as $L_x \propto \dot{E}^{1.13}$ with less than a quarter decade of scatter. Based on this empirical relation, we would expect a quiescent luminosity for SAX J1808 of

\textsuperscript{12}The Spitkovsky (2006) formula for $N_{\text{dipole}}$ differs substantially from the classically derived torque due to a rotating dipole in a vacuum, $N_{\text{vac}} = \frac{2}{5} \mu^2 (2\pi\nu/c)^3 \sin^2 \alpha$, especially for small $\alpha$: for $\alpha = 15^\circ$, the derived limit is approximately one fifth of the vacuum value.
5 \times 10^{30} \text{ erg s}^{-1}. However, this prediction is a factor of ten lower than the observed quiescent fluxes of 8 \times 10^{31} \text{ erg s}^{-1} and 5 \times 10^{31} \text{ erg s}^{-1} (Campana et al. 2002; Heinke et al. 2007), suggesting other mechanisms for quiescent emission are at work.

**Magnetic propeller torque**

The propeller effect offers another possible explanation for the observed spin down during quiescence. If the Keplerian corotation radius (defined as \( r_{\text{co}} = \left[GM/4\pi^2\nu^2\right]^{1/3} \approx 31 \text{ km} \)) is less than the magnetospheric radius \( r_0 \), at which point the infalling matter couples to the magnetic field, then the magnetic field will accelerate the matter, possibly ejecting it from the system (Illarionov & Sunyaev 1975). The torque exerted on the neutron star by propeller ejection of matter at a rate \( \dot{M}_{\text{ej}} \) depends on the details of the interaction between the pulsar magnetosphere and the accretion disk. However, we can parametrize this torque as

\[
N_{\text{prop}} = -n\dot{M}_{\text{ej}}(GMr_0)^{1/2}
= -n(r_0/r_{\text{co}})^{1/2}\dot{M}_{\text{ej}}(GMr_{\text{co}})^{1/2},
\]

(4.15)

where the detailed physics determines the dimensionless torque \( n \), which is zero for \( r_0 = r_{\text{co}} \) and of order unity for \( r_0 \gtrsim 1.1 r_{\text{co}} \) (Ekşi et al. 2005).

We can then roughly estimate the rate at which matter would need to be ejected from the system during quiescence to account for the observed spin down:

\[
\dot{M}_{\text{ej}} < -2.3 \times 10^{-12} n^{-1}(r_0/r_{\text{co}})^{-1/2}
\times \left( \frac{I}{10^{45} \text{ g cm}^2} \right) \left( \frac{M}{1.4 M_\odot} \right)^{-2/3} \left( \frac{\nu}{401 \text{ Hz}} \right)^{1/3}
\times \left( \frac{-\nu}{5.6 \times 10^{-16} \text{ Hz s}^{-1}} \right) M_\odot \text{ yr}^{-1}.
\]

(4.16)

As a consistency check, we note that this upper limit does not exceed the predicted long-term mass transfer rate for the binary, \( 1 \times 10^{-11} M_\odot \text{ yr}^{-1} \), which is driven by gravitational radiation emission due to the binary orbit (Bildsten & Chakrabarty 2001). Indeed, not all the mass lost by the donor star will necessarily reach the pulsar magnetosphere during quiescence and be propelled outward; most of it would queue up in the accretion disk and later reach the NS during an outburst. Galloway et al. (2006) found that the mass transfer is roughly conservative, albeit with enough uncertainty that propeller mass loss as large as the above \( \dot{M}_{\text{ej}} \) limit is not ruled out.
4.4 Discussion

Even if propeller spin down provides the dominant quiescent torque, the resulting ejection of matter from the system would not greatly affect the binary orbit. The timescale for propeller spin down is proportional to the timescale for the ejection of mass: \( \dot{P}_{\text{orb}}/P_{\text{orb}} \propto \dot{M}_\text{ej}/M_c \), where \( M_c \approx 0.05 \, M_\odot \) is the mass of the companion. Applying the above \( \dot{M}_\text{ej} \) gives \( M_c/\dot{M}_\text{ej} = 20 \, \text{Gyr} \), far longer than the observed orbital evolution timescale of \( P_{\text{orb}}/\dot{P}_{\text{orb}} = 66 \, \text{Myr} \).

More refined calculations using the arguments of Tauris & van den Heuvel (2006) yield a propeller timescale of 6 Gyr, still far too large. Clearly there are other contributions to the orbital evolution; we discuss some in §4.4.7.

Gravitational radiation torque

A variety of mechanisms have been proposed in which rapidly rotating neutron stars can develop mass quadrupoles that give rise to gravitational radiation from the neutron star itself. These mechanisms include \( r \)-mode instabilities (Wagoner 1984; Andersson et al. 1999), accretion-induced variations in the density of the NS crust (Bildsten & Cumming 1998; Ushomirsky et al. 2000), distortion of the NS due to toroidal magnetic fields (Cutler 2002), and magnetically confined mountains at the magnetic poles (Melatos & Payne 2005).

The loss of angular momentum due to gravitational radiation has been suggested as a mechanism to explain the absence of observed pulsars with spin frequencies faster than \( \approx 730 \, \text{Hz} \) (Chakrabarty et al. 2003b; Chakrabarty 2005) and makes millisecond pulsars a target for interferometric gravitational wave detectors.

The mass quadrupole moment of the star, \( Q \), determines the torque produced by gravitational radiation: \( N_{\text{gr}} = -\frac{32}{5} G Q^2 (2\pi\nu/c)^5 \). For our measured \( \dot{\nu} \), this sets an upper limit of

\[
Q < 4.4 \times 10^{36} \left( \frac{I}{10^{45} \, \text{g cm}^2} \right)^{1/2} \left( \frac{\nu}{401 \, \text{Hz}} \right)^{-5/2} \times \left( \frac{-\dot{\nu}}{5.6 \times 10^{-16} \, \text{Hz s}^{-1}} \right)^{1/2} \, \text{g cm}^2,
\]

or \( Q \lesssim 10^{-8} I \). The strain amplitude of the resulting gravitational waves, averaged over all NS orientations, is \( h_c = 115 G \nu^2 Q/d^4 \) (Brady et al. 1998), giving a characteristic strain at Earth of \( h_c = 6 \times 10^{-28} \). This strain is undetectable by current or planned gravitational wave experiments. For Advanced LIGO, with a strain sensitivity of \( \sim 3 \times 10^{-24} \, \text{Hz}^{-1/2} \) in the 100–400 Hz range (Fritschel 2003), even a search using an accurate phase model would require years of integration time. Note that the dependence of \( N_{\text{gr}} \) on the \( \nu \) is very
strong, so it is quite possible that gravitational wave emission produces larger spin downs in faster (≈700 Hz) rotators.

### 4.4.2 Pulse profile variability

The evolution of the pulse profile is clearly not purely stochastic. With multiple outbursts, we are able to note for the first time that the pulse profile seems to take on similar shapes at similar times in the outbursts, as illustrated in Figure 4.3. These characteristic changes in the pulse profiles suggest that the emitting regions of the NS are changing shape and position as the outbursts progress. The consistency of these changes, along with the consistency of the outburst light curves, suggests that as the accretion disk empties onto the star, the geometry of the disk, the accretion funnels, and the resulting hot spots evolve in a similar manner for each outburst.

The most striking example of ordered pulse-profile evolution is the strong relationship between the harmonic content and luminosity: $r_2 \propto L^{-1/2}$. Given the complexity of the system, its abidance by such a simple model is quite surprising. SAX J1808 is not alone in this behavior. At least two other millisecond pulsars, IGR J00291+5934 and XTE J1807−294, exhibit similar inverse correlations between the amplitude of their second harmonics and luminosity (Hartman et al. 2007, in prep.).

One possible explanation is recession of the accretion disk as the accretion rate drops, revealing the star’s previously occulted second hot spot. For rapidly rotating pulsars, partial occultation of the star by the accretion disk will be common. Assuming a mass of $1.4 M_\odot$, the co-rotation radius of SAX J1808 is $r_{co} = 31 \text{ km} \approx 3R$, where $R$ is the NS radius. Following the standard pulsar accretion model (e.g., Ghosh & Lamb 1979b), the inner edge of the accretion disk will be at roughly the Alfvén radius: $r_0 \approx r_A \equiv (2GM)^{-1/7} \dot{M}^{-2/7} \mu^{1/7}$. This truncation radius must be at $r_0 < r_{co}$ for infalling matter to reach the NS surface. There are clear problems with the application of this model, which was developed for higher-field pulsars with $r_0 \gg R$: the width of the transition region in which the magnetic field becomes dominant is on the same order as its distance to the star, muddling the definition of a truncation radius. Nevertheless, this simple model is still qualitatively instructive.

Since $r_{co} \approx 3R$, neutron stars in systems with inclinations $i \gtrsim 70^\circ$ will always be partially occulted during outburst. During the outbursts of SAX J1808, the peak fluxes at which the pulses are most sinusoidal are roughly a factor of 10 greater than the low fluxes at which the harmonics are more prevalent (cf. Fig. 4.5). As a result, the Alfvén radius will increase by a factor of $r_{A,\text{tail}}/r_{A,\text{peak}} \approx 10^{2/7} \approx 2$ as the source dims. Because the maximum Alfvén
radius is \( \approx 3R \) during accretion, the radius during the peak of the outbursts must be \( r_{A,\text{peak}} \lesssim 3.2R \). At this separation, the star will be partially occulted if \( i \gtrsim 45^\circ \). Thus the degree of occultation will depend on \( \dot{M} \) for a wide range of inclinations. For \( 45^\circ \lesssim i \lesssim 70^\circ \), the disk will partially occult the NS above some critical \( \dot{M} \). For \( i \gtrsim 70^\circ \), the NS will always be partially occulted, with the degree of occultation increasing as \( \dot{M} \) increases. Pulse profile modeling by Poutanen & Gierliński (2003) suggests that the system is at an inclination of \( i > 65^\circ \).

The observations that show clearly double-peaked pulse profiles happen exclusively in the final, flaring tail stage of the outbursts, typically during the fading portion of a flare. In view of this model, one could imagine that the accretion disk is most recessed as the flares fade. One difficulty with this model is that the increased \( r_2 \) observed at low luminosities is not solely due to the appearance of doubly peaked pulse profiles; many profiles in this regime show single pulses, but with substantially greater asymmetry than typically seen at higher luminosities.

Another possible cause is the expansion of the hot spots during high accretion due to diffusive effects. Simulations of accretion flows by Romanova et al. (2004) demonstrate that as the fluence increases, the cross-sections of the accretion funnels grow. Modeling by Muno et al. (2002c) establishes that the harmonic content of the pulsations decreases as the size of the hot spot increases.

### 4.4.3 Motion of the hot spot

During the 1998, 2002, and 2005 outbursts, we observed clear trends in the phase residuals that suggest that the emitting regions do not remain at a fixed longitude. During 2002 and 2005, an abrupt phase change in the fundamental at the end of the main body of the outburst produces an advance of the pulse peak that corresponds to a shift of the hot spot by \( \approx 50^\circ \) eastward.\(^\text{13}\) These shifts are simultaneous with and occur on the same 3–4 d timescale as the sudden drops in luminosity at the end of the main outbursts. During 1998 and 2002, the phase residuals of both harmonics begin gradually increasing during the flaring tails of the outburst, corresponding to a westward drift of the hot spots. Motion of the hot spot has also been suggested to explain phase residuals in GX 1+4 and RX J0812.4−3114 (Galloway et al. 2001) and XTE J1814−338 (Papitto et al. 2007).

These trends in the phase residuals almost certainly represent motion of the

\(^{13}\)For a more natural description, we adopt the Earth-based convention of longitude: earlier pulse arrivals \( \equiv \) prograde hot spot motion \( \equiv \) eastward shift, and vice versa.
observed hot spot rather than frequency glitches. Glitches are rapid changes in the spin frequency of the NS due to imperfect coupling between the crust and more rapidly rotating, superfluidic lower layers (e.g., Anderson & Itoh 1975). This interaction occurs well below the accretion layer, and it would not be expected to coincide with or have the same timescale as rapid changes in the accretion rate.

When discussing the motion of the hot spots, the longitudes of the magnetic poles provide natural meridians from which to measure phase. Since their movement would require the realignment of currents in the core and crust, the magnetic poles remain at fixed positions for timescales far longer than the outbursts. The suppression of regions of the field due to accretion also occurs on long timescales (Cumming et al. 2001).

For high-field pulsars, the magnetospheric radius is far from the star, and the accretion column follows field lines that reach the NS surface near the magnetic pole. This is not necessarily the case for low-field pulsars. A closer accretion disk will intersect more curved field lines, which terminate farther from the poles. In the previous section, we described how the Alfvén radius can move outward from roughly $1.5R$ to $3R$ as the accretion rate drops. As the disk recesses, it will intersect decreasingly curved field lines that are rooted closer to the poles, causing the hot spots at the bases of the accretion columns to also approach the poles.

This simple picture can explain the observed phase shift as the luminosity rapidly drops during the end of the 2002 and 2005 outbursts. In both cases, the luminosity decreases by about a factor of 4. A change in $\dot{M}$ by this magnitude would cause the Alfvén radius to move outward by a factor of 1.5 and the inner edge of the accretion disk to move outward by a similar amount. This change will almost certainly cause material removed from the inner edge of the disk to attach to a different set of field lines, with the larger radius favoring lines that attach closer to the pole. If the hot spot tends to be to the west of the pole, as seen in Romanova et al. (2004) for a magnetic pole angle of $\alpha = 30^\circ$ from the rotational pole, then the attachment to different field lines would produce an eastward drift as observed. That said, these MHD simulations appear to have strong, chaotic dependencies on their parameters. (For $\alpha = 15^\circ$, the hot spot is south of the magnetic pole; for $30^\circ$, west; and for $45^\circ$, north!) More work is needed to better model these observations.

This scenario does not explain why the shift of the pulse peak would solely be expressed by a change in the fundamental; during these episodes in 2002 and 2005 the phase of the harmonic remains relatively constant. However, a movement of the hotspot toward the magnetic pole would most likely change the shape of the hotspot, possibly in a way that would preserve the phase of
The slow drifts seen during the tails of the 1998 and 2002 outbursts are also difficult to explain. The flares during the tail cause the luminosity to change in a periodic manner, so we cannot expect a monotonic motion of the accretion disk’s inner edge. The net drift during the tail of the 2002 outburst is of the same magnitude as the rapid phase shift that happens right before the tail begins, suggesting the drift may be a relaxation of the accretion column back to its original location.

### 4.4.4 Comparison with previous spin frequency measurements

There have been a number of previous reports of short-term $\dot{\nu}$ measurements made during outbursts of several accreting millisecond pulsars including XTE J0929–314 (Galloway et al. 2002), SAX J1808 (Morgan et al. 2003; Burderi et al. 2006), XTE J1751–305 (Markwardt et al. 2003a), IGR J00291+5934 (Falanga et al. 2005b; Burderi et al. 2007), and XTE J1814–334 (Papitto et al. 2007). Some of the reported $\dot{\nu}$ values have been surprisingly large given the estimates of $\dot{M}$ during the outbursts, possibly violating a basic prediction of magnetic disk accretion theory: that accretion torques cannot exceed the characteristic torque $N_{\text{char}} = \dot{M}(GMR_{\text{co}})^{1/2}$ exerted by accreting Keplerian material at the corotation radius (e.g., Ghosh & Lamb 1979b).

In the particular case of SAX J1808, spin derivatives as large as a few times $10^{-13}$ Hz s$^{-1}$ near the outburst peak were reported (Morgan et al. 2003; Burderi et al. 2006), corresponding to accretion torques exceeding $N_{\text{char}}$ for this source. However, these studies calculated pulse phase residuals using only a single harmonic; Morgan et al. (2003) reported $\dot{\nu}$ detections using only the fundamental, while Burderi et al. (2006) measuring the phase from the second-harmonic alone after noting the sudden phase shift of the fundamental in the middle of the 2002 outburst. Our results in §3.6 indicate that both of these approaches are likely to be contaminated by pulse shape changes, at least in the case of SAX J1808.

Figure 4.9 illustrates this point. Taking the phases of the harmonic components as direct spin measurements can produce large values of $\dot{\nu}$ during the peak of the 2002 outburst. Fitting only the fundamental’s phase residuals during the first 10 d of the outburst, we find $\dot{\nu} = (-1.2 \pm 0.4) \times 10^{-13}$ Hz s$^{-1}$. On the other hand, using only the second harmonic for the same interval, we find $\dot{\nu} = (5.3 \pm 0.1) \times 10^{-13}$ Hz s$^{-1}$, in good agreement with the Burderi et al. (2006) measurement. Because the pulse shape is changing rapidly during this part of the outburst, the pulse arrival times cannot be accurately determined. We therefore cannot reliably use this part of the outburst to measure the spin
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

Figure 4.9: Fitting a frequency model using only the fundamental (black) or the harmonic (grey) produces non-zero $\dot{\nu}$ measurements during the peak of the 2002 outburst. The data points are the 512 s phase residuals relative to the best constant-frequency model for the 2002 outburst. The solid lines give the best constant-$\dot{\nu}$ models, fit solely to the fundamental or the second harmonic. The dashed line shows the constant-frequency model derived using both, combined via equation (4.9); this fit did not use the points prior to MJD 52565.

of the NS. Note that if we exclude this region of large pulse shape variability, the remaining phase residuals are consistent with a constant spin frequency over the outburst interval (§3.6). From an examination of all the outbursts of SAX J1808 (excluding regions of large pulse shape variability), our work sets an upper limit of $|\dot{\nu}| \lesssim 2.5 \times 10^{-14} \text{ Hz s}^{-1}$.

We thus conclude that the past measurements of short-term $\dot{\nu}$ in SAX J1808 are unreliable. The analysis technique we described in §2.3 can mitigate the effects of pulse shape variability to some extent, but attempts to measure $\dot{\nu}$ in accreting pulsars must properly account for these variability effects, and in some instances these effects may prevent such measurements. The $\dot{\nu}$ measurements reported in other accreting millisecond pulsars must all be reevaluated in this light; all the apparent violations of the $N \leq N_{\text{char}}$ limit predicted by theory may be owing to spurious measurements caused by pulse shape vari-
ability. However, at least some accreting millisecond pulsars are observed to have relatively stable pulse shapes, indicating that accurate short-term $\dot{\nu}$ measurements are possible and that previous measurement of these sources should be reliable.

### 4.4.5 Constraints on the magnetic field

We showed in §4.1.1 that the condition $N_{\text{dipole}} \leq N_{\text{obs}}$ implies that the magnetic dipole moment $\mu \lesssim 0.8 \times 10^{26}$ G cm$^3$. This limit is consistent with the range for $\mu$ implied by the observation of accretion-powered pulsations throughout the outbursts (Psaltis & Chakrabarty 1999). At low accretion rates, the field cannot be so strong that it centrifugally inhibits matter from reaching the NS; during times of high accretion, it must be strong enough to truncate the disk above the stellar surface in order for there to be pulsations. The dimmest observation in which we observed pulsations was in 1998, with a flux in the 2–25 keV band of $1.5 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$; the brightest was at the peak of the 2002 outburst, $2.62 \times 10^{-9}$ erg cm$^{-2}$ s$^{-1}$. These fluxes, along with an improved estimate of the Eddington luminosity from observations of photospheric radius expansion bursts (Galloway et al. 2006), give us new limits on the range of accretion rates at which pulsations have been detected, relative to the Eddington rate $\dot{M}_E$: $\dot{M}_{\text{min}} = 1.8 \times 10^{-4} \dot{M}_E$ and $\dot{M}_{\text{max}} = 0.03 \dot{M}_E$. (We have made the usual assumption that $L \propto \dot{M}$.) These limits allow us to update the range for $\mu$ derived in Psaltis & Chakrabarty (1999), equations (11) and (12):

$$0.2 \times 10^{26} \text{ G cm}^3 \lesssim \mu \lesssim 6 \times 10^{26} \text{ G cm}^3.$$  

(4.18)

Taken together with the $N_{\text{dipole}}$ limit, we obtain a fairly narrow allowed range for the magnetic dipole moment,

$$0.2 \times 10^{26} \text{ G cm}^3 \lesssim \mu \lesssim 0.8 \times 10^{26} \text{ G cm}^3,$$  

(4.19)

which corresponds to a surface dipole magnetic field strength of $(0.4–1.5) \times 10^8$ G. This field is relatively weak: the magnetic fields implied by the Australia Telescope National Facility Pulsar Catalog\textsuperscript{15} (Manchester et al. 2005) for millisecond pulsars range from $1.1 \times 10^8$ G to $14 \times 10^8$ G.

\textsuperscript{14}In deriving this range for $\mu$, we make the same conservative assumptions as Psaltis & Chakrabarty (1999): the Ghosh & Lamb (1991) boundary layer parameter ranges on $0.1 < \gamma_B(M) < 1$; the NS mass is $1.4 \, M_\odot < M < 2.3 \, M_\odot$; and the NS radius is $10 \text{ km} < R < 15 \text{ km}$.

\textsuperscript{15}http://www.atnf.csiro.au/research/pulsar/psrcat/

Pulsars associated with clusters were excluded to minimize the impact of line-of-site accelerations. Field strengths were approximated using equation (4.14)
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

4.4.6 Constraints on accretion torques

Even though we did not detect an accretion-induced $\dot{\nu}$ during the outbursts, our new upper limits on $|\dot{\nu}|$ provide far stronger constraints on the accretion physics of low-$B$ systems such as SAX J1808 than previous measurements. Following the earlier analysis of the 1998 outburst by Psaltis & Chakrabarty (1999), the lower limit on the spin frequency derivative predicted by accretion torque theory during an outburst with an average accretion rate of $\dot{M}_{\text{avg}} \approx \frac{1}{3} \dot{M}_{\text{max}} \approx 0.01 \dot{M}_E$ is

$$\dot{\nu} \gtrsim 2 \times 10^{-14} \eta \left( \frac{I}{10^{41} \text{ g cm}^2} \right)^{-1} \left( \frac{R}{10 \text{ km}} \right)^{3/2} \times \left( \frac{M}{1.4 M_\odot} \right)^{1/2} \left( \frac{\dot{M}_{\text{avg}}}{0.01 \dot{M}_E} \right) \text{Hz s}^{-1},$$

(4.20)

where $\eta$ is a dimensionless parameter encapsulating the disk-magnetosphere interaction. (Refer to Ghosh & Lamb 1979b for a discussion of the physics that goes into this parameter.) $\eta$ is strongly dependent on the magnetospheric radius. For $r_0 \approx r_{\text{co}}$, the NS will be in spin equilibrium with the accreted matter and $\eta$ will be small. From the $\dot{\nu}$ confidence intervals in Table 4.4, the probability that we would have missed detecting the resulting $2 \times 10^{-14}$ Hz s$^{-1}$ spin up is 0.15%, suggesting that $\eta < 1$ and the source is near spin equilibrium during the outbursts.

4.4.7 Discussion of the increasing $P_{\text{orb}}$

Our seven year baseline for timing analysis provides the most precise measurements yet of the orbital period of SAX J1808. We find that the orbital period is increasing at a rate $\dot{P}_{\text{orb}} = 3.5(2) \times 10^{-12}$ s s$^{-1}$. This $\dot{P}_{\text{orb}}$ lies somewhat outside the 90% confidence upper limit set by Papitto et al. (2005) using the 1998–2002 outbursts, most likely owing to the more limited baseline available in that analysis.

It is interesting to compare our measurement with theoretical expectations. For orbital periods $\lesssim 3$ hr, mass transfer is LMXBs is driven by angular momentum losses due to gravitational radiation from the binary (Kraft et al. 1962), since magnetic braking torques are thought to be ineffective in this regime (Rappaport et al. 1983; Spruit & Ritter 1983). For SAX J1808, the $\dot{M}$ predicted by this mechanism is consistent with observationally inferred long-term average value of $\dot{M} = 1 \times 10^{-11}$ $M_\odot$ yr$^{-1}$ (Bildsten & Chakrabarty 2001). For conservative mass transfer from a degenerate (brown dwarf) donor, this predicts orbital expansion on a time scale $P_{\text{orb}}/\dot{P}_{\text{orb}} = 3$ Gyr (see, e.g., Tauris &
van den Heuvel 2006). By contrast, our measured value of $P_{\text{orb}}/\dot{P}_{\text{orb}} = 66$ Myr is an order of magnitude more rapid.

The origin of the anomalously large $\dot{P}_{\text{orb}}$ in SAX J1808 is unclear, although we note that unexpectedly large $\dot{P}_{\text{orb}}$ values have also been observed in several other LMXBs including 4U 1820–30 (van der Klis et al. 1993), EXO 0748–676 (Wolff et al. 2002), and 4U 1822–371 (Hellier et al. 1990). As pointed out by Chakrabarty & Morgan (1998), the binary parameters of SAX J1808 are very similar to those of the so-called “black widow” millisecond radio pulsars, all of which are ablating their low-mass companions (see, e.g., Fruchter et al. 1990). If SAX J1808 does indeed turn on as a radio pulsar during X-ray quiescence (Burderi et al. 2003; Campana et al. 2004; see also §4.1.1), it may be a black widow system as well, consistent with its very low donor mass. As such, it is interesting to note that a large and variable $\dot{P}_{\text{orb}}$, both positive and negative, has been measured in two black widow pulsars (Arzoumanian et al. 1994; Doroshenko et al. 2001).

Although mass loss from the companion through an ablated wind would tend to increase $\dot{P}_{\text{orb}}$, the mass loss rate required to explain the observed $\dot{P}_{\text{orb}}$ in SAX J1808 is $\sim 10^{-8} M_\odot \text{ yr}^{-1}$ (Tauris & van den Heuvel 2006); this is unphysically large given our measured pulsar spindown rate (§4.1), which sets the pulsar luminosity available for irradiating the companion. This explanation for $\dot{P}_{\text{orb}}$ is also inadequate in the black widow pulsars, where the orbital period variability is quasi-cyclic on a $\approx 10$ yr time scale (Arzoumanian et al. 1994; Doroshenko et al. 2001). In those systems, it has been suggested that tidal dissipation and magnetic activity in the companion is responsible for the orbital variability, requiring that the companion is at least partially non-degenerate, convective, and magnetically active (Arzoumanian et al. 1994; Applegate & Shaham 1994; Doroshenko et al. 2001). If this mechanism is active in SAX J1808, we would expect quasi-cyclic variability of $P_{\text{orb}}$ to reveal itself over the next few years.

### 4.5 APPENDIX: Improved Optical Position for SAX J1808

An accurate source position is essential for high-precision pulsar timing. An incorrect position results in errors during the barycentering of X-ray arrival times, producing frequency offsets due to improperly corrected Doppler shifts (see, e.g., Manchester & Peters 1972). SAX J1808 lies only $\beta = -13.6^\circ$ below the ecliptic plane, so any errors during barycentering will be particularly pronounced. For example, a position error of $\epsilon = 0^\circ 2$ parallel to the plane
of the ecliptic produces frequency and frequency derivative offsets relative to 
ν₀ ≈ 401 Hz of

\[ \Delta \nu = \nu_0 \epsilon \left( a_\odot \cos \beta / c \right) (2\pi / P_\odot) \cos \tau = 40 \cos \tau \mathrm{ nHz} \] (4.21)

\[ \Delta \dot{\nu} = -\nu_0 \epsilon \left( a_\odot \cos \beta / c \right) (2\pi / P_\odot)^2 \sin \tau = -8 \times 10^{-15} \sin \tau \mathrm{ Hz s}^{-1} \] (4.22)

Here \( \tau = 2\pi t / P_\odot \) parametrizes the Earth’s orbit, with time \( t \) equal to zero when the Earth is closest to the source. These offsets are comparable with the expected timing uncertainties. Each outburst gives a baseline of about \( 2 \times 10^6 \) s over which we can typically measure pulse arrival times with an accuracy of better than 25 \( \mu \)s, or \( 1 \times 10^{-2} \) cycles, producing \( \sim 5 \) nHz frequency uncertainties. By similar logic, we should be sensitive to \( \dot{\nu} \)'s as small as \( \sim 3 \times 10^{-15} \) Hz s\(^{-1}\). In practice, the pulse shape noise observed in SAX J1808 makes the actual uncertainties somewhat greater than these back-of-the-envelope values, increasing the \( \nu \) uncertainty by a factor of \( \sim 2 \) and the \( \dot{\nu} \) uncertainty by a factor of \( \sim 10 \), but the frequency uncertainty is still substantially less than the offsets due to a \( 0''2 \) position error.

We observed the field of SAX J1808 with the Raymond and Beverly Sackler Magellan Instant Camera (MagIC) on the 6.5-m Baade (Magellan I) telescope on the night of 2001 June 13, using the \( r' \) filter. The seeing was \( 0''5 \). Figure 4.10 shows the results. After standard reduction, involving bias-subtraction and flatfielding, we attempted to register the field to the International Coordinate Reference System (ICRS). We examined three astrometric catalogs for this purpose: the Hubble Space Telescope Guide Star Catalog (GSC, which was used by Giles et al. 1999; Lasker et al. 1990), the USNO-B1.0 survey (Monet et al. 2003), and the Two-Micron All-Sky Survey (2MASS; Skrutskie et al. 2006). We selected stars from all three catalogs that were not saturated or blended on our image, and fit using the IRAF task \texttt{ccmap} for the position offset, rotation, and plate-scale. We found that we could obtain the best astrometry with 2MASS: with USNO and GSC, many stars that had consistent positions between 2MASS and our image had deviations of more than \( 0''2 \), the overall scatter was larger, and there were fewer stars. With 2MASS we fit using 70 stars across the 2' MagIC frame. With position residuals of \( 0''08 \) in each coordinate, we obtained a combined uncertainty of \( 0''08 / \sqrt{70} = 0''01 \). Therefore, our astrometric uncertainty is dominated by the \( \approx 0''15 \) position uncertainty of 2MASS.

To verify our position, we checked for stars on the MagIC image from the Second US Naval Observatory CCD Astrograph Catalog (UCAC2; Zacharias et al. 2004). These are highly accurate positions (individual uncertainties of 20–40 mas) for relatively bright (\( \approx 15 \) mag) stars taken with a CCD at a current epoch (1996–1998) and with proper motions. We found three unsaturated
4.5 APPENDIX: Improved Optical Position for SAX J1808

Figure 4.10: A 30" portion of our $r'$-band Magellan image. The counterpart of SAX J1808 is indicated by the tick marks: it is the north-west object of the close pair at the center. We also indicate three 2MASS stars that we used for astrometry with the circles. The changing grayscale levels across the image reflects poor correction for the four-amplifier readout of MagIC but does not affect our astrometry.

UCAC2 stars on our image: 16259696, 16259777, and 16259680. We measured their positions on our image and compared the positions derived from the 2MASS solution to those from UCAC2, updated to epoch 2001.45. We found no net shift, and the offsets are less than 0"16 in all cases. (We note that the stars are toward the edge of the image, where residual image distortions may be present, in contrast to SAX J1808 which is at the center of the image). Therefore we believe that our solution using 2MASS is indeed accurate to our stated uncertainty of 0"15.

We then measured the position of SAX J1808 on the image and transformed the position to the ICRS. The position that we find is: R.A. = 18h08m27.62, Decl. = −36°58′43″3, equinox J2000.0, with uncertainty 0″15. This is 1″5 from the Giles et al. (1999) position, twice their quoted 0″8 uncertainty. But with many more reference stars of higher quality over a smaller field (Giles et al. 1999 used 5 GSC stars over a 4′ field), and CCD data taken at a more recent epoch (1998 for 2MASS, vs. 1987–1996 for GSC and 1981 for USNO), this new position should be more accurate.

\[16\] The USNO does not recommend GSC for current use: see http://ad.usno.navy.mil/star/star_cats_rec.shtml#gsc2.2.
4. The long-term evolution of the spin, pulse shape, and orbit of the accretion-powered millisecond pulsar SAX J1808.4−3658

4.6 Derivation of Phase Uncertainties

Derivation of the uncertainties of the phase residuals, as given in equation (4.4), follows from our definition of the phases,

\[
A_k \exp(2\pi ik\phi_k) = 2 \sum_{j=1}^{n} x_j \exp(2\pi ijk/n),
\]

where we have divided our phases into \( n \) bins, each containing \( x_j \) photons. Inverting to solve for \( \phi_k \),

\[
\phi_k = \frac{1}{2\pi ik} \left( \ln \sum_{j=1}^{n} x_j E_{jk} - \ln \frac{A_k}{2} \right),
\]

where we define the constants \( E_{jk} \equiv \exp(2\pi ijk/n) \) for the sake of brevity.

For relatively low fractional amplitudes (certainly the case throughout this paper), each phase bin will contain approximately the same number of photons: \( x_j \approx N_{ph}/n \), with variances \( (\sigma x_j)^2 \approx N_{ph}/n \) due to Poisson counting statistics. These add in quadrature to give the variance in \( \phi_k \):

\[
\sigma_k^2 = \sum_{j=1}^{n} \left( \frac{\partial \phi_k}{\partial x_j} \right)^2 \sigma x_j^2 = \sum_{j=1}^{n} \left( \frac{1}{2\pi k} \frac{x_j E_{jk}}{x_j E_{jk}^2} \right)^2 \left( \frac{N_{ph}}{n} \right).
\]

Summing the exponentials, we have \( \left| \sum_{j=1}^{n} E_{jk}^2 \right| = \frac{1}{2} n \). From the definition of \( A_k \) in equation (4.23), \( \left| \sum_{j=1}^{n} x_j E_{jk} \right| = \frac{1}{2} A_k \). Substituting these in, we reach our estimate of the phase uncertainty: \( \sigma_k = \sqrt{2N_{ph}/2\pi kA_k} \).
The properties of low energy pulsations in SAX J1808.4-3658

A. Patruno, N. Rea, D. Altamirano, M. Linares, R. Wijnands, M. van der Klis


Abstract

*XMM-Newton* observed the accreting millisecond pulsar SAX J1808.4-3658 during its 2008 outburst. We present timing and spectral analyses of this observation, in particular the first pulse profile study, and the high resolution spectral analysis of this source during the outburst. Combined spectral and pulse profile analyses suggest the presence of a strong unpulsed source below 2 keV that strongly reduces the pulsed fraction. The higher energies are dominated by a hard pulsed component that generates strongly double peaked pulse profiles. The pulsations show a phase dependence with X-ray flux and the strong second harmonic at higher energies is possibly produced by a shock around the hot spot. We also studied the high-resolution grating spectrum of SAX J1808.4-3658, and found strong ISM absorption lines, that we use to infer the interstellar abundances and the first model independent determination of the interstellar column density toward the source.
5. The properties of low energy pulsations in SAX J1808.4-3658

5.1 Introduction

The accreting millisecond X-ray pulsar (AMXP) SAX J1808.4–3658 (J1808 from now on) was the first X-ray binary found to pulsate in the millisecond range (with a spin period of 2.5 ms, Wijnands & van der Klis 1998). It has been observed in outburst 6 times, roughly every 2.5 years since 1996.

The magnetic field is thought to channel part of the disk material onto the neutron star magnetic poles. The radiation emitted from the impact region (hot spot) and/or a slab of shocked material above it is then modulated at the neutron star spin period. This radiation is observed as pulsed emission that adds to the unpulsed emission coming from the accretion disk. A possible comptonizing medium surrounding the impact region can upscatter part of the radiation to higher energies (Poutanen & Gierliński 2003). The pulsations and the X-ray spectrum were observed during previous outbursts by RXTE and a first study of the 1998 outburst was performed using those data (Poutanen & Gierliński 2003, Ibragimov & Poutanen 2009). J1808 was never observed below 2 keV during an outburst (except in the 2000 and 2005 outbursts at very low luminosity levels, see Wijnands 2003 and Campana et al. 2008). This energy range is important to understand the pulse formation mechanism, because it is here that both the hot spot and the accretion disk thermal emissions are expected to peak. Also, many absorption lines from the interstellar medium are expected in this energy range. These lines are important to definitively determine the interstellar column density toward the source.

Broadband spectral analyses of this XMM observation were reported by Papitto et al. (2009) and Cackett et al. (2009), who both focused on the study of the iron line emission at 6.4 keV. Here we present the first simultaneous spectral and timing analysis of the pulsations of J1808 as observed with XMM-Newton during the 2008 outburst, with a particular attention to the lower energy range (< 2 keV).

5.2 X-ray observation

J1808 has been observed in outburst with XMM-Newton on 2008 October 1st (MJD 54740), with 63 ks of on-source exposure time. J1808 then was in the exponential decay stage of the outburst, with a relatively high flux level (see Hartman et al. 2009 for a description of the overall outburst lightcurve). The XMM-Newton Observatory (Jansen et al. 2001) includes three 1500 cm$^2$ X-ray telescopes with the European Photon Imaging Camera (EPIC), and a Reflecting Grating Spectrometer (RGS; den Herder et al. 2001). An Optical Monitor is also present (Mason et al. 2001). It is used to follow optical
5.2 X-ray observation

Figure 5.1: **Left panel:** fractional amplitude for the pulse profiles in 9 energy bands. The circles and triangles refer to the fundamental and second harmonic respectively. Vertical bars indicate 1 σ errors in amplitude and horizontal bars the energy range. The black squares are the fractional amplitudes of the global profile, with the rms of the fundamental and second harmonic added in quadrature. The pulsed fraction is much lower below ≈ 2 keV, reaching a minimum value of ≈ 0.4% rms in the 0.3-1 keV energy range. **Right panel:** pulse profiles in four different energy bands. The profiles are double peaked, with the strength of the second harmonic increasing at higher energies. Two cycles are plotted, with profiles normalized to the maximum intensity and shifted by arbitrary amounts. The bottom profile corresponds to the low energies (0.3-2 keV) and its fractional amplitude is considerably reduced (although still significantly detected).
counterparts and will not be considered in this work. The EPIC camera is composed of two MOS CCDs (Turner et al. 2001) and a pn CCD (Strüder et al. 2001). Each EPIC camera has a fixed, mode dependent frame read-out frequency, producing event lists in the 0.1-12 keV energy range. The RGS, is composed of a double array of gratings and produces high resolution spectra in the 0.33 to 2.5 keV energy range.

Data have been processed using SAS version 8.0.0, and we have employed the most recent calibration files (CCF) available at the time the reduction was performed (February 2009). Standard data screening criteria were applied in the extraction of scientific products. After removing solar flares and telemetry dropouts the net exposure time is 41 ks.

The MOS1 has the central CCD in full frame mode with thin filters, and is heavily piled-up. For this reason we do not consider the MOS1 data any further. The MOS2 was in small window mode and is also excluded from the analysis since the 1.5 ms time resolution is insufficient to study the pulsations and several bad columns plus the non-imaging capabilities of the timing mode lead to the production of bad spectral files.

The pn used thin filters, in timing mode in order to reduce pile-up and allow the high precision timing analysis required for an accreting millisecond X-ray pulsar (AMXP). We extracted the source photons from a rectangular region of the pn with RAWX coordinates 26-49. The background is obtained from a rectangular region of the same size, at RAWX 2-25. Only photons with PATTERN ≤ 4 were used. We also extracted first and second order RGS1 and RGS2 spectra, using the standard procedure reported in the XMM–Newton analysis manual.

5.3 Timing analysis

We have first corrected the event times to the barycenter of the Solar System (using the SAS tool barycen, and the optical position given in Hartman et al. 2008a) and then we applied the 2008 outburst timing solution published in Hartman et al. (2009) in order to predict the phases of each photon detected and reconstruct the pulse profiles (see Patruno et al. 2009 for a detailed explanation of the timing technique). For the timing analysis we use events in the 0.3–12 keV energy range. The pulse profiles are built by folding data segments of length ≈ 500 s for the pulse phase analysis. The pulses are then decomposed in two harmonics with period equal to the pulse frequency (fundamental, ν) and twice the pulse frequency (second harmonic, 2ν) plus a constant representing the non-pulsed emission.

We did not attempt to calculate a new timing solution since the precision
of the solution achievable with the short observation baseline of XMM is at least an order of magnitude lower than the what was done with the RXTE data (Hartman et al. 2009). The short baseline of the observation is also insufficient to model the timing noise that affects the pulse phases and that was extensively discussed in Hartman et al. (2008a, 2009) for J1808. If timing noise is present, a systematic error is introduced in the determination of the pulse phases and spin frequency (Patruno et al. 2009).

Therefore we decided to subtract the solution reported in Hartman et al. (2009) and obtain the phase residuals with respect to that constant pulse frequency plus Keplerian circular orbit model. The pulse phase residuals of the fundamental, drift by $\approx 0.1$ cycles during the observation. We also found a correlation between these pulse phase residuals and the 0.3-12 keV X-ray flux. We fitted the data with a linear relation that gives a $\chi^2$ of 11.4 for 10 degrees of freedom and a slope of $10^{-3}$ cycle/ct/s.

The second harmonic is significantly detected in only $\approx 1/3$ of the profiles, where a detection is defined as a ratio between the pulse amplitude and its statistical error larger than 3. When not detected, the second harmonic fractional amplitude upper limit was between 0.4 and 0.6% rms at the 98% confidence level.

To increase the signal to noise and calculate the harmonic content of the pulsations in the whole energy band, we folded the entire 41 ks data into one single pulse profile. The fractional rms amplitude of fundamental and second harmonic in the 0.3-12 keV energy band is 0.98(2)% rms and 0.33(3)% rms respectively. The quoted errors are 1σ uncertainties.

We then repeated the procedure by dividing the observations in 9 energy bands between 0.3 and 12 keV. The fractional amplitude of the pulse profile increases from 0.3 up to 3 keV, and then it remains constant within the errors up to 12 keV (Fig 5.1). The fundamental tracks the behavior of the total pulse profile. The second harmonic increases monotonically in the energy range considered. At energies above $\approx 6$keV the fractional amplitudes of the fundamental and second harmonic are comparable and the overall pulse profile is double peaked (Fig. 5.1, see also Hartman et al. 2009).

5.4 Spectral analysis

We performed spectral analysis using the EPIC-pn spectrum (extracted as reported in §2) in the 0.5–12 keV energy range, and the RGS1 and RGS2 in the 0.4–2 keV energy range for the first order, and 0.7–2.2 keV for the second order (Fig. 5.2). XSPEC version 11.3 was used for the spectral analysis. A multiplicative constant has been used in all the fits to account for inter-calibration.
5. The properties of low energy pulsations in SAX J1808.4-36.58

uncertainties between EPIC-pn and RGS. The relative offsets are less than 4%
in all the fits.

We first used solar abundances from Anders & Grevesse (1989) and cross-
sections from Balucinska-Church & McCammon (1992) for the photoelec-
tric absorption. We tried an absorbed power-law plus a multi temperature
blackbody model (phabs*(diskbb + bbody + powerlaw)) as suggested in Cackett et al. (2009) and Papitto et al. (2009).
The $\chi^2_{\nu} = 5$ was unacceptable, mainly because of unmodelled features in the
data points between 0.3–2 keV and 6–7 keV (Fig. 5.2).

The 6-7 keV energy range is where ionized fluorescence lines of Fe and Ni
are expected (George & Fabian 1991). Following Cackett et al. (2009) and
Papitto et al. (2009) who claimed the detection of a broad iron K$\alpha$ line in this
energy range, we fitted this feature with a diskline to model the continuum
($\chi^2_{\nu} = 3.3$). We refer to Cackett et al. (2009) and Papitto et al. (2009) for
discussion of this broad iron line.

To model the 0.3–2 keV features, we added a 1.5% systematic error which
takes into account the calibration accuracy of each of the used instruments
1). We then tried several photoelectric cross-sections and element abundances
and the phabs and tbabs models. The residuals are not very sensitive to
the photoelectric cross-section parameters, while they strongly depend on the
assumed abundances. The stronger low energy features in the residuals were
coming from absorption and emission features close to the oxygen K-edge at
0.543 keV (see Fig. 2). We decided to model only the continuum and the iron
K$\alpha$ line as a first step, ignoring the data between 0.5-0.6 keV. This energy
range and the single features it contains were then investigated separately by
using the RGS data (see § 5.4.1, and Table 1 for the continuum and Fe line
spectral results).

The two weak features at 1.8 keV and 2.2 keV (Fig. 5.2) are known to be due
to the instrumental Si and Au K edges, not yet perfectly calibrated, especially
when dealing with timing mode observations (see the EPIC calibration report
in the footnote). We do not find evidence of a 0.871 keV O VII edge (reported
in Papitto et al. 2009), and no other significant edges are detected in the 0.8-1
keV range.

After removing the O-Si-Au edges (0.5-0.6 and 1.6-2.3 keV, see Fig. 2 bot-
tom panel), we obtain $\chi^2_{\nu} = 1.41$ (4051 dof). Given the high quality X-ray
spectrum, the relatively high value of $\chi^2_{\nu} = 1.41$ is very likely due to inter-
calibration problems between the pn and the RGS spectra, and to the calibration
uncertainties of each camera which emerge when observing bright sources.
When using only the pn spectrum, we obtain a statistically acceptable fit with

1http://xmm2.esac.esa.int/docs/documents/CAL-TN-0018.pdf
5.4 Spectral analysis

\[ \chi^2_\nu = 0.95 \text{ for 235 dof. Therefore we decide to accept the } \chi^2_\nu = 1.41 \text{ and do not further complicate the spectral model.} \]

5.4.1 High-resolution spectroscopy

We inferred the ISM abundances in a model independent way, by separately fitting the absorption edges to the RGS data. We used only the RGS1 and RGS2 first order spectra, which are the best calibrated and have a higher number of counts. The continuum parameters are reported in Table 1. We used \texttt{vphabs} that allows to set fixed abundance parameters with respect to the solar composition and to isolate the single absorption edges. The strongest features were observed around the oxygen K-edge \((O_K; \text{see Fig.5.3})\). We fitted only the \(N_H\) parameter in the \texttt{vphabs} model, and fixed at zero the oxygen abundance to fit the \(O_K\) with an \texttt{edge} model. The best fit gives \(O_K = 0.5421 \pm 0.0003\) keV and \(\tau = 0.66 \pm 0.01\), with \(\tau_x = N_x \times \sigma\) and \(\sigma\) the photoelectric cross section.

We fitted also the iron L, neon K \((Ne_K)\), magnesium K \((Mg_K)\) and silicon K \((Si_K)\) edges that lie in the RGS band. The column density for each element is reported in Tab.5.2.

The absorption features close to \(O_K\) were fitted with Gaussian lines and were identified as 1s \(-\) 2p atomic transitions of O I, O II and O III. The O IV line lies too close to an instrumental edge and was excluded from the analysis. The levels higher than IV are not detected, with a 3\(\sigma\) upper limit of \(\sim 0.1\) eV on the equivalent width (EW). Similar features have been observed in other X-ray binaries like XB1254–690 (Díaz Trigo et al. 2009) and Cyg X-2 (Takei et al. 2002; Costantini et al. 2005).

From the single absorption edges we calculated the column density of each element \((\tau_x = N_x \times \sigma)\) assuming photoelectric cross sections from Gould & Jung (1991). The equivalent hydrogen column density is inferred from the best measured edge (oxygen-K), by using abundances from Wilms et al. (2000). We derived \(N_H = (1.4 \pm 0.2) \times 10^{21} \text{ cm}^{-2}\), consistent with the value derived in the direction of J1808 from both HI and HII measurements \((1.3 \times 10^{21} \text{ cm}^{-2} \text{ and } 1.14 \times 10^{21} \text{ cm}^{-2}; \text{Dickey & Lockman 1990 and Kalberla et al. 2005 respectively).} \)

From the measurement of the EW of the oxygen I, II, and III 1s \(-\) 2p transitions, we have an independent measure of the relative oxygen abundances, using the formula \(EW_\lambda = 8.85 \times 10^{-13} N_x \lambda^2 f_{ij}\), where \(\lambda\) is the wavelength of the line, and \(f_{ij}\) the oscillator strength for the transition (Spitzer 1978). We found an \(N_{oxygen}\) consistent with that inferred from the oxygen K-edge (see Tab. 2).
5. The properties of low energy pulsations in SAX J1808.4-3658

![Figure 5.2: Left panel: from top to bottom: EPIC-pn and RGS1 and RGS2 first and second order spectra for SAX J1808.4-3658; residuals of the best fit without modeling the Si-K and Au-K instrumental lines and the oxygen forest; residuals obtained when cutting away all the unmodelled lines (continuum spectral parameters do not change). Right panel: $\nu F_\nu$ plot of the model used to fit the continuum. The power-law flux dominates over the other spectral components.](image)

110
Figure 5.3: High resolution RGS1 spectrum of J1808 around the oxygen K-edge: OI, OII and OIII absorption lines for the 1s-2p transition. The O IV line falls too close to the instrumental edge to be measured.
5. The properties of low energy pulsations in SAX J1808.4-3658

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DiskBB+BB+PL+Diskline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_H$</td>
<td>$0.16 \pm 0.02$</td>
</tr>
<tr>
<td>Inner Disk $kT$ (keV)</td>
<td>$0.20 \pm 0.01$</td>
</tr>
<tr>
<td>Disk flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(1.15 \pm 0.21) \times 10^{-10}$</td>
</tr>
<tr>
<td>$kT$ (keV)</td>
<td>$0.33 \pm 0.01$</td>
</tr>
<tr>
<td>BB radius (km)</td>
<td>$10.6 \pm 3.2$</td>
</tr>
<tr>
<td>BB flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(1.26 \pm 0.15) \times 10^{-10}$</td>
</tr>
<tr>
<td>Photon Index $\Gamma$</td>
<td>$2.11 \pm 0.01$</td>
</tr>
<tr>
<td>PL flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(1.29 \pm 0.09) \times 10^{-9}$</td>
</tr>
<tr>
<td>$E_{Fe}$ (keV)</td>
<td>$6.45 \pm 0.08$</td>
</tr>
<tr>
<td>EW (eV)</td>
<td>$97.7 \pm 31.4$</td>
</tr>
<tr>
<td>$R_{IN}$ (km)</td>
<td>$20 \pm 2$</td>
</tr>
<tr>
<td>$R_{OUT}$ (km)</td>
<td>$193 \pm 15$</td>
</tr>
<tr>
<td>Incl. (deg)</td>
<td>$&gt; 44^a$</td>
</tr>
<tr>
<td>$\beta^b$</td>
<td>$-2.1 \pm 0.1$</td>
</tr>
<tr>
<td>Fe-line flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(6.0 \pm 0.2) \times 10^{-12}$</td>
</tr>
<tr>
<td>Absorbed Flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(1.50 \pm 0.08) \times 10^{-9}$</td>
</tr>
<tr>
<td>Flux (erg s$^{-1}$cm$^{-2}$)</td>
<td>$(1.84 \pm 0.08) \times 10^{-9}$</td>
</tr>
<tr>
<td>$\chi^2$ (dof)</td>
<td>$1.41 (4051)$</td>
</tr>
</tbody>
</table>

Table 5.1: Spectral parameters for J1808 in outburst, from combined pn and all RGS data. Errors are at 1\(\sigma\) confidence level. $N_H$ in units of $10^{22}$ cm$^{-2}$ and assuming abundances from Wilms et al. (2000). Fluxes are calculated in the 0.5-10 keV energy range, corrected for absorption. A distance of 3.5 kpc has been assumed to infer the BB radius. a. The lower limit is quoted at 95% confidence level. b. $\beta$ is the power law index of the emissivity.

<table>
<thead>
<tr>
<th>Edge</th>
<th>Energy (keV)</th>
<th>$\tau$</th>
<th>$\sigma$</th>
<th>$N_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_K$</td>
<td>$0.5421 \pm 0.0003$</td>
<td>$0.66 \pm 0.01$</td>
<td>5.642</td>
<td>$11.7 \pm 0.2$</td>
</tr>
<tr>
<td>$Fe_L$</td>
<td>$0.712 \pm 0.005$</td>
<td>$0.08 \pm 0.02$</td>
<td>4.936</td>
<td>$1.6 \pm 0.4$</td>
</tr>
<tr>
<td>$N_eK$</td>
<td>$0.865 \pm 0.004$</td>
<td>$0.10 \pm 0.02$</td>
<td>3.523</td>
<td>$2.8 \pm 0.5$</td>
</tr>
<tr>
<td>$M g K$</td>
<td>$1.281 \pm 0.007$</td>
<td>$0.06 \pm 0.01$</td>
<td>2.191</td>
<td>$2.7 \pm 0.4$</td>
</tr>
<tr>
<td>$Si K$</td>
<td>$1.79 \pm 0.007$</td>
<td>$0.14 \pm 0.01$</td>
<td>1.476</td>
<td>$9.4 \pm 0.6$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1s-2p</th>
<th>Energy (keV)</th>
<th>EW (eV)</th>
<th>$N_{oxygen}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O I$</td>
<td>$0.527 \pm 0.001$</td>
<td>$1.2 \pm 0.2$</td>
<td>$9 \pm 2$</td>
</tr>
<tr>
<td>$O II$</td>
<td>$0.531 \pm 0.001$</td>
<td>$1.2 \pm 0.2$</td>
<td>$8 \pm 2$</td>
</tr>
<tr>
<td>$O III$</td>
<td>$0.537 \pm 0.001$</td>
<td>$0.6 \pm 0.1$</td>
<td>$11 \pm 2$</td>
</tr>
</tbody>
</table>

Table 5.2: Interstellar medium edges and lines in the direction of SAX J1808.4-3658. Photoelectric cross sections ($\sigma$) are in units of $10^{-19}$ cm$^2$ (from Gould & Jung 1991), while the column densities $N_x$ are in units of $10^{17}$ cm$^{-2}$.
5.5 Discussion

We observed for the first time the pulsations in the 0.3-2 keV energy range for J1808. The pulse profiles show an unexpected behavior during the 2008 XMM–Newton observation: the pulse fractional amplitudes sharply decrease below \( \approx 2 \) keV.

Our spectral analysis required a multi temperature blackbody at low energies that we identify with the source of unpulsed emission that causes the steep decrease of pulse fractional amplitudes below 2 keV. We interpret the higher energy blackbody as the emission coming from the hot spot of the neutron star. The power law is also a pulsed component, since we measured \( \approx 2\% \) rms pulse amplitudes up to 12 keV were the power law emission dominates. The unpulsed disk emission has to dominate below 2 keV to explain the sudden drop of the pulse amplitudes at those energies. Therefore the empirical power-law model is unphysical, since in the model below 2 keV the disk contributes only \( \approx 30\% \) of the power-law flux (Fig 5.2). A self consistent physical scenario would require a sharp decrease of the power-law component below \( \approx 2 - 3 \) keV.

A Comptonization model with a shocked plasma in a slab geometry is expected to cut off below \( \approx 3 \) keV and was already proposed by Gierliński & Poutanen (2005) for the AMXP XTE J1751-305. The hot blackbody would then be produced by the pulsating emission of the hot spot, the cold blackbody by the unpulsed radiation of the accretion disk and the hard component by the pulsed comptonized radiation of the shock around the hot spot.

The origin of the second harmonic can be related to a different pulse profile of the hard emission compared to the lower-energy blackbody as was suggested by Gierliński et al. (2002) and Poutanen & Gierliński (2003) for RXTE observations of J1808, and by Gierliński & Poutanen (2005) for XMM and RXTE observations of XTE J1751-305. This might result from a different angular distribution of the Comptonized radiation, which is expected as it is produced in the optically thin accretion shock, but not at the stellar surface as the blackbody emission.

The reason why the fundamental is correlated with the X-ray flux while the second harmonic is not, can then be related with different formation processes for the fundamental and the second harmonic. If the hot spot contributes only to the fundamental frequency while the comptonization region contributes to both the fundamental and the second harmonic, the fundamental pulse phase may track the hot spot position. The phase of the second harmonic instead will be affected by the comptonization process and might come from an extended region around the hot spot, covering a large area of the neutron star.
surface. The hot spot can instead come from a well defined region on the neutron star surface, and can move according to the X-ray flux fluctuations thus producing the pulse phase wandering correlated with X-ray flux (Romanova et al. 2004, Lamb et al. 2008b).

We also measured the first model independent column densities of several elements in the line of sight of J1808 (Tab. 2). The most precise measurement comes from the oxygen column density, giving an the equivalent hydrogen column density $N_H = 1.4 \times 10^{21} \text{ cm}^{-2}$ for solar abundances. This determination can be particularly important in studies of quiescence and cooling where the number of free parameters is high and the uncertainties on $N_H$ can alter the reliability of the results (see for example Heinke et al. 2009 for cooling studies of SAX J1808.4-3658 and Yakovlev & Pethick 2004 for a discussion of the problem). These measurements confirm the expected abundances of Dickey & Lockman (1990) for solar composition.
Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

A. Patruno, J.M. Hartman, R. Wijnands, D. Chakrabarty, M. van der Klis

submitted to The Astrophysical Journal

Abstract

We present a coherent timing analysis of the 2003 outburst of the accreting millisecond pulsar XTE J1807–294. We find an upper limit for the spin frequency derivative of $|\dot{\nu}| < 5 \times 10^{-14}$ Hz/s. The sinusoidal fractional amplitudes of the pulsations are the highest observed among the accreting millisecond pulsars and can reach values of up to 27% (2.5-30 keV). The pulse arrival time residuals of the fundamental follow a linear anti-correlation with the fractional amplitudes that suggests hot spot motion over the surface of the neutron star both in longitude and latitude. An anti-correlation between residuals and X-ray flux suggests an influence of accretion rate on pulse phase, and casts doubts on the use of standard timing techniques to measure spin frequencies and torques on the neutron star.
6. Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

6.1 Introduction

An open problem in the field of accreting millisecond pulsar (AMXP) is how to devise a reliable method to measure spin and orbital parameters. Since the discovery of the first AMXP (Wijnands & van der Klis 1998) considerable improvements have been made, leading to the measurement of accurate orbital and spin parameters for 9 of the 10 known AMXPs (see Wijnands 2004, Poutanen 2006, di Salvo et al. 2007 for a review and Patruno et al. 2009 for the last source discovered). Current methods (see e.g. Taylor 1992) are based on folding procedures to reconstruct the pulse profiles of the accreting neutron star and on direct measurement of the pulse phase variations due to orbital Doppler shift and spin changes (for example due to torques). The pulse phases are fitted using $\chi^2$ minimization techniques. However, a substantial complication sometimes arises due to the presence of a strong unmodelled noise component in the pulse phases that, when ignored, might affect the reliability of the method. Two possible strategies have been used in the literature to try and overcome this: (i) harmonic data selection (Burderi et al. 2006, Riggio et al. 2008, Chou et al. 2008, Papitto et al. 2007) and (ii) use of a minimum variance estimator (Boynton & Deeter 1985a, Hartman et al. 2008a). In the first case the pulse profiles are decomposed into their harmonic components: generally one sinusoid at the fundamental frequency (or first harmonic, $\nu$) and one at the second harmonic ($2\nu$) and are analyzed separately, measuring two independent sets of orbital and spin parameters. The harmonic with the weakest noise content is selected for the measurement of the spin and orbital parameters and the noisier one is discarded (see e.g., Burderi et al. 2006). Although this use of the most “stable” harmonic reduces the $\chi^2$, this selection throws away part of the information and in that sense is not optimal. The hypothesis behind the selection of the most stable harmonic is that, for unknown reasons, that harmonic better tracks the spin of the neutron star. Burderi et al. (2006) speculated that the second harmonic might be more stable because it arises from accretion onto both the polar caps and hence is insensitive to the flux ratio between poles.

In the second method, both harmonics are used and weighted to minimize the effect of phase noise (Boynton & Deeter 1985a, Hartman et al. 2008a). However, also in this second situation in practice data selection is performed. If the phases of both harmonics change differently, the possibility of defining pulse arrival times breaks down and the data where this happens have to be excluded from the analysis (Hartman et al. 2008a). Because both methods employ different data selections, different results are obtained when analyzing the same source. For example in the case of SAX
6.2 Data reduction and reconstruction of the pulse profiles

J1808.4–3658, the pulse frequency derivative \( \dot{\nu} \), measured from only the second harmonic was \( 4.4 \times 10^{-13} \text{ Hz s}^{-1} \) for the first 14 days of the 2002 outburst and \( -7.6 \times 10^{-14} \text{ Hz s}^{-1} \) for the rest of the outburst (Burderi et al. 2006). In Hartman et al. (2008a) we considered the same source and gave an upper limit of \( |\dot{\nu}| < 2.5 \times 10^{-14} \text{ Hz s}^{-1} \) for all the four outbursts for which high resolution timing data was available. The reason for this discrepancy is that while Burderi et al. (2006) used only the information carried by the second harmonic and rejected the results of the fundamental frequency, we used both harmonics but excluded the initial data where the phase variations where stronger and discrepant between harmonics (Hartman et al. 2008a). So these differences arise as a consequence of different data selections. In this paper we try to better characterize the timing noise such as observed in AMXPs focusing on a source where the noise is strong: XTE J1807-294 (J1807 from now on) which has been in outburst for \( \approx 120 \) days in 2003 (Markwardt et al. 2003a).

6.2 Data reduction and reconstruction of the pulse profiles

We reduced all the pointed observations from the RXTE satellite taken with the Proportional Counter Array (PCA, Jahoda et al. 2006) that cover the 2003 outburst of J1807. The PCA instrument provides an array of five proportional counter units (PCUs) with a collecting area of 1200 cm\(^2\) per unit operating in the 2-60 keV range and a field of view with a FWHM of \( \sim 1^\circ \).

We constructed the X-ray lightcurve using the counts in PCA Absolute channels 5-67 (\( \approx 2.5 - 29 \) keV).

We constructed our pulse profiles by folding 512 s long chunks of lightcurve in profiles of \( N = 32 \) bins, with the ephemeris of Riggio et al. (2008). In this folding process we used the TEMPO pulsar timing program to generate a series of polynomial expansions of the ephemeris that predict the barycentered phase of each photon detected. The total number of photons detected in a single profile bin is \( x_j \pm \sqrt{x_j} \), with the error calculated from counting statistics and \( j = 1, \ldots, N \). Since the pulse profile shape changes throughout the outburst, it is not possible to base the analysis on a stable template profile. Therefore we decided to analyze the pulse profile harmonic components separately.

To calculate the pulse fractional amplitudes and phases we decomposed each profile as:

\[
x_j = b_0 + \sum_k b_k \cos \left\{ 2\pi \left[ \frac{k(j - 0.5)}{N} - \phi_k \right] \right\}
\]
by using standard $\chi^2$ minimization techniques. The term $b_k$ is the amplitude of the sinusoid representing the $k$-th harmonic, and $b_0$ is the unpulsed flux component. We choose the first peak of each sinusoid in the profile as the fiducial point for each harmonic. Defining the $k$-th harmonic frequency to be $k \cdot \nu$, the unique pulse phases $\phi_k$ of each harmonic range from 0 to 1. The $i$-th pulse time of arrival (TOA) of the $k$-th harmonic is then defined as: $t_{k,i} = \frac{\phi_k}{k \cdot \nu} + \Delta t_i$. Here $\Delta t_i$ is the time of the middle of the $i$-th folded chunk. With these definitions, a positive time shift is equivalent to a lagging pulse TOA, while a negative shift corresponds to a preceding pulse TOA. This is the convention that will be used later to define pulse phase residuals.

The fractional sinusoidal amplitude of the $i$-th pulse profile and the $k$-th harmonic is calculated as:

$$R_{i,k} = \frac{N \times b_k}{N_{ph,i} - B_i}$$

where $N_{ph,i}$ and $B_i$ are the total number of photons and the background counts (calculated with the FTOOL pcabackest) in the $i$-th pulse profile. The error on the fractional amplitude $R_{i,k}$ is calculated propagating the errors on $b_k$ and $N_{ph,i}$. The error on $B_i$ is negligible with respect to the other errors and will not be considered further.

We define a pulse profile harmonic to be significant if the ratio between the amplitude $b_k$ and its statistical error $\sigma_{b_k}$ is larger than 3.3 when using a folding time of 512 s. The choice of 3.3 guarantees that the number of false detections expected when considering the global number of pulse profiles ($\approx 850$), is less than one. The length of the folding time was then changed to 300 and 3000 s to probe different timescales (see §3), and the significance threshold rescaled to 3.5 and 3$\sigma$ respectively, according to the new number of pulse profiles.

After obtaining our set of TOAs for all the significant harmonics we chose to describe the phase $\phi$ of the $k$-th harmonic (we omit the $k$ index from now on) at the barycentric reference frame, as a combination of six terms:

$$\phi(t) = \phi_L(t) + \phi_Q(t) + \phi_O(t) + \phi_M(t) + \phi_A(t) + \phi_N(t)$$

where $\phi_L(t)$ is a linear function of the time ($\phi_L(t) = \phi_0 + \nu t$, with $\phi_0$ an initial reference phase), $\phi_Q(t)$ is a parabolic function of time ($\phi_Q(t) = \frac{1}{2} \dot{\nu} t^2$), and $\phi_O(t)$ is the keplerian orbital modulation component. The term $\phi_M(t)$ is the measurement error component, and is given by a set of independent values and is normally distributed with an amplitude that can be predicted by propagating the Poisson uncertainties due to counting statistics. The term $\phi_A(t)$ is the astrometric uncertainty position error, and the last term, $\phi_N(t)$, is

118
the so-called timing noise component that defines all the phase variations that remain. The timing noise includes, but is not limited to, any phase residual that can be described as red noise and possible extra white noise in addition to that described by the measurement error component \( \phi_M(t) \).

One of the key points when dealing with timing noise is how to distinguish a true spin frequency change of the neutron star from an effect that mimics it. In general, \( \phi_Q \) and \( \phi_N \) can both be due to torques, both not be due to torques, or one can, while the other is not. In the first case the torque is not constant and has a fluctuating component. In the second case there is a process different from a torque affecting the pulse phases. In the third case, if \( \phi_Q \) is due to a torque, it is constant, while if \( \phi_N \) is due to a torque then the torque is not constant.

In the presence of timing noise (\( \phi_N \)) the formal parameter errors estimated using standard \( \chi^2 \) minimization techniques are not realistic estimates of the true uncertainties, as the hypothesis behind the \( \chi^2 \) minimization technique is that the source of noise is white and its amplitude can be predicted from counting statistics. In the presence of an additional source of noise, such as the timing noise, the apparently significant measurement of a parameter can simply reflect the non realistic estimation of the parameter errors. To solve this, we adopted the technique we already employed in Hartman et al. (2008a), who used Monte Carlo (MC) simulations of the timing residuals to account for the effect of timing noise on the parameter errors. The technique uses the power density spectrum of the best-fit timing residuals of a \( \dot{\nu} \) model, as output by TEMPO. Then thousands of fake power density spectra are produced, with Fourier amplitudes identical to the original spectrum and with random uncorrelated Fourier phases. The Fourier frequencies are then transformed back to the time domain into fake residuals, and thousands of \( \nu \) and \( \dot{\nu} \) values are measured to create a Gaussian distribution of spin frequencies and spin frequency derivatives. The standard deviations of these distributions are the statistical uncertainties on the spin frequency and derivative. For a detailed explanation of the method we refer to Hartman et al. (2008a).

6.3 Results

6.3.1 Measurement of the spin frequency and its derivative in the presence of timing noise

We fitted the phases of each harmonic with a circular keplerian model (\( \phi_O \)) plus a linear term (\( \phi_L \)) and a quadratic term (\( \phi_Q \)). All the residual phase variation we observe after removing these three terms is treated as noise (\( \phi_M \) and \( \phi_N \)).
6. Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

The ν and ħ measured for each of the two harmonics is given in Tables 1 and 2, respectively. The errors on the pulse frequency and its derivative are calculated performing 10^4 MC simulations as described in § 6.2. At long periods (days), red noise dominates the power spectrum, while at short periods (hours), the uncorrelated Poisson noise dominates. The red noise power spectrum is not very steep, and has a power law dependence \( P(\nu) \propto \nu^\alpha \) with \( \alpha \approx -0.5 \).

The source position we used comes from Chandra observations whose 68% confidence level error circle is 0''4 in radius. The astrometric uncertainty introduced in this way on the frequency and frequency derivative is 3 \( \times 10^{-8} \) Hz and 0.7 \( \times 10^{-14} \) Hz s\(^{-1} \), respectively (calculated with eqs. A1 and A2 from Hartman et al. (2008a), which added in quadrature to the MC statistical errors gives the final errors reported in Tables 1 and 2. The final pulse frequency derivative significances for the fundamental and the second harmonic are \( \approx 2.7\sigma \) and \( \approx 1.5\sigma \), respectively.

We note that the significance of the frequency derivative for the fundamental increases above the 3 sigma level when the statistical errors are calculated with standard \( \chi^2 \) minimization techniques, consistently with Riggio et al. (2008). These errors calculated with \( \Delta \chi^2 = 1.0 \) are 2 \( \times 10^{-16} \) Hz s\(^{-1} \) and 1.6 \( \times 10^{-15} \) Hz s\(^{-1} \) for the fundamental and second harmonic respectively. So, a significant ħ is present which is, however, consistent with being part of the (red) timing noise.

The timing residuals obtained after removing a ħ = 0 model are plotted in Figure 6.1 for both the harmonics (see Tables 1 & 2 for the pulse frequencies used in the fits). Our orbital solution is consistent for the two harmonics and with the orbital parameters published in Riggio et al. (2008). For the fundamental we find:

- orbital period: 2404.4163(3) s
- projected semi-major axis: 4.830(3) lt-ms
- time of ascending node: MJD 52720.675601(3)

where the quoted errors are calculated with the \( \chi^2 \) minimization technique and correspond to \( \Delta \chi^2 = 1 \). Since the pulse phase residuals are approximately white and consistent with the expected Poissonian uncertainty, on timescales equal to and shorter than the orbital period, the orbital parameter errors are a good approximation of the true uncertainties.
6.3 Results

Figure 6.1: a) Timing residuals for a constant spin frequency and a circular keplerian orbit. The fundamental (blue circles) and the second harmonic (red squares) phases were measured using an integration time of 512 s per pulse profile. b) Sinusoidal fractional amplitude of the fundamental (blue asterisks: flaring; black circles: non-flaring) and second harmonic (red squares) during the whole outburst. During the flares, the fundamental sinusoidal fractional amplitude grows up to \( \approx 27\% \), which is the highest value ever observed for an AMXP. c) XTE J1807–294 lightcurve of the 2003 outburst. The count rate was normalized to the Crab (Kuulkers et al. 1994) using the data nearest in time and in the same PCA gain epoch (e.g., van Straaten et al. 2003). The blue circles and the black asterisks identify the 4 non-flaring and the 3 flaring states, respectively, as defined in Chou et al. (2008).

### 6.3.2 Relation between timing residuals and X-ray flux

In this section we analyze the relation between the pulse arrival time residuals relative to a constant pulse frequency (\( \dot{\nu} = 0 \)) model and X-ray flux. Riggio et al. (2008) found that the residuals of the fundamental show a strong correlation with the X-ray flux, while the second harmonic shows only a marginal correlation. Since large pulse phase shifts are often observed (in both harmonics) in coincidence with the flaring states, we investigate the possibility that at least part of the observed timing noise is correlated with the presence of X-ray flux variations.

In this section we show that both the harmonics are consistent with being correlated with X-ray flux. First we focus on the entire data set, then we split
the data in intervals choosing the same 7 chunks as Chou et al. (2008); see Figure 6.1c), distinguishing non-flaring states following the exponential flux decay of the overall outburst, and flaring states, comprising the six spikes in the lightcurve. In Figure 6.2 we plot the residuals vs. the count rate for both the fundamental and the second harmonic.

We applied a Spearman rank correlation test to the flux anti-correlation for each harmonic. We accept the null hypothesis (no correlation in the data set) if the probability \( p > 1\% \). If we do not make any data selection, the Spearman test shows no correlation in either harmonic. However, a clear split in the data is apparent at around 7 mCrab: below this threshold the residuals seem to follow a correlation with the flux, while above this threshold an anti-correlation is visible for both harmonics. The Spearman coefficients for the points above the threshold are \( \rho = -0.8 \) and \( \rho = -0.65 \) for the fundamental and the second harmonic respectively (\( p < 1\% \)). A few outliers are visible in the plot, such as for example the four points of the second harmonic at about \( \approx -0.3 \) cycles. These points correspond to data taken during some of the flaring states. If we consider only the non-flaring states, the Spearman coefficients become \( \rho = -0.9 \) and \( \rho = -0.76 \) respectively (\( p < 1\% \)).

The fact that we see a change from correlation to anti-correlation around 7 mCrab is due to the fact that at that flux level in the decay of the outburst the timing residuals reach the peak of the parabolic function that dominates the residuals (at MJD \( \approx 52745 \), see Figure 6.1). This is a consequence of the fitting procedure, which selects the constant reference pulse frequency that minimizes the \( \chi^2 \) of the timing residuals. As the observed pulse frequency is increasing,
Figure 6.2: Phase residuals vs. X-ray flux for the fundamental (blue asterisks for the flaring intervals and open black circles for the non-flaring intervals) and the second harmonic (red open squares) relative to a $\dot{\nu} = 0$ model. Each pulse is a 512 s folded chunk of lightcurve. The dashed line at around 7 mCrab splits the diagram in two regions: in the left one the points follow a correlation, in the right one they follow an anti-correlation. The four points of the second harmonic that lay outside the relations correspond to the large jump observed during the second flare.

the reference frequency is too fast for the rising part of the residuals, and too slow for the decreasing part.

We have seen in § 6.3.1 that the measured pulse frequency increase is consistent with being part of a red noise process and that true neutron star spin variations may or may not be the cause. We can choose a higher reference pulse frequency than the one used to produce Figure 1a, and turn the correlation-anti-correlation dichotomy in the flux-residual diagram into only an anti-correlation, at the cost of increasing $\chi^2$ by a factor $\approx 10$. A $\nu$ higher by $10^{-7}$ Hz makes the split in the data disappear and increases the degree of correlation between flux and timing residuals.

All the correlations and anti-correlations disappear or are strongly reduced for the timing residuals relative to the best-fit finite constant-$\dot{\nu}$ model.
6.3.3 Pulse profiles

In this section we focus on the shape of the pulse profiles and their relation with other observables, such as the phase, the timing noise and the X-ray flux.

The fractional amplitude-residual diagram

We have seen in the previous section that for some data selections the X-ray flux correlates with the timing residuals relative to a $\dot{\nu} = 0$ model, but not when a finite $\dot{\nu}$ is admitted. As already noticed by Zhang et al. (2006) and Chou et al. (2008), the fractional amplitude of the pulsations shows six spikes coincident with the six flares in the lightcurve. Therefore, a correlation might also exist between the fractional amplitude of the pulsations and the arrival time residuals. Using a $\dot{\nu} = 0$ model, and again using a Spearman rank test, we found a correlation coefficient $\rho = -0.61$ ($p < 1\%$) for the fundamental, while no significant correlation exists for the second harmonic. The second harmonic is also inconsistent with following the same correlation as the fundamental. Repeating the test for a $\dot{\nu}$ model we still find no correlations for the second harmonic, but the anti-correlation found for the fundamental becomes stronger ($\rho = -0.80$, $p < 1\%$). In Figure 6.3 we show the fractional amplitude vs. residual diagram (relative to a $\dot{\nu}$ model). The anti-correlation is evident. It is interesting that the small number of points (circled in the figure) that are outliers all belong to the first 2.5 days of the outburst.

We then analyzed the flaring and non-flaring states separately. The non-flaring state shows a weak anti-correlation with a $\dot{\nu} = 0$ model ($\rho = -0.43$, $p < 1\%$) which becomes slightly stronger with a $\dot{\nu}$ model ($\rho = -0.51$, $p < 1\%$). The flaring state shows an anti-correlation relative to a $\dot{\nu} = 0$ model ($\rho = -0.58$, $p < 1\%$) that becomes much stronger for a $\dot{\nu}$ model ($\rho = -0.81$, $p < 1\%$).

We found no energy dependence in this fractional amplitude-timing residual anti-correlation (amplitude anti-correlation from now on) when we repeated the analysis in 6 different energy bands from 2.5 to 30 keV. The same is true for the second harmonic: no correlation was found in any energy band.

The X-ray flux and the fractional amplitude

In our previous paper (Hartman et al. 2008a), we found an anti-correlation between the fractional amplitude of the second harmonic and the X-ray flux in SAX J1808.4–3658. We also noted that the fractional amplitude of the fundamental behaved unpredictably. Something similar applies to J1807, where no correlation is found for the fundamental while a strong anti-correlation exists between the observed count rate and the fractional amplitude of the second
6.3 Results

Figure 6.3: Timing residuals vs. fractional amplitude diagram. The blue asterisks refer to the flaring states, while the black circles are the non-flaring states, both referring to the fundamental frequency. The second harmonic is plotted as red open squares. Each pulse was built using 512 s of integration time. The residuals are relative to a finite $\dot{\nu}$ model. The green circled outliers of the anti-correlation for the fundamental, all belong to the first 2.5 days of the observations. The second harmonic amplitude is uncorrelated to timing residuals.

harmonic ($\rho = -0.79$, $p < 1\%$, see Figure 6.4). The behavior of the fundamental is inconsistent with this relation. By analogy with Hartman et al. (2008a) we fitted a simple power-law model ($R_2 \propto f_x^\gamma$, where $f_x$ is the X-ray flux) to the data, which gives a power law index $\gamma = -0.41 \pm 0.04$ with a $\chi^2$/dof of 90.2/117. Interestingly, the power law index we found for SAX J1808.4$-$3658 (Hartman et al. 2008a) was in agreement with this. So, a difference in behavior exists between the fractional amplitude of the fundamental frequency and of the second harmonic. They respond differently to both the flux and the arrival time residuals.
Figure 6.4: The fractional amplitude of the second harmonic is anti-correlated with the flux and scales with a power law of index $\gamma = -0.41 \pm 0.04$, close to the power law index found in a similar relation for SAX J1808.4-3658.

Fractional amplitude

We focus now on the energy dependence of the pulse profiles. We consider again all the data available and the subgroups of flaring and non-flaring states. Chou et al. (2008) already reported on the energy dependence of the fundamental frequency during the non-flaring state. Here we explore also the flaring state and the energy dependence of the second harmonic. Looking at Figure 6.5 two interesting features are immediately apparent:

1. the fractional amplitude energy dependence is the same for both harmonics and regardless of the state of the source (flaring, non-flaring), up to a constant factor
2. the fractional amplitude of the fundamental increases by a factor of $\approx 1.8$ during the flaring state with respect to the non-flaring state, while it remains approximately constant for the second harmonic.

Another important property of the pulses is the time dependence of the fractional amplitude. In the middle panel of Figure 6.1 we plot the fractional
amplitude of the pulsations in the 2.5 – 30 keV band. As can be seen, during the last of the six flaring states the fractional amplitude of the fundamental increases up to ≈ 27%, which is the highest ever observed for an AMXP.\textsuperscript{1} Selecting a narrower band between 2.5 and 10 keV the maximum fractional amplitude does not appreciably change. During the non-flaring stage the fractional amplitude decreases smoothly from ≈ 9% down to ≈ 4%. The second harmonic amplitude on the contrary increases from ≈ 2% up to ≈ 5%.

\textbf{Harmonic content}

We decomposed each pulse profile in its harmonic components to look for the presence of higher harmonics. While the detection of the second harmonic is quite common among the AXMPs, higher harmonics have never been detected, with the exception of a possible third harmonic in SAX J1808.4−3658 (Hartman et al. 2008a). In J1807 we detected a third harmonic at better than 3\(\sigma\), in several different stages of the outburst, with a maximum fractional amplitude of ≈ 1.5% at MJD around 52560. To increase the S/N, we folded chunks of data of length 3000 s. The number of > 3\(\sigma\) detections of the third harmonic was of 11 out of 163 chunks searched. We searched the same chunks for a fourth harmonic, and found 5-10 significant detections above 3\(\sigma\) in the whole outburst, depending on the binning. When detected, the fourth harmonic has a fractional amplitude 0.5 – 2.0%.

There were no observations where we detected all 4 harmonics at the same time. During the second and third flares, we found a second and fourth but not a third harmonic, during the first two flares we found a second and third but not a fourth harmonic.

For the third and fourth harmonics we count respectively 8 and 5 detections during the flaring states and 3 and 2 detections in the non-flaring states.

The fractional amplitude of the third harmonic also decreases with the flux, although the slope of the power law is much smaller (\(\gamma = -0.017 \pm 0.004\)). The fourth harmonic has no significant flux dependence, but its power law slope is also consistent with the \(\gamma\) obtained for the third harmonic.

Of course this result has to be taken with caution, since we are suffering from low number statistics with only ≈ 20 detections of the third and fourth harmonic altogether.

\textsuperscript{1}In this paper we are quoting sinusoidal fractional amplitudes, which are \(\sqrt{2}\) larger than the rms fractional amplitudes

127
6. Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

![Graph showing energy dependence of pulse fractional amplitudes.](image)

**Figure 6.5:** Energy dependence of the pulse fractional amplitudes. The squares and the triangles refer to the flaring and non-flaring states respectively. The circles comprise the whole outburst. The bottom curves, overlapped in the plot, are the fractional amplitudes of the second harmonic which remains stable in both states. The pulses of the fundamental in the flaring states have a fractional amplitude which is about 1.8 times larger than during the non-flaring states. Up to a constant factor, the fractional amplitude has the same energy dependence for both harmonics and for both flaring and non-flaring states.

### 6.3.4 Short-term $\dot{\nu}$ measurements

Using the fundamental frequency, we measured short-term pulse frequency derivatives using the seven sub groups of data as defined in §3.2. These measurements are useful to investigate the time dependence of the pulse frequency derivative with time. This test is possible in J1807 because of its very long outburst duration (more than 120 days, of which $\approx 106$ days with detectable pulsations).

The $\dot{\nu}$ values and their uncertainties were first calculated with standard $\chi^2$ minimization techniques. All measured $\dot{\nu}$ values during the non-flaring states had a positive sign, whereas a negative sign was measured for all three flaring
states. The measured $\dot{\nu}$ values are shown in Figure 6.6. There is no clear trend, and most importantly no correlation between $\dot{\nu}$ and the average X-ray flux in either the flaring and the non-flaring states. This test cannot be performed on the second harmonic, since the smaller number of detections prevents a meaningful analysis of data subsets for this purpose.

We then calculated the statistical uncertainties on the $\dot{\nu}$ for each sub group of data by using the MC method as explained in §6.3.1. All the $\dot{\nu}$ values were consistent with being part of the same red noise process, consistently with what was calculated for the long term $\dot{\nu}$ value of §6.3.1.

### 6.4 Discussion

We have analyzed the outburst of XTE J1807-294 and we have calculated statistical errors by means of MC simulations as we previously did for SAX
J1808.4-3658 (Hartman et al. 2008a). We found that with our statistical treatment of the red noise observed in the timing residuals of both the fundamental and the second harmonic, the significance of the spin up is reduced below 3σ for both the fundamental and the second harmonic. The fact that the spin frequency derivative is not significant does not mean that there is not a component in the residuals that can be fitted with a parabola. It just means that the parabola is consistent with having the same origin as the power at other low frequencies: both the parabola and the remaining fluctuations are consistent with being part of the realization of the same red noise process in the timing residuals. It is a separate issue whether or not this process is due to true spin changes and torques on the neutron star.

Our observed parabola in the timing residuals combined with the stochastic and astrometric uncertainty implies that any spin frequency derivative has a magnitude smaller than $|\dot{\nu}| \lesssim 5 \times 10^{-14} \text{Hz s}^{-1}$ at the 95% confidence level.

Evidence against the spin-up interpretation of the phases comes from the lack of any correlation between the observed X-ray flux and the measured $\dot{\nu}$ (see § 6.3.4). If standard accretion torque theory applies, then the magnetospheric radius ($r_m$) should decrease as the mass accretion rate $\dot{M}$ increases, following a power-law $r_m \propto \dot{M}^{-\alpha}$ when $r_m < r_{co}$, with $\alpha = 2/7$ in the simplest case, where $r_{co}$ is the corotation radius. This implies that also the instantaneous $\dot{\nu}$ has a power-law dependence on the mass accretion rate, and (when $r_m < r_{co}$) it is:

$$\dot{\nu} = \frac{\dot{M} \sqrt{GMr_m}}{2\pi I} \simeq 1.6 \times 10^{-13} \text{Hz s}^{-1}$$

$$\times \left( \frac{\dot{M}}{10^{-10} M_{\odot} \text{yr}^{-1}} \right) \left( \frac{\nu}{\text{Hz}} \right)^{-1/3} \left( \frac{r_m}{r_{co}} \right)^{1/2} \tag{6.4}$$

see Bildsten et al. (1997). Here $\dot{M}$ is the average mass accretion rate, $M$ the neutron star mass and $I$ the neutron star moment of inertia. We have observed no such a correlation between the flux and the instantaneous pulse frequency derivative, neither in the flaring nor in the non-flaring states. One possible explanation is that the X-ray flux is not a good tracer of the mass accretion rate. If it is, standard accretion theory does not apply and the most logical conclusion is that the observed timing residuals are not due to torques. The possibility that the X-ray flux is not a good tracer of the mass accretion rate is a long standing issue in the X-ray binary pulsar field and has no simple solution. If the X-ray flux is completely unrelated to the mass accretion rate, then no conclusions can be drawn on the effect of the accretion on the pulse phase.
By using eq. (6.4) we can calculate the spin frequency derivative expected for J1807 from standard accretion theory, assuming a distance of 8 kpc and converting the average X-ray luminosity into an average mass transfer rate through $L_x \approx \eta c^2 \dot{M}$. We assume an efficiency $\eta = 0.1$ for the conversion of gravitational potential energy into radiation. In this way we obtain an average mass accretion rate $\dot{M} \approx 3 \times 10^{-11} M_\odot \text{yr}^{-1}$ (averaging over the outburst). Assuming $r_m \approx r_{co}$ we have an expected $\dot{\nu} \approx 10^{-14} \text{Hz s}^{-1}$, which is below our calculated upper limit of $5 \times 10^{-14} \text{Hz s}^{-1}$. However, the short term $\dot{\nu}$ values calculated in §3.4 exceed the theory value by 1–2 orders of magnitude and therefore are very unlikely due to accretion torques.

The possibility that we are not observing the effect of a torque on the neutron star is also suggested by the fact that looking at the shape of the lightcurve one can immediately infer the sign of the measured pulse frequency derivative in the timing residuals. This is a consequence of the flux anti-correlation. If the lightcurve is concave, then the average $\dot{\nu}$ is positive, while if the lightcurve has a convex shape the average $\dot{\nu}$ will be negative. This explains why $\dot{\nu} > 0$ in the non-flaring states and $\dot{\nu} < 0$ in the flaring states. It suggests a direct influence of the accretion rate on phase, which could be effectuated through the hot spot position on the neutron star surface. Extending this interpretation to the average $\dot{\nu}$ over the entire outburst, we also favor the interpretation of a moving hot spot for that long term trend, discarding the hypothesis of a torque to explain the parabola observed in the pulse phase residuals.

Chou et al. (2008) also suggested that the lagging arrival times observed during the flaring states cannot be explained with a torque model, since they correspond to a sudden change from a spin up to a spin down. These authors also suggested that motion of the hot spot can be responsible for both the phase shifts and the increase of the fractional amplitude during the flaring states. Chou et al. (2008) assumed a fixed position of the hot spot during the non-flaring states. However, it is unlikely that the hot spot is fixed on the surface during the non-flaring state, as we have shown (see § 6.3.4) that the magnitude of the short-term $\dot{\nu}$ is too large to be compatible with standard accretion theory.

Ibragimov & Poutanen (2009) recently proposed a receding disk as a possible explanation for the timing noise and pulse profile variability observed in the 2002 outburst of SAX J1808.4-3658. In this model the antipodal spot can be observed when the inner accretion disk moves sufficiently far from the neutron star surface as a consequence of decreasing flux. We observed a strong overtone and pulse phase drifts since the early stages of the outburst, when the disk should be closest to the neutron star. Therefore it is not clear whether our
6. Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

observations can be explained by this model or not, and further investigations of the problem are required.

Two hot spots with different and variable intensities can produce a phase shift and a changing pulsed amplitude, even if the location of both hot spots is fixed on the neutron star surface (Burderi 2008). This possibility needs also further investigation since a self-consistent model has not yet been presented.

We observed (1) a relation between flux and time of arrivals for both the flaring and non-flaring state (§ 6.3.2). This relation was consistent with being the same for the two states. We also observed (2) an anti-correlation between pulse fractional amplitudes and time of arrivals during the flaring state. Finally, (3) this amplitude anti-correlation became stronger when using a long term $\dot{\nu}$. The amplitude anti-correlation was weak in the non-flaring state, regardless of the $\dot{\nu}$. In the context of a hot spot motion model for the time of arrival variations, these findings constrain the kinematics of this motion.

Lamb et al. (2008a) demonstrated that variations in the pulse fractional amplitudes should be anti-correlated with their time of arrival if the hot spot is close to the neutron star spin axis and the hot spot wanders by a small amount in latitude.

Lamb et al. (2008a) showed that even a small displacement in longitude of the emitting region, when close to the spin axis, produces a large phase change, but no amplitude variation. A motion in latitude produces both phase and amplitude changes due to the hot spot velocity variation affecting Doppler boosting and aberration. An anti-correlation between the pulse arrival times and the pulse amplitudes would be an indicator of the above. Combining this with our observational findings 1-3 above we conclude within the moving hot spot model for the phase variation that:

1. the hot spot moves with flux in both flaring and non-flaring state, since the relation between flux and arrival times is observed in both cases and is consistent with being the same,

2. the amplitude anti-correlation in the flaring state implies an hot spot moving in latitude. The hot spot cannot move mainly in latitude during the non-flaring state since a weak amplitude anti-correlation is observed and the fractional amplitude changes by only a factor $\approx 2$ in 106 days.

3. The long term $\dot{\nu}$ must be related with a motion in longitude since during the flaring state the amplitude anti-correlation becomes much stronger when a $\dot{\nu}$ model is used to fit the time of arrivals. This is also compatible with the non-flaring state, since the amplitude anti-correlation remains weak with or without a $\dot{\nu}$.

4. Finally a motion in longitude during the flaring state or in latitude during
6.5 Conclusions

In this paper we analyzed the 2003 outburst of XTE J1807-294 and found that the pulse frequency derivative previously reported in literature is consistent with being part of a red noise process. No significant spin frequency derivative is detected when considering this red timing noise as a source of...
uncertainty in the calculation of statistical uncertainties, and an upper limit of $5 \times 10^{-14} \text{Hz s}^{-1}$ can then be set for any spin frequency derivative. The average accretion torque expected from standard accretion theory predicts a long-term spin frequency derivative which is still compatible with the derived upper limit and cannot therefore be excluded from current observations.

We propose hot spot motion on the neutron star surface as a simpler model able to explain all the observations reported in this work, as well as the presence of a pulse frequency derivative. If this explanation is correct, similar flux and amplitude anti-correlations should be observed in other AMXPs.
An alternative interpretation of the timing noise in accreting millisecond pulsars

A. Patruno, R. Wijnands, M. van der Klis

Accepted by The Astrophysical Journal Letters

Abstract

The measurement of the spin frequency in accreting millisecond X-ray pulsars (AMXPs) is strongly affected by the presence of an unmodeled component in the pulse arrival times called 'timing noise'. We show that it is possible to attribute much of this timing noise to a pulse phase offset that varies in correlation with X-ray flux, such that noise in flux translates into timing noise. This could explain many of the pulse frequency variations previously interpreted in terms of true spin up or spin down, and would bias measured spin frequencies. Spin frequencies improved under this hypothesis are reported for six AMXPs. The effect would most easily be accounted for by an accretion rate dependent hot spot location.
7. An alternative interpretation of the timing noise in accreting millisecond pulsars

7.1 Introduction

Precise orbits and spin parameters have now been reported in 9 accreting millisecond X-ray pulsars (AMXPs; see Wijnands 2004; Poutanen 2006; di Salvo et al. 2007 for reviews, Patruno et al. 2009 for the AMXP most recently found). Yet, controversy still surrounds the interpretation of the observed pulse time-of-arrival (TOA) records. The reason for this is the presence of a red “timing noise” component in the TOA residuals. While on the time scales of hours relevant to determining the orbit this noise has only a moderate effect, its amplitude is large on the timescales of weeks to months required to measure the pulse frequency $\nu$ and its time derivative $\dot{\nu}$. The origin of the timing noise is unknown, there is no satisfactory model for it, and no agreement on how to deal with it when measuring $\nu$ and $\dot{\nu}$. Some authors fit a constant $\nu$ (Hartman et al. 2008a, H08 from now on), while others fit a constant $\dot{\nu}$ or more complex models, interpreted as due to accretion torques (Falanga et al. 2005b; Burderi et al. 2006; Papitto et al. 2007; Chou et al. 2008; Riggio et al. 2008). Both methods leave unmodeled timing noise in the residuals, and in determining orbital and spin parameters arbitrary rejection is performed of data segments, or of pulse profile harmonics, showing 'too much' noise (see Patruno et al. 2009b).

In two AMXPs it was already noted that on short (<10 d) timescales X-ray flux and TOA residuals correlate (J1814, Papitto et al. 2007) or anticorrelate (J1807, Riggio et al. 2008), and on the timescales of weeks similar to the duration of an AMXP outburst these correlations were found to be stronger for a constant $\nu$ model than for constant $\dot{\nu}$ (Watts et al. 2008, Patruno et al. 2009b).

Here we show that such correlations are common in AMXPs and can be interpreted in the sense that a given flux level induces a given, constant, TOA offset, so that trends in flux bias the measured $\nu$ and $\dot{\nu}$ values. Our findings then suggest that the timing noise is not dominated by accretion torques but instead by accretion rate dependent variations in hot spot location on the neutron star surface.

7.2 Observations and data reduction

We use all RXTE PCA public data for 6 AMXPs (Table 1). We did not analyze HETE J1900.4–2455 and SAX J1748.9–2021 as their weak and intermittent pulsations require special analysis, nor SWIFT J1756.9–2508 and Aql X-1, whose pulse episodes were too brief to be useful.

We refer to Jahoda et al. (2006) for PCA characteristics and RXTE absolute
7.3 Phase-flux correlations

We used all available Event and GoodXenon data, rebinned to 1/8192 s and in the 2.5–16 keV band that maximizes S/N. We folded 512-s data chunks, keeping only those with S/N > 3–3.3σ, giving < 1 false pulse detection per source. We detect both a fundamental (ν) and a first overtone (2ν) in our pulse profiles of J1808, J1807, and J1814 and only a fundamental in J1751, J00291, J0929. The former three sources show strong pulse shape variability, so that the fiducial point defining the pulse TOA becomes ill defined (cf. Patruno et al. 2009b). Therefore, we measured the TOAs of fundamental and overtone separately, and then separately fitted them with a Keplerian orbit plus a linear and possibly a parabolic term representing ν and ˙ν. The first three sources also have strong timing noise and without modeling this noise both models give reduced χ² ≫ 1, the latter three have weaker timing noise and reduced χ² closer to 1, but both models remain statistically unsatisfactory.

### 7.3 Phase-flux correlations

Figure 7.1a (bullets) shows the TOA residuals Δt from a standard fit of a constant ν model, expressed as a phase residual in units of pulse cycles: Δφ ≡ νΔt. The phases show structures that are clearly anti-correlated with short term flux variations (top trace; the concavity at day 2-10, the bump around day 10, the slower decay after day 12). However, there is no correlation with the long term flux decay. Indeed, plotting the phases against flux (Fig. 7.1b) no correlation is seen. The reference (ephemeris) pulse frequency ν selected by a standard χ² fit is the one that distributes the residuals evenly among positive and negative values; this choice is arbitrary and other choices, introducing a net slope in the plot of residuals vs. time, are equally valid (cf. Patruno et al. 2009b). With this in mind we now investigate the hypothesis that not only the short-term variations in flux correlate with phase, but also the long term trend in flux correlates with a similar trend in phase over the entire outburst. If true, the best fit pulse frequency is not exactly the spin frequency, but contains a bias that can in principle be removed.

Figure 7.2 provides similar plots as Fig. 7.1a for each outburst, but now choosing ν such as to maximize the phase-flux correlation by minimizing the χ² of a linear fit to phase vs. flux (Fig. 7.3). The difference with the standard technique can be seen by comparing Fig. 7.1a with the top right panel of Fig. 7.2.
Figure 7.1: (a) Phase residuals (∆t × ν) from a standard fit of a constant frequency model to the fundamental frequency TOAs in the 1998 outburst of J1808 (bullets) and simultaneous 2.5–16 keV light curve (crosses) in arbitrary units. Short term correlations are clearly present, but the long term trend in flux is not seen in phase. (b) Phase vs. flux for the same data. (c) X-ray light curve and phase residuals of J1807. The phase residuals are shown upside down and in arbitrary units to better visualize the correlation of the short term phase fluctuations with the X-ray flux.
Carefully scrutinizing Figs. 7.2 and 7.3, it is clear that short term correlations and anticorrelations such as in Fig. 7.1a are ubiquitous in our sample, and that with few exceptions (discussed below) with a proper choice of $\nu$ (Table 1) these can be matched with a correlation on long timescales. In J1814, the slope of the short term correlation (measured by first subtracting best fit parabolae from flux and phase records) is $-16.3 \pm 0.5$ cycle/mCrab, and that of the long term one $-18.9 \pm 0.3$ cycle/mCrab. So, it looks like it is phase that correlates to flux here, not its second derivative $\dot{\nu}$ as in standard accretion torque models. If we remove this X-ray flux effect from the TOAs of J1814 and then fit a $\dot{\nu}$ model, $\dot{\nu}$ is a factor 15 lower than using the standard method.

The phase-flux correlation we find is positive for some sources and negative for others, and in different outbursts of J1808 both signs occur, but always has the same sign on both long and short time scales. The full phase range implied by the best correlation to the flux decay is always less than one cycle. In J1814 and in the '98 outburst of J1808 the correlation is the same for fundamental and overtone, while in J1807 for the overtone it has a slope $\approx 0.25$ that of the fundamental. In the '00, '02 and '05 outbursts of J1808, the phase of the fundamental correlates with flux whereas the 1st overtone exhibits a weak anti-correlation. However, since the 1st overtone has a low S/N and short term fluctuations are difficult to detect, we decided to exclude the 1st overtone from our analysis.

In addition, during the J1808 re-flaring state (Wijnands 2004) at the end of the '02 and '05 outbursts (but not the – sparsely sampled – '00 one) the correlations break down: other modes of accretion from the disk might play a role here (see Patruno 2008). We excluded these re-flaring states when reporting $\nu$ in Table 1. In Fig. 3 we plot only the points included in the fit; the lines correspond to the frequencies reported in Table 1. In J1807, J1808 and J0929 the pulse phases deviate from the linear correlation below some flux threshold.

The $\chi^2$ values in Table 1 are still unsatisfactory, however, they are a statistically highly significant factor 2 -- 5 smaller in J1807, J1814 and in the 1998 outburst of J1808, and similar in the other six outbursts, to the $\chi^2$ one obtains from standard methods when fitting a $\dot{\nu}$ model. So, accounting for a dependence of phase on flux by a simple uniform linear relation produces a significantly better fit than a $\dot{\nu}$ model for 3 of our 9 outbursts. In Fig. 1c we show light curve and inverted phase residuals of J1807 together; clearly there is a good correlation on all timescales, but the relation is not linear. Indeed, Fig. 7.3 suggests that still better results might be obtained allowing for the more complex (curved or broken) phase-flux relations seen there, but that is beyond scope of this Letter.
An alternative interpretation of the timing noise in accreting millisecond pulsars.

<table>
<thead>
<tr>
<th>Source name</th>
<th>Short Outburst date [yr]</th>
<th>ν (F, Hz)</th>
<th>ν (1st overt., Hz)</th>
<th>χ²/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>XTE J1807</td>
<td>2003</td>
<td>190.62350712(3)</td>
<td>190.62350712(3)</td>
<td>12806.1/765 (F), 1724.5/146 (1st)</td>
</tr>
<tr>
<td>XTE J1814</td>
<td>2003</td>
<td>314.35610872(3)</td>
<td>314.35610874(3)</td>
<td>1348/602 (F), 1086.6/555 (1st)</td>
</tr>
<tr>
<td>SAX J1808.4−3658</td>
<td>1998, 2000, 2002, 2005</td>
<td>0.52(3), 0.31(3), 0.21(2), 0.06(2)</td>
<td>0.50(5)</td>
<td>114/47 (F), 93.6/45 (1st)</td>
</tr>
<tr>
<td>XTE J0929−314</td>
<td>2002</td>
<td>185.10525437(2)</td>
<td>422.1/206</td>
<td></td>
</tr>
<tr>
<td>XTE J1751−305</td>
<td>2002</td>
<td>435.31799405(5)</td>
<td>441.6/306</td>
<td></td>
</tr>
<tr>
<td>IGR J00291+5934</td>
<td>2004</td>
<td>598.89213048(3)</td>
<td>1251.9/521</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1: Inferred spin frequencies

All the uncertainties quoted correspond to χ² = 1. F = fundamental frequency; 1st = 1st overtone

The frequencies of J1808 are relative to an offset frequency of 400.975210 Hz. The 1st overtone can be measured only for the 1998 outburst. The χ²/dof refers to the 1998 outburst.

Source name | ν (1st overt., Hz) | ν (fund., Hz) | χ²/dof |
-------------|-------------------|---------------|--------|
XTE J0929−314 |                  |               |        |
XTE J1751−305 |                  |               |        |
IGR J00291+5934 |              |               |        |
7.4 Discussion

In J1808, $\nu$ decreases between outbursts; a linear fit to the four mean pulse frequencies gave $\dot{\nu} = -0.56 \pm 0.20 \times 10^{-15}$ Hz/s with $\chi^2$/dof=9.7/2 (H08). Our new fit has $\dot{\nu} = -1.9 \pm 0.2 \times 10^{-15}$ Hz/s, approximately 3 times that measured by H08. Our $\chi^2$/dof=6.4/2, reducing to 1.6/2 including the astrometric uncertainty (cf. eq. A1, A2 in H08). While other than H08 we use only the fundamental, whose phase-flux correlation is always detected, the different $\dot{\nu}$ comes from removing the flux bias from the phases.

7.4 Discussion

Short term (<10 d) correlations or anticorrelations, depending on outburst, between pulse phase and X-ray flux are ubiquitous in our AMXPs, and a considerable fraction, up to $\approx 97\%$ of the variance of the timing noise, can be explained from the X-ray flux variability if we assume that phase depends directly on flux. These correlations can be extended smoothly and maintaining sign to the longest accessible time scales (weeks), i.e., to include a direct correlation of phase with flux as it decays in each outburst, by a proper choice of pulse reference frequency. This strongly suggests that there is a direct physical link between instantaneous flux level and phase that is very different from the correlation between X-ray flux and spin frequency derivative predicted by standard accretion theory (e.g., Bildsten et al. 1997), and that the pulse frequency derivatives measured in AMXPs are (mostly) not the direct result of accretion torques. Of course, the phase-flux correlation we observe might still (and in fact is plausible to) arise through a common parameter, i.e., accretion rate $\dot{M}$.

The fact that the observed range of phase residuals is less, and usually much less, than one cycle over each outburst allows an interpretation where a given flux level induces a given constant phase offset by affecting the location of the hot spot on the neutron star surface. Hot spot motion was recently discussed for AMXPs in various contexts. It was observed in numerical simulations (Romanova et al. 2003, 2004), and it was noted that for hot spots near the rotation pole small linear shifts can cause large pulse phase shifts (Lamb et al. 2008b). If the accretion disk inner radius is related with the X-ray flux, then at different flux levels the accreting gas will attach to different magnetic field lines and hence fall on different locations on the surface, producing a hot spot that moves in correlation with flux (and might also change in shape and size, cf. H08, Lamb et al. 2008b). The hot spot motion will bias the average frequency over the outburst if the hot spot longitude gradually varies from a different value at the beginning of the outburst to the end, and can mimic the effect of a spin frequency derivative if the light curve is concave or convex. Details
in the accretion geometry might make the sign of the correlation positive or negative for near-polar hot spots, but this remains to be calculated. The observed X-ray flux threshold below which the pulse phase deviates from the linear correlation (as in J1814, J0929, J1808, and J1807) might indicate the onset of a different mode of accretion (such as a propeller).

True underlying spin up or spin down episodes during an outburst cannot be excluded from these observations. However, our analyses indicate that if phase depends on flux as we suggest any true spin frequency derivative \( \dot{\nu} \) must be much lower than previously claimed, as the measured \( \dot{\nu} \) values primarily result from the X-ray lightcurve concavity. We then interpret the best fit reference frequencies derived under the hypothesis of a uniform phase-flux correlation (Table 1) as our best estimates of the true spin frequency in each outburst. With our new set of spin frequencies for J1808 we infer a long term spindown which, interpreted as due to the magnetic dipole torque, implies a dipole moment of \( \mu = 1.4 \pm 0.1 \times 10^{26} \text{ G cm}^3 \) (see Spitkovsky 2006 for a derivation of the non-vacuum magnetic dipole torque) and a magnetic field strength at the poles in the range \( B = 2.0-2.8 \pm 0.2 \times 10^8 \text{ G} \). This value is slightly smaller than the previous estimate of H08 and agrees with that inferred from optical and quiescent X-ray observations (\( B \approx 2 - 10 \times 10^8 \text{ G} \), Burderi et al. 2003, Di Salvo & Burderi 2003) and with the value expected from standard accretion theory (Wijnands & van der Klis 1998, Psaltis & Chakrabarty 1999).

An open question is why in J1808 in 1998 and in J1807 the correlation is different for overtone and fundamental. A difference in behavior between fundamental and overtone was first noted by Burderi et al. (2006) for the 2002 outburst of J1808, who suggested that due to competing contributions from two poles to the pulse profile the overtone more closely tracks the spin. However, if, as our results suggest, the timing noise in the fundamental can be explained from the flux variations through a moving hot spot model, then after correcting for this the fundamental must reflect the spin. We note that, e.g., an \( \dot{M} \) dependent competition between fan and pencil beam contributions to the pulse profile would instead primarily affect the phase of the overtone.

In conclusion, our analysis of a large record of AMXP data suggests that \( \dot{M} \) induced hot spot motion dominates the observed pulse phase residual variations and that this effect needs to be taken into account when measuring the spin of these neutron stars.
Figure 7.2: Phase residuals of the fundamental frequency and 2.5–16 keV light curves; as Fig. 7.1a, but with $\nu$ that optimizes the phase-flux correlations, see text. Phase-flux correlations can be seen on all timescales.
Figure 7.3: Phase-flux correlations for fundamental (black circles) and overtone (gray squares). Best linear fits to the fundamental phases are shown; deviations mostly occur at low flux. All phase residuals are TOA residual multiplied by pulse frequency, so overtone and fundamental scales match.

7. An alternative interpretation of the timing noise in accreting millisecond pulsars.
Abstract

We present a simultaneous periodic and aperiodic timing study of the accreting millisecond X-ray pulsar SAX J1808.4–3658. We analyze five outbursts of the source and for the first time provide a full and systematic investigation of the enigmatic phenomenon of the 1 Hz flares observed during the final stages of some of the outbursts. We show that strong links between pulsations and 1 Hz flares exist, and suggest they are related with hydrodynamic disk instabilities that are triggered close to the disk-magnetosphere boundary layer when the system is entering the propeller regime.
8.1 Introduction

The low mass X-ray binary transient SAX J1808.4-3658 (hereafter J1808) was discovered with Beppo-SAX in 1996 (in ’t Zand et al. 1998) and was found to be an accreting millisecond X-ray pulsar (AMXP, Wijnands & van der Klis 1998) in 1998 when observed with the Rossi X-ray Timing Explorer (RXTE). Since then, four other outbursts have been observed with RXTE. The X-ray lightcurves of the 5 outbursts under consideration are remarkably similar in shape and duration. The typical outburst duration is several weeks with a recurrence time of \( \sim 2.5 \) yr; after the ’98 one, outbursts occurred again in ’00, ’02, ’05 and ’08. The accretion rate increases steeply in the first 2–5 days of the outburst (fast rise), then it stays relatively high for a few days (peak), reaching at most a few percent of the Eddington rate. After this, the X-ray flux has a slow decay lasting several days, before entering a fast decay stage in which the flux drops in 3–5 days. The source then enters a low flux state characterized by 3–5 d flares separated by intervals of very low luminosity, the re-flaring state that can last for months, followed by quiescence.

J1808 has shown 401 Hz pulsations during all the outbursts, at all the luminosities observable by RXTE (\( \gtrsim 10^{34} \text{ erg s}^{-1} \)), even during the re-flares (Hartman et al. 2008a, 2009). The re-flaring state was observed to last \( \sim 60 \) days (MJD 53550 – 53610) in the 2005 outburst, followed by a low luminosity state approximately 10 times brighter than quiescence that lasted for another \( \sim 60 \) days (Campana et al. 2008). In the 1998 outburst the RXTE observations stopped immediately after the beginning of the re-flares, while in the 2000 outburst, only the re-flaring state was observed (for \( \sim 100 \) days, see Wijnands et al. 2001). Thanks to the good sensitivity of the XMM-Newton and Swift-XRT satellite, Wijnands (2003) and Campana et al. (2008) measured a minimum luminosity of \( \sim 5 \times 10^{32} \text{ erg s}^{-1} \) between the flares in the re-flaring state in the 2000 and 2005 outburst (assuming a distance of 3.5 kpc). In 2002 (Wijnands 2004) and 2008 (Hartman et al. 2009) the re-flaring state was observed with RXTE for approximately 1 month.

Campana et al. (2008) interpreted the observed low luminosities as a signature of the onset of the propeller regime. The propeller regime is characterized by a Keplerian velocity in the innermost region of the accretion disk that is slower than the rotational velocity of the neutron star magnetosphere. In its original formulation (Illarionov & Sunyaev 1975) it was proposed to suppress the accretion flow onto the neutron star surface. The gas, carrying part of the neutron star angular momentum, was thought to be expelled and spin down the neutron star. Ghosh & Lamb (1979a,b) proposed that spin down and accretion could occur simultaneously and recent MHD simulations (Romanova
et al. 2005, Ustyugova et al. 2006) show that two different propeller regimes are possible: a strong propeller, characterized by a strong outflow of gas, and a weak propeller, with no outflows. In both cases a magnetically channeled accretion flow onto the neutron star surface is still expected, consistent with the 401 Hz pulsations observed.

In the 2000 and 2002 re-flaring states, J1808 shows a modulation at a repetition frequency of $\sim$ 1 Hz that completely dominates the lightcurve (Wijnands 2004). This $\sim$ 1Hz modulation appears as sudden intensifications of the X-ray flux that are obvious in the power spectra and sometimes are directly detected in the lightcurve. No reports were made of 1 Hz modulations in other outbursts.

The mechanism of the re-flares and the 1 Hz modulation is still unclear, but it might be related to the onset of instabilities expected for sources near the propeller stage (Ustyugova et al. 2006). It has been suggested that the fast decay and the re-flares are related to cooling and re-heating fronts propagating through the disk (Dubus et al. 1999). The heating fronts change the accretion disk structure from a neutral to an ionized state, increasing the viscosity and the mass transfer rate through the inner accretion disk. If the inner disk structure is influenced by this process, the disk-magnetospheric boundary and/or the accretion process can be modified as well, possibly producing hydrodynamic instabilities in the accretion flow (Goodson et al. 1997; Spruit & Taam 1993; Bildsten & Cutler 1995).

Whether the 1 Hz modulation is created by such instabilities is still an open question. J1808 provides a unique opportunity to study this, since it shows X-ray pulsations that can be observed simultaneously with the aperiodic variability. One of the reasons why the pulsations can play an important role in understanding the mechanism behind the 1 Hz modulation is the pulse behavior observed during the 2002 outburst. A drift of $\sim$ 0.2 cycles was observed in the pulse phases of the fundamental frequency (but not in the first overtone, Burderi et al. 2006). The pulse phase starts to drift in coincidence with the beginning of the fast decay and ends when the re-flares appear. The interpretation of this drift is controversial.

Burderi et al. (2006) concluded that the phase drifts appear in coincidence with the onset of instabilities induced by accretion of matter onto a weakly magnetized star, such as motions of the hot spot on the neutron star surface. Hartman et al. (2008a) concluded that the observed phase drift might have been due to a motion of the hot spot toward the magnetic pole as the inner accretion disk recedes with decreasing luminosity (and thus decreasing mass accretion rate). In either case the drift of the pulse phase seems connected with some fundamental change in the accretion flow.
In this paper, we present the first comprehensive analysis of the 1 Hz modulation and its relation to the re-flares and 401 Hz pulsations in all outbursts of J1808. We discuss possible explanations for the onset of the 1 Hz modulation and possible reasons why it has been observed only in J1808 until now. We suggest the onset of accretion flow instabilities when J1808 enters the propeller stage as the origin of the 1 Hz modulation.

8.2 X-ray observations and data reduction

8.2.1 RXTE observations

We reduced all the pointed observations with the RXTE satellite’s Proportional Counter Array (PCA, Jahoda et al. 2006) that cover the outbursts of J1808.

The aperiodic timing analysis was done using GoodXenon data with a time resolution of $2^{-20}$ s and Event data with a time resolution of $2^{-13}$ s. The data were binned into 1/8192 s bins including all 256 energy channels. We performed fast Fourier transforms (FFTs) of 128 s data segments, fixing the frequency resolution and the lowest available frequency to $1/128$ Hz; the highest available Fourier frequency (Nyquist frequency) was 4096 Hz. No background subtraction or dead-time correction was made prior to the FFTs. The Poisson level was subtracted from the resulting power spectra. Following Klein-Wolt et al. (2004) we first estimated the Poisson noise using the Zhang et al. (1995) formula and then (after inspecting the high frequency range and finding no unexpected features) shifted it to match the level between $\sim 3000$–$4000$ Hz, where no intrinsic power should be present, but only counting statistics noise. Then we normalized the power spectra using the rms normalization (van der Klis 1995). In this normalization, the integral over the power spectrum is equal to the fractional rms amplitude squared. The power density units are $(\text{rms/mean})^2 \text{Hz}^{-1}$ and the fractional rms amplitude in one specific band is:

$$\text{rms} = \left[ \int_{\nu_1}^{\nu_2} P(\nu) \, d\nu \right]^{1/2}$$

(8.1)

The errors on the fractional rms are calculated by using the dispersion of points in the data. We consider a measurement as a non-detection in a specific band, when the ratio between the fractional rms and its standard deviation is smaller than 3. In this case we quote upper limits at the 98% confidence level.

The periodic 401 Hz pulsations were analyzed by constructing pulse profiles folding chunks of lightcurve in profiles of $N = 32$ bins, with the ephemeris of J1808 provided by Hartman et al. (2009). In this folding process we used
8.2 X-ray observations and data reduction

<table>
<thead>
<tr>
<th>RXTE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Outburst (year)</td>
<td>Start (MJD)</td>
</tr>
<tr>
<td></td>
<td>End (MJD)</td>
</tr>
<tr>
<td></td>
<td>Time (ks)</td>
</tr>
<tr>
<td></td>
<td>Program IDs</td>
</tr>
<tr>
<td>1998</td>
<td>50914.8</td>
</tr>
<tr>
<td></td>
<td>50939.6</td>
</tr>
<tr>
<td></td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>30411</td>
</tr>
<tr>
<td>2000</td>
<td>51564.1</td>
</tr>
<tr>
<td></td>
<td>51604.6</td>
</tr>
<tr>
<td></td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>40035</td>
</tr>
<tr>
<td>2002</td>
<td>52562.1</td>
</tr>
<tr>
<td></td>
<td>52604.8</td>
</tr>
<tr>
<td></td>
<td>664</td>
</tr>
<tr>
<td></td>
<td>70080</td>
</tr>
<tr>
<td>2005</td>
<td>53522.7</td>
</tr>
<tr>
<td></td>
<td>53587.9</td>
</tr>
<tr>
<td></td>
<td>267</td>
</tr>
<tr>
<td></td>
<td>91056, 91048</td>
</tr>
<tr>
<td>2008</td>
<td>54731.9</td>
</tr>
<tr>
<td></td>
<td>54776.1</td>
</tr>
<tr>
<td></td>
<td>236</td>
</tr>
<tr>
<td></td>
<td>93027</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SWIFT</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>54756.6</td>
</tr>
<tr>
<td></td>
<td>54778.3</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>0003003435-0003003404</td>
</tr>
</tbody>
</table>

Table 8.1: RXTE and Swift-XRT observations analyzed for each outburst

the TEMPO pulsar timing program (v 11.005) to generate a series of polynomial expansions of the ephemeris that predict the barycentered phase of each photon detected. The length of each data chunk was chosen according to the length of the RXTE observation.

We then split each pulse profile into a fundamental (at $\nu$) and a first overtone (at $2\nu$) using standard $\chi^2$ fits. For a detailed description of the method used for measuring the pulse time of arrivals (TOAs) we refer to Patruno et al. (2009). A set of pulse phase residuals was then obtained by subtracting a Keplerian circular orbit and constant pulse frequency model, using the ephemeris of Hartman et al. (2009).

8.2.2 Swift-XRT observations

During the 2005 and 2008 outbursts, the Swift X-ray Telescope (Swift-XRT) observed the re-flaring state of J1808. We refer to Campana et al. (2008) for the study of the Swift-XRT 2005 observations. Here we consider 10 pointed observations (see Table 2) covering a total of $\sim$ 15 ks that were taken during the 2008 outburst (MJD 54757–54778) and were reduced by using the XRT pipeline (v. 0.12.0). Each observation lasted between 1 and 3 ks. The data were collected in photon counting (PC) mode, except for observation 00030034041 which was taken in windowed timing (WT) mode.

We extracted source and background events for each observation using circular regions with radii of 20 arcseconds, and extracting photons with energies between 2–10 keV and 0.5–10 keV.
8.3 Results

8.3.1 The X-ray lightcurves and the re-flaring state

In the 2008 outburst re-flaring state, the lightcurve reached very faint luminosities, with a large portion of the re-flares below the sensitivity limit of RXTE, but above the detection threshold of Swift-XRT. The flux reached a minimum level of $2.0 \times 10^{-13}$ erg s$^{-1}$ cm$^{-2}$ in the 0.5–10 keV band, which corresponds to a luminosity of $\sim 3 \times 10^{32}$ erg s$^{-1}$ at 3.5 kpc. This luminosity is of the same order of magnitude as that observed by Wijnands (2003) and Campana et al. (2008) during the 2000 and 2005 re-flares.

A property of all the re-flares is the periodicity on time scales of a few days, creating the “bumps” observed in the lightcurve (see Fig. 8.1). The fast decay and the re-flares in the 2002, 2005 and 2008 show a similar duration. As noted in § 8.1, in '98 the observations stopped too early and the '00 ones started too late to allow a similar comparison.

The precise determination of the re-flare periods suffers of biases due to occasional poor sampling and to the sensitivity limit of RXTE. Therefore, although the data certainly allow this, there is no significant evidence that differences in fast decay time scales between the 3 outbursts have an effect on the subsequent re-flare periodicity time scales.

In the 2008 outburst it is hard to calculate the re-flare time scale because the flux is below the sensitivity limit of RXTE in the majority of the observations. The Swift sampling was approximately 1 observation every 2 days, too long compared to the fast decay time scale of 3 days to unambiguously exclude shorter time scales. However, the fast decay and the re-flare time scales are again compatible with being the same.

In 2002 and 2005 the re-flares’ peak luminosity tends to decrease with time (see Fig. 8.1). This is not observed in the 2000 (see Wijnands et al. 2001) and 2008 re-flares, which show an erratic change of peak luminosities. In the re-flaring state, the luminosity can change by $\sim 3$ orders of magnitude on time scales of $\leq 1.5$ days (see Fig. 8.1 at MJD $\sim$ 54756 and Wijnands 2003 for a similar observation in the 2000 outburst).
Figure 8.1: Outburst lightcurves in the 2-10 keV energy band. The count rate was normalized to the Crab intensity measurements nearest in time (Kuulkers et al. 1994) and in the same PCA gain epoch (e.g., van Straaten et al. 2003). Each black circle and triangle corresponds to one RXTE and Swift-XRT observation, respectively (see Table 1 for a complete list of the observations used). The open squares indicate observations when the 1 Hz QPO was detected, while the open circles indicate the RXTE observations with an X-ray flux below the detection threshold of $\sim 1$ mCrab. The vertical dashed lines divide the lightcurve into intervals beginning with the fast decay, and separating the re-flares. The duration of each re-flare is indicated in days and is similar to that of the fast decay (also indicated). In the 2008 outburst only few observations were above the detection threshold and all the vertical lines refer to the fast decay time scale.
**8.3.2 The fast decays and the pulse phase drifts**

During the 2002 outburst, the pulse phase of the fundamental was observed to drift by 0.2 cycles in just a few days (Burderi et al. 2006). Hartman et al. (2008a) showed that a similar drift was present in 2005. Here we show that both phase drifts start and end in coincidence with (or very close to) the beginning and the end of the fast decay.

In Fig. 8.2 we plot the pulse timing residuals of the fundamental and the 2-10 keV X-ray flux in mCrab. In the 2002 outburst the pulse phase drift starts exactly when the fast decay begins. In the 2005 outburst we see the phase drift beginning close to the beginning of the decay, although given the large scatter in the pulse phases it is difficult to define the exact moment when the phases begin to drift. Both the 2002 and 2005 pulse phase drifts end in coincidence with the end of the fast decay. Both the 2002 and 2005 pulse phases drift by ~0.2 cycles.

The slope of the fast decay is consistent with being the same in both outbursts (see also Fig. 8.3). The pulse phases in both outbursts during this time drift at the same speed of ~0.07 cycle/day. The overall pulse phase behavior in the 2002 and 2005 outbursts is therefore consistent with being identical.

In the 2000 outburst a similar test cannot be performed since RXTE missed the decaying portion of the outburst, and the observations covered just the re-flaring state. In the 1998 and 2008 outbursts no clear phase drift is observed.
Figure 8.3: X-ray lightcurve of four outbursts, plotted up to the end of the fast decay stage. The curves are aligned to the beginning of the fast decay. The 2008 outburst has the shortest decay time. The 2008 slow decay, peak and fast rise stages are very similar to those of the 2005 outburst. The end of the 1998 outburst fast decay is limited by a non detection (open circle); it is the outburst that shows the dimmest luminosities before the beginning of the re-flares.

We refer to Hartman et al. (2008a, 2009) for a detailed discussion of the coherent timing analysis of those two outbursts.

### 8.3.3 QPO parameters and flux

We will not in this paper provide a complete description of all the aperiodic variability observed in the 5 outbursts. For a description of the aperiodic timing features observed in AMXPs, we refer to van Straaten et al. (2005) and to the review of Wijnands (2006). Here we focus our attention on the 1 Hz modulation and give a brief description of the power spectra observed from the fast decay stage on, in order to provide context. A description of our quantitative analysis of the 1Hz modulation follows in the next sections.

We examined the entire data set of all 5 outbursts for evidence of the 1 Hz modulation. Close to the end of the 2002 and 2005 pulse phase drifts, when the fast decay stage is almost over, the X-ray lightcurve clearly shows this strong modulation with a repetition frequency of $\sim 1$ Hz (Fig. 8.4). It
shows up as a strong quasi periodic oscillation (QPO) peak around 1 Hz in the power spectrum (Fig. 8.5, see also Wijnands 2004 for a similar plot for the 2002 outburst). Later during the re-flares of these outbursts the modulation occasionally recurs, as discussed in more detail in § 8.3.4. The 1 Hz modulation also appears and disappears sporadically during the re-flares of the 2000 but was not detected in the 1998 and 2008 outbursts (cf. § 8.1).

![Figure 8.4](image-url)  

**Figure 8.4:** 1 Hz modulation of the X-ray lightcurve (background subtracted), as observed in a re-flare of the 2005 outburst. The figure shows a 15 s chunk of lightcurve with a time resolution of 0.1 seconds. In this observation the 1 Hz modulation has the highest measured fractional rms amplitude (125% rms, ObsId:91418-01-02-05).

We calculated the fractional rms amplitude\(^1\) of the 1 Hz QPO by integrating the power in the 9.95 Hz wide band 0.05 – 10 Hz. This frequency band was chosen because it is here that the power is observed in the large majority of the observations. Some power above 10 Hz is observed in some cases as an extended tail of the 1 Hz QPO (Fig. 8.5), but selecting an upper frequency of 20 Hz does not change the results significantly. Clearly, using this method we sum up all the power from the different components in the power spectrum:

\( \text{Fractional rms amplitude is standard deviation divided by mean flux. Values of rms larger than 100\% indicate that the 1 Hz flares have a large amplitude and a short duty cycle.} \)

For a light curve composed exclusively of square flares with duty cycle \( f \), \( \text{rms} = \sqrt{\frac{2}{f}} \), arbitrarily large for \( f \to 0 \)

---

\(^1\)Fractional rms amplitude is standard deviation divided by mean flux. Values of rms larger than 100% indicate that the 1 Hz flares have a large amplitude and a short duty cycle.
power of the 1 Hz QPO, its harmonics, and the broad band noise. So everything in the range 0.05–10 Hz is included, and therefore this method is not optimal to measure the power generated by one single component. However, the rms calculated in this way is empirical and independent from any model used to fit the data. The fractional rms amplitude in the 0.05–10 Hz band was in the range 10–125% in all observations where the modulation was detected.

The QPO is usually quite broad, and its shape cannot be satisfactorily fitted by single or multiple Lorentzians, as it has a sharp fall-off in power at lower frequencies. In some observations a second harmonic peak is visible (Fig. 8.5). We model these features with Gaussians of the form:

\[ P(\nu_i) = A \cdot \exp \left[ \frac{(\nu_i - B)^2}{C^2} \right] \]  \hspace{1cm} (8.2)

where \( \nu_i \) are the Fourier frequencies, A is a normalization factor, B is the centroid frequency (\( \nu_0 \) or 2\( \nu_0 \)) and C is related to the FWHM through: FWHM = 2C \cdot [\ln(2)]^{1/2}. The typical value for the FWHM of the two Gaussians is \( \sim 1 \text{Hz} \) for all the three outbursts. The Gaussian that we use to fit the 1 Hz modulation usually has a quality factor \( Q = \nu_0 / \text{FWHM} < 2 \). As shown in the bottom panel in Fig. 8.5, when \( Q < 1 \) the power spectrum does not show a clear peak, but it does show a break in the noise. Here we call the 1 Hz modulation “QPO” regardless of its quality factor. When the 1 Hz QPO is fitted by 2 Gaussians \( \nu_0 \) is defined as being the frequency of the Gaussian with the highest peak power in the power spectrum. With this choice \( \nu_0 \) is consistent with being always the fundamental frequency of the 1 Hz modulation (varying between 0.8 and 1.6 Hz), as the harmonic peak at 2\( \nu_0 \) is always lower in maximum power.

We calculated the fractional rms amplitude and centroid frequency of the 1 Hz QPO and the X-ray flux for the 2000, 2002 and 2005 outbursts. The fractional rms amplitude as observed in 2002 and 2005 is shown in Fig. 8.6 (middle panel). The fractional rms amplitude shows abrupt changes over time and does not follow a clear correlation with flux. In the 2000 re-flaring state, the 1 Hz modulation was detected in 13 observations out of 46.

We plotted together all the points for the three outbursts in Fig. 8.7. There is a clear increase of the frequency with flux and an anti-correlation between frequency and QPO fractional rms amplitude. Both relations have considerable scatter. We explored the fractional rms dependence on the X-ray flux using a similar plot, and we found no clear dependence. There is no correlation between the pulse phase and fractional rms, \( \nu_0 \) or the X-ray flux. We then calculated the upper limits (quoted at the 98% confidence level) for all the observations after the end of the fast decay where the QPO was not detected.
Figure 8.5: Power spectra (power × frequency) of the re-flares of the 2005 outburst. The four plots show different manifestations of the 1 Hz modulation in the power spectrum. The upper panel (ObsId: 91418-01-02-05, MJD~53550.1) shows a strong coherent 1 Hz QPO and its overtone clearly separated. They blend together in the second panel (ObsId: 91418-01-01-00, MJD~53542.5) where some power is also observed at frequencies higher than ~10 Hz. The third panel (ObsId: 91056-01-04-01, MJD~53541.5) shows a QPO with a steep cutoff at frequencies larger than ~10 Hz. The bottom panel (ObsId: 91418-01-03-04, MJD~53556.7) shows still power, but as an incoherent feature without a clear peak.
Figure 8.6: Upper panels: pulse phases of the fundamental for the 2002 and 2005 re-flaring states. Central panels: Fractional rms amplitude of the 1 Hz QPO (black triangles) and non-detections at 98% confidence level (open triangles). Bottom panels: 2-10 keV X-ray lightcurves. The observations where the 1 Hz QPO is significantly detected are marked with open squares, while measurements where the X-ray flux was below the sensitivity limit are marked with open circles. The pulse phase shows jumps of 0.1-0.2 cycles on a time scale of hours or less, when the 1 Hz QPO disappears or when its rms amplitude reaches a low level. This suggests a link between the presence of the 1 Hz QPO and the accretion flow onto the neutron star surface.
Figure 8.7: 1 Hz QPO centroid frequency vs. X-ray flux (left), 1 Hz QPO centroid frequency vs. 1Hz QPO fractional rms amplitude (right panel) for the 2000 (asterisks), 2002 (circles) and 2005 (open triangles) outbursts. The QPO centroid frequency is clearly correlated with the X-ray flux and increases with decreasing fractional rms. The two relations are similar in all three outbursts.

The upper limits were calculated per observation. In the 1998 data two complications occur: the observations ended immediately after the beginning of the re-flares, and the fast decay reached very faint fluxes (< 1 mCrab) at its minimum. Only in one observation J1808 was detected; then the upper limit was 55% rms. A 1 Hz modulation therefore cannot be completely excluded for this outburst. In 2000, 2002, 2005 and 2008 the most constraining upper limits are 15%, 9%, 19% and 19% rms, respectively.

Across all five outbursts, the 1 Hz QPO was observed in a rather narrow range of luminosities during the re-flares (2–15 mCrab, 2–10 keV), in contrast with the larger range of luminosities covered by the outbursts (∼ 1 – ∼ 80 mCrab). The Swift-XRT and XMM-Newton observations had insufficient time resolution or an insufficient number of counts to probe the presence of the 1 Hz QPO below 1 mCrab.

8.3.4 The appearance of the 1 Hz QPO

In the 2002 outburst, the 1Hz QPO appears immediately after the pulse phase drift is complete, but with the flux still decreasing in the fast decay stage. In 2005 the 1 Hz QPO appears in a similar position (see black vertical lines in Fig. 8.2). In both the 2002 and 2005 outbursts, data gaps prevent the observation of the exact moment when the 1 Hz QPO appears. Taking account of the gaps, the flux level of the first 1 Hz QPO appearance is consistent between the two outbursts.

To track the appearance of the 1 Hz QPO on time scales as short as a few
hundred seconds, we investigated the observation available in which the 1 Hz QPO first appears in the 2005 outburst. The observation (Obs-Id 91056-01-04-03) shows two chunks of data separated by a gap of 5000 s (chunk A with a length of 3200 s and chunk B with length 3500s) that we analyzed separately.

In chunk A the power spectrum has already changed with respect to the earlier observations: the power at higher frequencies has disappeared, and there is only power in the range 0.05–10 Hz. Probably we are witnessing the onset of the 1 Hz modulation. The average power spectrum shows a fractional amplitude of $(21 \pm 1\%)$ rms in the 0.05–10 Hz band (Fig 8.8). In chunk B we find a 1 Hz QPO clearly present with a fractional amplitude of $(68 \pm 4\%)$ rms.

We then calculated power spectra from 256 s data segments and determined the fractional rms amplitude in the 0.05-10 Hz band. (see Fig. 8.9). Significant power is detected in three segments during the observations of chunk A, meaning that some low level activity is already present in the same frequency range where the QPO will later appear. In chunk B power is nearly always detected; the onset of the 1 Hz QPO must have occurred within the 5000 s gap. At MJD 53540.48 the amplitude shows an abrupt decrease from $\sim 60\%$ rms down to $< 30\%$ (98% confidence level) on a time scale of 256 s.

We also calculated the power in a number of frequency bands in the range

Figure 8.8: Power spectrum of chunk A (see text for the definition), preceding the appearance of the 1Hz QPO in the 2005 outburst. Some power is present between 0.05 and 10 Hz, with a total fractional rms amplitude of $21 \pm 1\%$.
Figure 8.9: Temporal evolution of the 0.05-10 Hz fractional rms amplitude in observation 91056-01-04-03, showing the 1 Hz QPO appearance during the 2005 outburst. Each point corresponds to a 256 s long observation. Chunks A and B correspond to two different RXTE orbits. The black circles are $> 3\sigma$ detections, while the open squares are 98% confidence upper limits. The rms amplitude of chunk A is always below 30% rms, with 3 significant detections where the 1 Hz QPO is a broad incoherent feature in the 0.05-10 Hz band. In chunk B, the 1 Hz QPO fractional amplitude can reach values of up to $\sim 85\%$ rms and it fluctuates on very short time scales ($\sim 256$ s).

1/128 to 256 Hz for all power spectra of chunk A. The power is always consistent with zero at the three sigma level in all the frequency bands except for the 0.05-10 Hz band in the three segments mentioned above.

8.3.5 Energy dependence of the 1 Hz QPO

In Fig. 8.10 we show the energy dependence of the 1 Hz QPO. The points in the figure refer to the observation (one per outburst) for which the fractional rms of the 1 Hz QPO in the 2-60 keV energy band was the highest (110.5% rms in 2000, ObsId 40035-01-04-01; 117% rms in 2002, ObsId 70080-03-15-00; 125% rms in 2005, ObsId 91418-01-02-05). In these observations the characteristic frequencies of the 1 Hz QPO and its overtone form clear peaks in the power spectrum and $Q > 2$. 

160
The energy spectrum is hard, rising by a factor of 1.5-1.7 between 2 and 17 keV. Upper limits (98% confidence level) in the 17-60 keV band were: 114% rms for 2000, 157% rms for 2002 and 155% rms for 2005.

![Energy spectrum graph](image)

**Figure 8.10:** Energy dependence of the 1 Hz QPO fractional rms amplitude. The rms increases with energy for all the three outbursts. The plot shows rms amplitudes up to 17 keV, where a significant amount of counts is detected. In the range 17-60 keV only upper limits can be calculated for the three outbursts, which are consistent with the rms increase observed in the 2-17 keV range.

### 8.3.6 Jumps in 401 Hz pulse phases related to strength of 1 Hz QPO

During the first re-flare of the 2002 outburst, the 1 Hz QPO becomes very broad and then disappears. When the QPO becomes very broad and its fractional rms amplitude suddenly drops from $\sim 50\%$ down to 10%, a jump of 0.2 cycles is observed in the pulse phases of the fundamental (jump A, see Fig. 8.6). Two similar pulse phase jumps of 0.1 cycles are observed (jump B and C) when the QPO is not detected, with rms amplitude upper limits of $\sim 30$ and $\sim 10\%$ rms (98% confidence level). We call these phase changes “jumps” as opposed to the “drifts” observed during the fast decay stage (§ 8.3.2), since the 0.1 – 0.2 cycle jumps are sudden, occurring on time scales of a few hours or less, while the 0.2 cycle drifts take 4 – 5 days.
In the 2005 outburst we see something very similar, although the re-flares have a sampling that is much worse: the observations are rather sparse and are separated by at least 1 day. In the first re-flare the QPO fractional amplitude drops in a similar position as in the 2002 outburst, i.e., in the decreasing portion of the re-flare. As soon as the QPO is undetectable, the pulse phase jumps by approximately 0.2 cycles (jump A). Then two other jumps occur; on one occasion the rms amplitude was $\sim 10\%$ and on the other occasion the QPO was not detected (upper limit of $\sim 50\%$ at 98\% confidence level).

In both the 2002 and 2005 outbursts, the pulse phase in the jumps is close to the pulse phase prior to the beginning of the fast decay stage (see \S 8.3.2).

In the 2000 outburst the observations are rather sparse, with gaps of several days between observations. However also in this case when the QPO rms amplitude is low or the QPO is undetectable, the pulse phases are observed to jump by $\sim 0.1$–0.2 cycles, consistent with the 2002 and 2005 pulse phase behavior.

8.3.7 Dependence of 401 Hz pulsation on the 1 Hz phase

The 1 Hz modulation might affect the formation of the pulsations. Possible differences in the pulse properties can be unveiled by analyzing the pulsations coming from different phases of the 1 Hz modulation. Since the modulation has a frequency that changes with time, it is hard to unambiguously define its phase.

We therefore used a crude but simple approach, defining “peaks” and “valleys” of the 1 Hz modulation in the lightcurve relative to the average flux. We first calculated the average X-ray flux per observation. Then we split the X-ray lightcurve into an upper half and a lower half containing peaks and valleys, respectively. We checked that this method was properly dividing the lightcurve by a visual inspection. We then folded all the peaks and the valleys into two different 401 Hz pulse profiles at the spin frequency of the neutron star, and repeated the procedure for each observation where the 1 Hz QPO was detected.

The pulse phases are always consistent with being the same for peaks and valleys, within the pulse phase uncertainty of down to 0.01 cycles. The absolute pulse amplitude is higher in the peaks than in the valleys but by a smaller fraction than the mean flux, so that the pulse fractional amplitude is higher in the valleys by a factor varying between 1.2 and 6.
8.4 Discussion

Our analysis of the five outbursts of SAX J1808.4-3658 has shown that the fast decay, the re-flares and the 1 Hz modulation are related phenomena, which can also affect the 401 Hz pulsations. The drifts in pulse phase and the re-flares observed on time scales of days might be related with some change in the disk structure which is connected to the fast decay, which takes place on a similar time scale. The fast decay presumably is either a thermal or a viscous one, and additionally depends on the length scale on which significant changes occur within the disk, longer length scales corresponding to longer time scales. The 1 Hz QPO properties had previously only been very briefly discussed in the literature, and left the mechanism responsible to be identified. We therefore studied the 1 Hz modulation properties in detail and related them to the behavior of the X-ray flux and the X-ray pulsations. Any suitable model for the 1 Hz modulation has to explain the following key properties that we reported in § 8.3:

- It appears at the end of the fast decay, after the pulse phases have drifted by 0.2 cycles (Fig 8.2)
- It appears at a flux level that is consistent in the 2002 and 2005 outbursts (§ 8.3.2)
- It appears only in a narrow range of luminosities (2–15 mCrab, § 8.3.3).
- It is very coherent in some observations while in some others it is a broad incoherent feature (Fig. 8.5)
- Its amplitude has a connection with the 401 Hz pulse phases (§ 8.3.6, Fig. 8.6).
- Its fractional rms amplitude can be as high as 125% and then 10% for the same X-ray luminosity (Fig. 8.6)
- Its amplitude is energy dependent, rising with energy (Fig. 8.10)
- Its rms amplitude is very high, up to 170% in the 10-16 keV band (Fig. 8.10)
- Its centroid frequency is quite stable and slightly increases with flux (Fig. 8.7)
- Its centroid frequency decreases with increasing fractional rms amplitude (Fig. 8.7)
Flares that might be similar to the J1808 re-flares are seen in other low mass X-ray binaries (LMXBs): the black hole candidate XTE J1650-500 (Tomsick et al. 2003), and the neutron star transient SAX J1750.8-2900 (Linares et al. 2008). This behavior therefore may not be unique to J1808, although this is the only AMXP in which such re-flares have been observed.

However, the 1 Hz modulation does appear to be unique to J1808. The most similar QPOs, in terms of frequency and high fractional amplitude, are found in the systems that show Type II bursts. The Rapid Burster (Lewin et al. 1976; Hoffman et al. 1978; Marshall et al. 1979) has QPOs in the range 0.04–4 Hz in the persistent emission, and from 2–5 Hz in during its Type II bursts (Tawara et al. 1982; Lewin 1987; Stella et al. 1988; Dotani et al. 1990; Lubin et al. 1991, 1992; Rutledge et al. 1995). Similar frequencies (0.04–0.4 Hz) have also been reported in the aftermath of Type II bursts from the bursting pulsar GRO J1744-28 (Kommers et al. 1997).

### 8.4.1 Accretion onto a magnetized neutron star

In order to understand what might be causing the 1 Hz QPOs, some general background on magnetically channeled accretion will be useful. To sustain channeled accretion at the maximum accretion rate of a few percent Eddington, the magnetic field for J1808 must be \( \gtrsim 10^7 \) G. The upper limit on the field, determined from timing, assuming that the spin down comes from magnetic dipole radiation from a rotation powered pulsar, is \( 1.5 \times 10^8 \) G (Hartman et al. 2008a).

In a discussion of magnetized accretion, reference is often made to the corotation radius \( r_c \), the radius at which matter in a Keplerian orbit would have the same angular velocity as the star.

\[
r_c \sim 17 \left[ \frac{\nu_s}{1 \text{ kHz}} \right]^{-2/3} \left[ \frac{M}{1.4 M_\odot} \right]^{1/3} \text{ km}
\]

(8.3)

where \( \nu_s \) is the spin frequency of the neutron star and \( M \) is its mass. For J1808, assuming a mass of \( 1.4 M_\odot \), \( r_c \sim 31 \) km. The other radius of relevance is the magnetospheric radius \( r_m \): the radius at which the magnetic field becomes dynamically important in controlling the inflow of matter. For spherically symmetric accretion, one can estimate \( r_m \) by setting the magnetic pressure equal to the ram pressure of free fall (Lamb et al. 1973):
where we have assumed a dipole field $B \sim \mu/r^3$, $\mu$ being the magnetic moment. $R$ is the stellar radius and $\dot{M}$ the accretion rate. In the case where accretion occurs from a disk this expression will be slightly modified by the rotational energy of the disk (Spruit & Taam 1993). Rotation of the central star can also affect the location of $r_m$ (Lovelace et al. 1999).

This simple order of magnitude estimate yields 18 km for SAX J1808 at the peak of the outburst. This increases as the accretion rate falls, becoming comparable to $r_c$ once the accretion rate drops to 1% of the Eddington rate. The precise value of $r_m$ will depend on details of inner disk physics as discussed in Psaltis & Chakrabarty (1999).

When $r_m < r_c$ accretion should proceed without difficulty, with the magnetic field channeling material out of the disk and onto the magnetic poles (Pringle & Rees 1972). Once $r_m > r_c$, however, the situation becomes more complex. Initially it was thought that for $r_m > r_c$ accretion would cease, with matter being expelled from the system. In this “propeller” stage, the neutron star would spin-down (Illarionov & Sunyaev 1975). Further study by Spruit & Taam (1993) showed that $r_m$ actually has to exceed $r_c$ by a reasonable margin for material to be expelled. Steady accretion is in fact possible when $r_m > r_c$, even though the neutron star should spin down. In this stage the inner edge of the disk stabilizes near $r_c$ and the density at the inner disk rises allowing angular momentum to be transferred outwards (Spruit & Taam 1993; Rappaport et al. 2004). This type of disk structure predicts spin-down without requiring penetration of the disk by the magnetic field far beyond $r_c$, as was proposed in early works on the topic (Ghosh & Lamb 1979b).

In the next Section we will examine in turn the various mechanisms that might be responsible for the observed 1 Hz QPO.

### 8.4.2 Candidate mechanisms: surface instabilities

We consider two possible mechanisms to generate oscillations on a neutron star surface which require accretion but are independent from the accretion flow process: oscillatory modes and marginal stable nuclear burning.
Surface oscillations

There are various oscillatory modes of the neutron star surface layers that could lead to periodic brightness variations. The most likely candidates would be an ocean g-mode (a vibration driven by thermal buoyancy McDermott & Taam 1987; McDermott et al. 1988). Bildsten & Cutler (1995) estimated g-mode oscillation frequencies for non-rotating neutron stars and found values of a few Hz, with higher order modes having lower frequencies. Rapid rotation changes these frequencies and modulates them at the neutron star spin frequency. To reach a frequency as low as \( \sim 1 \) Hz would require a pattern speed in the rotating frame that is both close to the stellar spin frequency and retrograde. This would necessitate either a very high \( n \) mode (\( n \) being the number of radial nodes) if \( m = 0 \), or a low order retrograde radial mode with \( m > 0 \) and very high \( l \), \( l \) and \( m \) being the standard angular quantum numbers (Bildsten et al. 1996). The high fractional rms amplitudes of the 1 Hz oscillation however are too high to be explained by this model, and if this was the right mechanism, they should also be observed in other stars since no unique triggering conditions are known in J1808. No other surface modes (discontinuity g-modes, r-modes of any kind, or crust interface modes) are thought to have frequencies in the observed range on such a rapidly rotating star.

Marginally stable nuclear burning

J1808 has shown thermonuclear X-ray bursts during the 2002, 2005 and 2008 outbursts. The matter accreted onto the surface of the neutron star may burn stably, unstably (generating X-ray bursts), or in a marginally stable fashion (for a review see Bildsten 1998b). Marginally stable burning, which occurs in a narrow range of accretion rates close to the boundary between stable and unstable ignition, will result in quasi-periodic oscillations in brightness. This type of phenomenon, operating at local accretion rates where helium ignition is on the boundary between stability and instability, is thought to generate the mHz QPOs (see Revnivtsev et al. 2001 and Altamirano et al. 2008 for observational results and Heger et al. 2007 for a theoretical study of the mHz QPOs). The time scale for the quasi-periodic variations associated with marginally stable burning is set by the accretion time scale and the thermal time scale. For hydrogen ignition, the time scale for marginally stable nuclear burning, computed as in Heger et al. (2007) using the appropriate accretion and thermal time scales, is clearly too slow (\( \sim 10 \) minutes). For helium ignition, the time scales are shorter but still too slow (\( \sim 100 \) s), thus ruling out the marginally stable nuclear burning scenario.
8.4.3 Candidate Mechanisms: disk/magnetosphere instabilities

There are numerous ways of obtaining variability from the disk and its interactions with the stellar magnetosphere. Many have been explored, however, as a means of explaining the kHz QPOs - and so have frequencies that would be very far off 1 Hz. For this reason we will neglect many of the mechanisms for disk variability that have been discussed in the literature (see van der Klis 2006 for an extensive review of these mechanisms) and focus on those that might have frequencies in the right range.

Disk obscuration

Disk warping or shadowing was suggested as a candidate for the $\sim 1$ Hz QPOs seen in the three dipping systems (Homan et al. 1999; Jonker et al. 1999, 2000). The dipping QPOs have fractional amplitudes of $\sim 10\%$ (Homan et al. 1999; Jonker et al. 1999, 2000). In the dipping sources the energy dependence of the QPO amplitude was flat, supporting the shadowing hypothesis. In J1808, there is a clear energy dependence and the 1 Hz QPO amplitude is much higher than that seen in the dippers. Furthermore, there is no evidence for dipping in the X-ray lightcurve of J1808. So, the 1 Hz QPO in J1808 is unlikely to be similar to the dipper QPOs.

Interchange instabilities

The role of interchange instabilities in admitting matter to the magnetosphere is discussed in detail by Arons & Lea (1976) and Elsner & Lamb (1977). The magnetic pressure prevents incoming matter from crossing magnetic field lines, but if it is energetically favorable (due for example to gravity) for material to be ‘inside’ the field lines rather than ‘outside’, then interchange instabilities can act to move the material inside. This is often referred to as the Rayleigh-Taylor instability, a term that refers to the instability that occurs when a more dense fluid overlies a less dense fluid in a gravitational field. Where a plasma is supported against gravity by a magnetic field, this is more correctly referred to as the Kruskal-Schwarzschild instability (Kruskal & Schwarzschild 1954).

The conditions necessary for the onset of Kruskal-Schwarzschild instabilities in the situation where $r_m \neq r_c$ have since been studied using MHD simulations (Romanova et al. 2008; Kulkarni & Romanova 2008). For misalignment angles $\theta \lesssim 30^\circ$ accretion can proceed either stably via funnel flows, or unstably via interchange instabilities. In the unstable situation matter accretes via a number of ‘tongues’ that penetrate the magnetopause in the equatorial plane. If a certain number of tongues dominate, quasi-periodic oscillations can emerge in the light curves. Funnel flows can co-exist with accretion by
tongues (Kulkarni & Romanova 2008), although their presence should reduce the amplitude of the persistent pulsations by reducing the azimuthal asymmetry (Romanova et al. 2008). This might be consistent with the continued presence of accretion-powered pulsations, with an amplitude that depends on the phase of the 1 Hz QPO.

In the cases studied by Romanova et al. (2008) interchange instabilities set in above a critical accretion rate, making them unlikely as a cause for the 1 Hz QPO (which appears only below a critical rate). Interchange instabilities cannot be completely ruled out, however. In the standard case studied by Romanova et al. (2008) magnetic pressure and gravity dominate the force equations. When \( r_m \sim r_c \), however, magnetic pressure equals gravity. At this point other terms start to dominate the force equations and the character of the interchange instability will change (Baan 1977; Spruit & Taam 1993). Baan (1977) showed that sporadic penetration of the magnetosphere is possible in this regime. However detailed numerical simulations of the type performed at higher accretion rates have not been done, and the effect on funnel flows (and hence the amplitude of the accretion-powered pulsations) is not known. Without further study, periodicity due to interchange instabilities operating in the regime where \( r_m > r_c \) cannot be ruled out as a mechanism for the 1 Hz QPO.

Magnetic reconnection instabilities

As matter moves within the disk (radially and azimuthally) it can drag magnetic field lines along with it. The sheared field lines can temporarily impede accretion until reconnection establishes a normal flow again. The resulting quasi-periodic accretion flow would lead to a corresponding quasi-periodicity in the lightcurve, provided that the accretion funnel and hot spot can respond on a 1 s time scale.

Magnetic reconnection is one of the most plausible mechanisms for Type II bursts (cf. § 8.4). Type II bursts are thought to occur in systems where magnetic inhibition causes accreting matter to build up in a reservoir outside the magnetosphere. Once a sufficient over-density of material has accumulated, instabilities cause a catastrophic breach of the magnetospheric hammock, resulting in sudden bursts of accretion (Lewin et al. 1976). Dramatic changes in QPO properties immediately before Type II bursts, with no detectable change in the spectrum, have been interpreted as indicating that the QPOs in the persistent emission are generated within the fuel reservoir (Dotani et al. 1990). To what extent this phenomenon is relevant to our 1 Hz QPO is unclear. In the following discussion we explore several mechanisms that can produce 1 Hz oscillations.
Aly & Kuijpers (1990) used reconnection to explain the QPOs observed in the Rapid Burster, and discussed how the disk would be broken up in ‘blobs’ by magnetic reconnection instabilities. They predict a frequency a few times the beat frequency between the rotation rate of the star $\nu_s$ and the Keplerian frequency at the inner edge of the disk $\nu_K$. This implies that the frequency of the QPOs should pass through zero at the point where $r_m \sim r_c$.

An argument in favor of this mechanism is that it has been observed in simulations (Goodson et al. 1997, 1999; Goodson & Winglee 1999, and Romanova et al. 2005). Romanova et al. (2005) observe the phenomenon in both the propeller and non-propeller regimes, with the quasi-periodicity far stronger in the propeller regime. Unfortunately Romanova et al. (2005) do not model the emission mechanisms, so whether outflows or discrete accretion would genuinely lead to high amplitude QPOs in the X-ray emission is not clear. Ustyugova et al. (2006), using MHD simulations, showed that rapidly rotating accreting stars have strong periodicity of this type linked to strong outflows. Observationally, there is evidence for jet formation from the radio detections in the decay of the 1998 outburst and the peak of the 2002 outburst (Gaensler et al. 1999, Rupen et al. 2002), although the latter cannot be related with the propeller onset since it occurs at the maximum fluxes observed for J1808.

The frequency, for this mechanism, should depend on accretion rate (as observed, Fig. 8.7). It should however occur at all accretion rates: current models suggest no means of confining this mechanism to a narrow range of accretion rates as we observe for the 1 Hz modulation ($\S$ 8.3.3). Another argument against this mechanism is that this type of instability should occur independently of field misalignment angle, so should be seen in all the AMXPs and the non-pulsating LMXBs (assuming that they all have an external magnetic field).

**Thermal/viscous/radiation-driven instabilities**

The inner region of an $\alpha$ disk (Shakura & Sunyaev 1973) is unstable to thermal and surface density perturbations (Pringle et al. 1973; Lightman & Eardley 1974). Both types of instability can arise when radiation provides the major contribution to the total pressure (Shakura & Sunyaev 1976). Frank et al. (2002) derive the various time scales in operation:

$$\tau_{\phi} \sim \tau_{z} \sim \alpha \tau_{\text{th}} \sim 10^{-4} \left[ \frac{M}{1 M_\odot} \right]^{-1/2} \left[ \frac{R}{10 \text{ km}} \right]^{3/2} \text{s} \quad (8.5)$$
\[ \tau_{\text{visc}} \sim 3 \alpha^{-4/5} \left[ \frac{M}{10^{16} \text{ g/s}} \right]^{-3/10} \left[ \frac{M}{1 M_\odot} \right]^{1/4} \times \left[ \frac{R}{10 \text{ km}} \right]^{5/4} \text{s} \] (8.6)

The time scales are defined as follows: \( \tau_\phi \) is the dynamical time scale in the disk; \( \tau_z \) is the time scale on which deviations from hydrostatic equilibrium in the z-direction get smoothed out; \( \tau_{\text{th}} \) is the thermal time scale, that is the time scale for re-adjustment to thermal equilibrium, if, say, the dissipation rate is altered; and \( \tau_{\text{visc}} \) is the time scale on which matter diffuses through the disk due to the effect of viscous torques. Note that the canonical value suggested by numerical simulations for \( \alpha \) is 0.01. Thus the dynamical time scale is of the order \( 10^{-4} \) s, the thermal time scale is 0.01 s and the viscous time scale is of the order 100 s assuming that the appropriate length scale is comparable to the inner disk radius \( R \sim r_c \sim 10 \) km. A study by King et al. (2007) shows that for many LMXBs \( \alpha \sim 0.1 \), an order or magnitude larger than the value suggested by numerical simulations of disks. Therefore the observed time scales are expected to be shorter than those calculated from numerical simulations. If the instability were confined to a narrow inner annulus of the disk, then this might be consistent with a 1 Hz frequency, since the viscous time scale falls for shorter length scales. In addition, if the instability triggers at the onset of the propeller regime when the inner edge of the disk puffs up to sustain accretion, this might explain the rarity of the phenomenon.

Taam & Lin (1984) examine the thermal-viscous instability of radiation-dominated disks in more detail, looking at the global response of the disk. They find that some areas that would be unstable using a local analysis are stabilized when considering the global problem. However, suppression of the instability is not complete, and hence accretion could still be unstable and proceed in a sporadic fashion: this is suggested as a mechanism for Type II bursts. Stella et al. (1988) discuss this instability as the origin of the low frequency QPOs in the Rapid Burster. They give a description of how sporadic accretion could launch a wave that would lead to a periodic thickening of the disk, hence causing QPOs.

The viscous-thermal instability operates at high accretion rates (> 10% Eddington) when radiation pressure starts to play a major role. The 1 Hz QPO we observe in J1808, by contrast, sets in at accretion rates of less than a few percent Eddington, where radiation pressure can be neglected (Psaltis & Chakrabarty 1999). For this reason the viscous-thermal instability is unlikely to be responsible for our QPO.
The other instability that might be relevant is the ionization instability that is thought to put the system into outburst in the first place (see Lasota 2001 for a detailed review). We know that the 1 Hz QPO appears at the end of the outburst, where current accretion disk models predict a transition from the hot to the cold state. The ionization instability might enter a marginally stable oscillatory state when the disk is on the verge of flipping between hot and cold regimes at luminosities of \( \sim 1\% \) Eddington. The question is then why it would not occur always when the source is in the required luminosity range, and why not also in other transient LMXBs. Fine tuning by requiring this marginal state to coincide with for example the onset of the propeller stage and the associated changes in the disk structure as discussed above might resolve this. Therefore, the ionization instability, although unlikely, cannot be ruled out.

Another class of instabilities, that have been discussed in relation to the 5–10 Hz normal-branch oscillations (NBOs, van der Klis 2006) seen in the luminous Z sources, are radiation-driven instabilities (Fortner et al. 1989; Miller & Park 1995). These occur when the systems are close to the Eddington limit, where radiation pressure and gravity become comparable. The frequency is set by the infall time from the largest radius at which the instability occurs: \( \sim 300 \) km for the Z sources. J1808 accretes at a rate far below the Eddington limit, so this type of instability cannot be responsible for the 1 Hz QPO.

### Spruit-Taam instability

The Spruit-Taam instability involves radial perturbations at the magnetospheric boundary (the inner edge of the accretion disk according to Spruit & Taam 1993). It relies on the viscosity in the disk to work, and does not require any shearing of the magnetic field. As discussed in Section 8.4.1, once \( r_m \sim r_c \), accretion is only possible if the density at the inner edge of the disk rises. A small perturbation of the disk radius away from \( r_m = r_c \) will be immediately damped and the disk radius will return to the “equilibrium” position \( r_m = r_c \) where inner disk edge and magnetosphere have the same angular velocity.

However, in the early propeller regime there exists a marginal state. When the inner accretion disk is at equilibrium, a given \( r_m \) corresponds to a specific density. If \( r_m \) moves inward a little bit, the boundary layer is ‘over-dense’ compared to the equilibrium for that smaller \( r_m \), and so quickly empties out. The rapid flow of matter from the inner edge of the disk causes the rest of the co-rotating transition zone to empty out too. The rise in local \( \dot{m} \) pushes \( r_m \) in still further, reinforcing the perturbation. Eventually however the innermost layers are devoid of matter, since the viscous time scale further out in the disk is too slow to have replenished the inner regions. At this stage the boundary
The time scale on which this instability operates is related to the viscous time scale just outside the corotating transition zone of the disk, since this sets the time scale for replenishment of the reservoir. However, this relation is not simple as it also depends on parameters like the average accretion rate and the steepness of the transition between disk and magnetosphere.

According to eq.(8.6), the viscous time scale at $r_c$ for J1808 is $\sim 100$ s, 2 orders of magnitude too long to explain the 1 Hz QPO. However this neglects the dependence of the time scale on the additional parameters and assumes that the appropriate length scale is comparable to the inner disk radius $r_c$. If a shorter length scale were involved then a shorter viscous time scale would be possible (Spruit & Taam 1993). To obtain a 1 second time scale, the length scale of the region of activity would need to be $\sim 1$ km, comparable to the scale of the inner “puffed up” regions in the steady state disk models calculated by Rappaport et al. (2004). The advantage of this mechanism is that the oscillatory state is expected to be active only in a very narrow range of radii from $r_m \simeq r_c$ to $r_m = 1.5r_c$ and hence a small range of accretion rates, as we observed for the 1 Hz QPO (see § 8.3.2).

The Spruit-Taam instability could modulate the accretion flow at high amplitude, which would fit the observations of very high fractional rms amplitudes for the QPO. The instability would also be compatible with the continued presence of accretion-powered pulsations, since accretion could still be funneled even if the inner edge of the disk were oscillating. Finally, the frequency of the instability has a weak dependence on the mass accretion rate (see Fig. 4 in Spruit & Taam 1993), rising or falling whether $r_m$ is greater or less than $r_c$. This weak dependence has been observed in J1808 (Fig. 8.7) with the frequency rising with X-ray flux, thus suggesting $r_m > r_c$.

8.4.4 The mechanism for the 1 Hz QPO

In summary, most of the mechanisms examined cannot, based on our current understanding of how they work, explain key features of the 1 Hz QPO. The most plausible mechanisms are all associated with the onset of the propeller regime. There are a number of other pieces of evidence (cf. § 8.1) that also point to major changes in the accretion environment at the luminosity where the 1 Hz QPO sets in (Wijnands et al. 2001, Wijnands 2003, Campana et al. 2008) - changes which might be explained by the onset of the propeller. In addition there are timing results suggesting a major change in disk structure around this time, such as the $\sim 0.2$ phase drift in the fundamental (arguing for a major change in the disk environment around this time), the change in
the soft lag behavior (Hartman et al. 2008b), and the (debated) detection of an accretion torque (Burderi et al. 2006; Hartman et al. 2008a).

The mechanism proposed by Spruit & Taam (1993) seems to be the most promising candidate to explain the 1 Hz QPO, although the precise details of the time scales for this instability in the situation when funnel flows are relevant remain to be worked out. It has a precise onset point associated with the early propeller regime, should remain relatively stable in frequency as accretion rate varies slightly and is only expected in a narrow range of accretion rates.

Other mechanisms may also play a role, perhaps in concert with the Spruit-Taam instability. In § 8.4.3 we mentioned that new classes of interchange instabilities might operate near the propeller transition, perhaps leading to sporadic accretion. In § 8.4.3 we also discussed the possibility of the ionization instability triggering on short length scales in the inner regions of the disk once the source enters the propeller regime. This possibility is particularly plausible if the disk is already close to the transition from outburst to quiescence. The ionization instability might reinforce the Spruit-Taam instability mechanism, and could also fine-tune the onset conditions for the 1 Hz QPO (see § 8.4.5).

It is hard to understand why the 1 Hz QPO does not appear during the faint re-flares in the 2008 outburst. 8 out of 57 observations were in the 2-15 mCrab range during the re-flaring state. The reason why the 1 Hz QPO is not observed in these 8 observations is an open problem. Although poorly constrained, the 1998 outburst exhibited a similar behavior, and on several occasions during the 2000, 2002 and 2005 outbursts the 1 Hz QPO also remained undetected even for fluxes in the 2-15 mCrab range, with fractional rms amplitude upper limits of \( \sim 10\% \). Clearly the 1 Hz QPO mechanism is not always triggered even in the 2–15 mCrab range in J1808.

### 8.4.5 J1808 and the other AMXPs

In order to enter the propeller regime, J1808 needs to be at the point where \( r_m \sim r_c \). Equating the crude expressions given in eq. (8.3) and (8.4), we obtain a relation between accretion rate, magnetic field and spin rate. Figure 8.11 shows the conditions for propeller onset for particular combination of these parameters. Clearly this is very approximate, since it is based on the simplest estimates of \( r_m \) and \( r_c \), and ignores dependencies on mass and radius, but sufficient to understand whether the propeller scenario is a realistic possibility.

We plot the mass accretion rate values of three well known AMXPs: XTE J1807-294 (spin frequency 190 Hz), J1808 (401 Hz) and IGR J00291+294 (599 Hz). The first object was chosen because its spin frequency is one of the lowest known among AMXPs and its outburst spans a wide range of luminosities.
IGR J00291+294 was chosen because its neutron star has the highest spin frequency known among AMXPs.

Figure 8.11: The accretion rate at which the propeller sets in for particular combinations of magnetic field and spin rate for the three AMXPs (dashed curves). The vertical arrows show the range of accretion rates over which the AMXPs have been observed and refer to X-ray fluxes. They have to be considered lower limits on the true mass accretion rates which are set by bolometric fluxes. The estimated bolometric fluxes at the peak of the outburst (where the mass accretion should be the highest) are marked with a black square (see text for details). The thick dashed vertical line in J1808 indicates the range of mass accretion rates at which the 1 Hz QPO has been observed.

The mass accretion rates used in Fig. 8.11 are calculated for X-ray fluxes in the 2–10 keV energy band, by assuming a neutron star mass of 1.4 $M_\odot$ and an efficiency of 10% for the conversion of rest mass energy of the accreted material into X-ray flux. Since these mass accretion rates do not refer to bolometric fluxes, they have to be considered lower limits. We also marked the bolometric peak luminosity of each source (as reported in Gierliński et al. 2002, Falanga et al. 2005a,b) for assumed distances of 8.5 kpc (IGR J00291+294 and XTE J1807-294) and 3.5 kpc (J1808). The very broad range of luminosities of J1808 are observed thanks to the deeper observations of Swift-XRT (§ 8.3.1, Campana et al. 2008) and XMM-Newton (Wijnands 2003).

For all three sources, the conditions for propeller onset should be encoun-
tered if the field strength is $\sim 10^8 \text{ G}$. For J1808, with a spin of 401 Hz and an accretion rate that runs from a few percent of Eddington at peak, down to less than 0.001 % in the dips between the re-flares, the system must always enter the propeller at the same level of accretion rate unless $B \lesssim 10^6 \text{ G}$. The range of magnetic fields is $(B \sim 0.4\ldots1.5 \times 10^8 \text{ G})$ as reported by Hartman et al. (2008a, 2009). The range of accretion rates for which the 1 Hz QPO appears (inferred from the 2-15 mCrab X-ray flux, § 8.3.2) lies just below this range. This coincidence is quite impressive since the accretion rates are lower limits.

By looking at Fig. 8.11 we can infer that the main reason why the 1 Hz QPO has been observed in J1808 and not in other AMXPs might be related with the proximity of J1808 (3.5 kpc) with respect to the other AMXPs ($\sim 8.5 \text{ kpc}$). If we suppose that the instability is triggered in IGR J00291+5934 at the same mass accretion rates as in J1808, then the expected luminosity would be below the detection threshold of RXTE. None of the known AMXPs has been extensively monitored at low flux levels by Swift-XRT or XMM-Newton, both of which would be able to (easily) test this scenario.

If J1808 enters the strong propeller regime during the re-flares, a large outflow of gas is expected. Hartman et al. (2008a) and di Salvo et al. (2008) observed an anomalously large orbital period derivative which requires a mass loss from the system of $\sim 10^{-9} M_\odot \text{ yr}^{-1}$. The onset of a strong propeller with outflows of matter from the system can in principle play a role in this.

A strong propeller can also in principle explain the long term spin down of the neutron star in J1808 as proposed by Hartman et al. (2008a)

### 8.5 Conclusions

We have performed the first complete study of the 1 Hz modulation, its relation to the 401 Hz pulsations and the re-flares of SAX J1808.4-3658 as observed over 10 years. Several common features are observed in the 2000, 2002 and 2005 outbursts while the 1998 and 2008 outbursts have different properties, the most remarkable one being the absence of a strong 1 Hz modulation.

We focused on the origin of the 1 Hz oscillation that sometimes dominates the re-flare lightcurve and we found that all viable candidate mechanisms are connected with the onset of the propeller stage. The most promising model discussed is the Spruit-Taam instability which explains the stable 1 Hz frequency, its high amplitude and the narrow flux range of its occurrence in a natural way, and is also compatible with the simultaneous presence of 401 Hz pulsations.

Many open issues remain. It is not clear yet why the pulse phase drifts by 0.2 cycles during the fast decay just before the onset of the 1 Hz modulation.
It is also unclear why the pulse phase jumps on time scales of hours or less when the 1 Hz modulation has low amplitudes. However, it is likely that this pulse behavior reflects changes in the accretion flow onto the surface triggered by flow changes associated with the 1 Hz QPO mechanism.

The reason why 1 Hz modulation has not been observed in other AMXPs might be the larger distance of the other sources making them undetectable in the relevant luminosity range. Future monitoring of low level flux states of other AMXPs will be very important to extend our comprehension of the 1 Hz modulation and further probe the onset of the propeller stage.
9 Samenvatting in het Nederlands

9.1 De accreterende milliseconde röntgenpulsars

De meeste sterren die we waarnemen in het heelal zijn meervoudige systemen die bestaan uit twee of meer sterren die bij elkaar worden gehouden door zwaartekracht. Doordat zwaartekracht over grote afstanden werkzaam is, is het lastig om exact te definiëren wanneer iets een "geïsoleerde" ster of een meervoudig systeem is. Echter, meervoudige systemen bestaan meestal uit sterren die veel dichter bij elkaar staan dan de gemiddelde afstand tussen geïsoleerde sterren. Sommige van deze meervoudige systemen zijn dubbelsteren bestaande uit twee sterren die zo dicht bij elkaar staan dat ze tijdens hun leven in direct contact met elkaar kunnen komen. Wanneer één van de twee sterren tamelijk zwaar is (ongeveer 10-20 maal zo zwaar als de zon) kan deze exploderen als een supernova, één van de meest energetische gebeurtenissen in het heelal waarbij een enorme hoeveelheid energie en gas de ruimte in worden geblazen. Ondanks dat een supernova explosie een catastrofale gebeurtenis is kan de andere ster in het dubbelstersysteem deze dramatische periode overleven en door zwaartekracht verbonden blijven aan het overblijfsel van de geëxplodeerde ster.

Wat overblijft na de supernova-explosie is één van de meest raadselachtige en fascinerende objecten in het universum: een neutronenster of een zwart gat. Neutronensterren zijn bolvormige objecten met een straal van ongeveer 10 km waarvan de afmeting heel klein is vergeleken met de straal van een normale ster (de zon heeft bijvoorbeeld een straal van ongeveer 1.000.000 km) en welke heel snel roteren met een frequentie van ongeveer 0.1 tot zelfs 700 keer per seconde. Het meest indrukwekkende is dat de massa van deze objecten vergelijkbaar is met die van de zon (namelijk tussen ongeveer 1.4 en misschien wel 2-2.5 maal zo groot). Een neutronenster bevat dus ongeveer dezelfde hoeveelheid materie als de zon, maar dan in een volume dat 1.000.000.000.000.000 zo klein is!
Aangezien de dichtheid van een object gelijk is aan zijn massa gedeeld door het volume dat hij inneemt, is het duidelijk dat een neutronenster een veel hogere dichtheid heeft dan ieder ander object dat we kennen. Een neutronenster heeft een dichtheid vergelijkbaar met (en zelfs groter dan) de dichtheid van een atoomkern. Om de indrukwekkende dichtheid van een atoomkern te begrijpen: een theelepeltje van atoomkernen zou ongeveer 100.000.000.000 kg wegen! Aan de hand van dit voorbeeld is het niet moeilijk te begrijpen dat als zo’n kleine hoeveelheid materie al een massa van honderd miljard kilogram kan hebben, een bol met een straal van 10 km, zoals een neutronenster, een verbazingwekkende hoeveelheid materie kan bevatten. Bovendien hebben neutronensterren ook enorm hoge magneetvelden, die ieder magneetveld dat ooit in aardse laboratoria is geproduceerd ver te boven gaan. De oorsprong van deze magneetvelden is nog steeds onduidelijk, maar ze komen overeen met de magneetvelden van de aarde met dit verschil dat hun sterkte tenminste honderd miljoen en zelfs wel een biljard maal hoger is dan de aardse magneetvelden.

Op dit punt bestaat het dubbelstersysteem uit een neutronenster en een normale ster die om elkaar heen draaien. Het systeem kan zich stilhouden, zelfs voor een hele lange tijd (in de orde van miljarden jaren), terwijl de sterren in het dubbelstersysteem bewegen zoals de aarde, de andere planeten en de zon in ons zonnestelsel bewegen. Op een bepaald punt in de evolutie kan hier een radicale verandering in komen wanneer de begeleidende ster te dicht bij de neutronenster komt of wanneer de atmosfeer van de begeleidende ster te veel uitzet (het uitzetten van de buitenste atmosferische lagen is een algemeen verschijnsel dat bij elke ster op een zeker moment in de evolutie optreedt). De zwaartekracht van de neutronenster trekt het gas uit de buitenste atmosferische lagen van de begeleidende ster naar zich toe. Het onttrokken gas valt niet in een rechte lijn naar de neutronenster toe, maar beweegt draaiend naar binnen, vergelijkbaar met hoe water in een gootsteen soms via een spiraal naar het putje beweegt, en vormt een schijf. De schijf strekt zich uit van een punt dicht bij het oppervlak van de neutronenster (enkele honderden meters daar vandaan) naar een buitengebied waarvan de positie afhangt van de onderlinge afstand tussen de neutronenster en de begeleidende ster. De omvang van de schijf varieert van enkele duizenden tot honderdduizenden kilometers of in de meest extreme gevallen zelfs wel miljoenen kilometers. In dit proefschrift heb ik me geconcentreerd op dubbelstersystemen die dit stadium hebben bereikt en waar de neutronenster waarschijnlijk tamelijk oud is (tot enkele miljarden jaren) en de begeleidende ster een massa heeft die kleiner is dan de massa van de zon.

Wanneer het materiaal in de schijf naar binnen beweegt neemt de rotatie-
snelheid toe om dezelfde reden als deze toeneemt bij een ronddraaiende schaatser die zijn uitgestrekte armen naar zijn lichaam toe beweegt. Wanneer het gas de binnenste delen van de schijf bereikt, dicht bij het oppervlak van de neutronenster, ondervindt het de kracht van het enorme magnetisch veld van de neutronenster: het wordt afgebogen en in de richting van de magnetische polen van de neutronenster geleid. Dit proces, genaamd accretie, lijkt erg op wat er gebeurt op onze planeet wanneer geladen zonnedeeltjes worden afgebroken door het aardse magnetisch veld in de richting van de magnetische polen, wat het spectaculaire fenomeen poollicht (noorderlicht) veroorzaakt. Echter, vergeleken met het poollicht op aarde, heeft het gas dat het oppervlak van de neutronenster raakt een veel hogere energie en is het magnetisch veld vele malen krachtiger. De fotonen die bij de inslag vrijkomen hebben daardoor ook een hogere energie, die typisch in het röntgengebied ligt. Deze röntgenfotonen worden dan gedetecteerd met röntgensatellieten die in de ruimte om de aarde heen bewegen.

Gezien het feit dat de neutronenster ronddraait terwijl deze röntgenstraling uitzendt, zal een waarnemer op afstand de röntgenfotonen detecteren met een modulatie die gelijk is aan de rotatiesnelheid van de neutronenster. Een vergelijkbaar effect kan worden waargenomen door iemand die kijkt naar een vuurtoren die met een bepaalde snelheid ronddraait. De waarnemer zal een lichtflits zien zodra de vuurtoren in zijn richting schijnt, maar ziet niets wanneer deze in de tegenovergestelde richting wijst. Als de waarnemer een referentieklok heeft en meet hoeveel ‘pulsen’ van licht hij/zij kan tellen in een seconde, dan kan deze de rotatiesnelheid van de vuurtoren bepalen. Dit eenvoudige principe is niet veel anders van wat gebruikt is in dit proefschrift: we meten de aankomsttijd van elk röntgenfoton dat door een neutronenster wordt uitgezonden en reconstrueren de pulsaties om zo de rotatiesnelheid van de neutronenster te bepalen. Dit is de reden waarom neutronensterren die op deze manier röntgenpulsaties vertonen accreterende röntgenpulsars worden genoemd. Met deze simpele techniek kan men niet alleen de rotatiesnelheid van de neutronenster bepalen, maar ook kleine variaties daarin die worden veroorzaakt door tal van andere belangrijke effecten zoals de baanperiode van het dubbelsternsysteem, de variatie in de positie van het gebied dat de röntgenstraling uitzendt en de intrinsieke instabiliteit in de rotatie van de neutronenster. Een andere fundamentele eigenschap die in principe kan worden gemeten is de verandering van de rotatiesnelheid van de neutronenster wan-

\[1\text{De reden dat de röntgentelescopen zich buiten de aardatmosfeer bevinden is dat kosmische röntgenstraling wordt geabsorbeerd door atomen in de atmosfeer. Dit is wellicht teleurstellend voor een röntgen-astronoom, maar van fundamenteel belang om leven op aarde mogelijk te maken, omdat röntgenstraling erg schadelijk is voor het DNA in cellen.} \]
neer het gas vanuit de schijf op het oppervlak van de neutronenster terecht komt: omdat het gas in de schijf sneller rond kan draaien dan de neutronenster wordt de "overtollige" snelheid overgedragen op de neutronenster, wiens rotatie daardoor wordt versneld. Dit proces kan een neutronenster versnellen tot spectaculaire snelheden van honderden rotaties per seconde. Aangezien de rotatieperiode (d.w.z. de tijd die het kost om één volledige rotatie uit te voeren) van een neutronenster die honderden malen per seconde ronddraait slechts een paar milliseconden is, worden sommige systemen accreterende milliseconde pulsars genoemd.

Niet alleen aan de hand van pulsaties die ontstaan door dit simpele mechanisme van accretie kunnen we belangrijke fysica bestuderen, maar ook door middel van een meer gewelddadig proces dat plaatsvindt op sommige neutronensterren: thermonucleaire röntgenflitsen. Dit zijn thermonucleaire explosies die op het oppervlak van een neutronenster plaatsvinden wanneer waterstof en helium zich ophopen tijdens de accretie van materie van de schijf. Deze uitbarstingen kunnen worden waargenomen als intense flitsen van röntgenstraling die tientallen tot honderden seconden kunnen duren en de totale röntgenhelderheid van het systeem doen toenemen met een factor 100 − 1000. Tijdens deze uitbarstingen worden vaak röntgenpulsaties waargenomen. Deze pulsaties hebben een andere oorsprong dan degene die eerder zijn beschreven, de plek waar de röntgenstraling vandaan komt is namelijk onbekend: of deze bij de magnetische polen, de equator of een ander gebied wordt geproduceerd is nog steeds niet duidelijk.

Uiteraard komen de waargenomen hoogenergetische fotonen niet alleen van het oppervlak van de neutronenster tijdens accretie of een röntgenflits, maar ook van het binnenste deel van de accretieschijf. Deze gebieden zijn zeer heet, ze worden namelijk opgewarmd door de viscositeit van het materiaal dat in de schijf roteert met snelheden die de snelheid van het licht benaderen. De bijbehorende temperaturen kunnen oplopen tot 10.000.000 K en worden waargenomen als röntgenemissie die optelt bij de pulserende röntgenstraling die van het oppervlak van de neutronenster komt. Zowel de pulserende als de niet-pulserende straling kan worden bestudeerd aan de hand van röntgenspectra: de waarnemer meet de energie van elk foton en telt hoeveel fotonen van een bepaalde energie worden ontvangen. Het basisprincipe is gelijk aan dat van dispersie van optisch licht dat een regenboog produceert: de meest energetische fotonen (violet) en de minst energetische fotonen (rood) worden gescheiden en waargenomen met een continue reeks van tussenliggende waarden.

Tenslotte kunnen de eigenschappen van de schijf, de magnetosfeer van de neutronenster en de sterke zwaartekracht worden bestudeerd via oscillaties
die in de schijf ontstaan. Deze zogenoemde quasi-periodieke oscillaties hebben een behoorlijk gecompliceerde fenomenologie en vinden hun oorsprong in verschillende mechanismen. Echter, een simpele intuïtieve verklaring is niet veel anders dan het vuurtoreneffect dat eerder in de tekst is uitgelegd: het hete gas roteert in de accretieschijf en als het gas niet volledig homogen is, bijvoorbeeld doordat een klont materiaal iets heter is dan de omgeving, kan dit een modulatie veroorzaken in de röntgenstraling. Deze frequenties zijn niet zo stabil als de rotatiefrequentie van de neutronenster, omdat de rotatie van het gas gemakkelijk beïnvloed kan worden door de omgeving en omdat ze betrekking kunnen hebben op diverse gebieden van de schijf die op iets verschillende snelheden roteren. Er zijn ook andere mogelijkheden om dergelijke oscillaties te veroorzaken, zoals globale oscillaties in de schijf (vergelijkbaar met de vibratie van een elastisch lichaam) of veranderingen in de structuur van de binnenste delen van de schijf wanneer deze een interactie aangaat met het magneetveld van de neutronenster.

9.2 De inhoud van dit proefschrift

In dit proefschrift hebben we de complexe fenomenologie van accreterende milliseconde pulsars onderzocht.

In § 2 bestuderen we de röntgenpulsaties die worden gevormd tijdens thermonucleaire röntgenflits in één bepaalde accreterende milliseconde pulsar. We hebben daarbij gevonden dat tijdens de röntgenflitsen de pulsaties van de magnetische polen afkomstig zijn, in tegenstelling met eerdere bevindingen.

In § 3 onderzoeken we de baan en de pulsaties van een andere milliseconde röntgenpulsar: de waargenomen pulsaties bleken te verdwijnen en te verschijnen in de loop van de tijd en dit "flikkerende" gedrag is nog steeds onbegrepen.

In § 4 analyseren we 7 jaar aan waarnemingen van de eerst ontdekte accreterende milliseconde pulsar, waarbij we vinden dat zowel de baan als de rotatiesnelheid van de neutronenster een afwijkende evolutie in de tijd laten zien.

In § 5 bestuderen we deze bron nogmaals, maar deze keer concentreren we ons op het spectrum en de pulsaties die werd waargenomen in de herfst van 2008, wat voor het eerst het pulsatiegedrag bij lage energieën blootlegde.

In § 6 beschouwen we een andere accreterende milliseconde pulsar en laten we zien dat de waargenomen variaties in de rotatiesnelheid van de neutronenster slechts schijn zijn en mogelijk worden veroorzaakt doordat het gebied dat röntgenstraling uitzendt zich verplaatst over het oppervlak van de neutronenster.
In § 7 laten we zien dat het optreden van deze variaties een algemene eigenschap is binnen de accreterende milliseconden pulsars en gerelateerd kunnen worden aan veranderingen in de hoeveelheid gas die van de schijf op het neutronenster oppervlak valt.

In § 8 bestuderen we 10 jaar aan waarnemingen van een accreterende milliseconde pulsar en brengen we de pulsaties die aan het oppervlak van de neutronenster ontstaan in verband met oscillaties in de schijf, wat demonstreert dat de pulsaties en de variabiliteit van de schijf gerelateerde processen zijn.
10 List of publications

10.1 Refereed publications

Radio pulsars around intermediate-mass black holes in superstellar clusters

The ultraluminous X-ray source in M82: an intermediate-mass black hole with a giant companion

Discovery of Coherent Millisecond X-Ray Pulsations in Aquila X-1

Intermittent Millisecond X-Ray Pulsations from the Neutron Star X-Ray Transient SAX J1748.9-2021 in the Globular Cluster NGC 6440

The Long-Term Evolution of the Spin, Pulse Shape, and Orbit of the Accretion-powered Millisecond Pulsar SAX J1808.4-3658

Optical emission from massive donors in ultraluminous X-ray source binary systems

183
*Weighing the black holes in ultraluminous X-ray sources through timing*

*Coherence of Burst Oscillations and Accretion-Powered Pulsations in the Accreting Millisecond Pulsar XTE J1814-338*

*Phase-Coherent Timing of the Accreting Millisecond Pulsar SAX J1748.9-2021*

*Broad Relativistic Iron Emission Line Observed in SAX J1808.4-3658*

11. Kaur, R.; Wijnands, Rudy; Patruno, Alessandro; Testa, Vincenzo; Israel, Gianluca; Degenaar, Nathalie; Paul, Biswajit; Kumar, Brijesh 2009, MNRAS 394, 1597
*Chandra and XMM-Newton observations of the low-luminosity X-ray pulsators SAX J1324.4-6200 and SAX J1452.8-5949*

*SAX J1808.4-3658: high resolution spectroscopy and decrease of pulsed fraction at low energies*

*An alternative interpretation of the timing noise in accreting millisecond pulsars*

### 10.1.1 Papers submitted to refereed journals

Accretion torques and motion of the hot spot on the accreting millisecond pulsar XTE J1807-294

1 Hz flaring in SAX J1808.4–3658: flow instabilities near the propeller stage

Discovery of burst oscillations near the spin frequency in the intermittent accretion-powered millisecond pulsar HETE J1900.1-2455

10.2 Non-refereed publications

The Dynamical Fingerprint of Intermediate Mass Black Holes in Globular Clusters

Near-IR and Optical Observations of the Ongoing Outburst of SAX J1808.4-3658

Radio non-detection of SAX J1808.4-3658

Motion of the hot spot and spin torque in accreting millisecond pulsars
_Pulse timing of the ongoing outburst of SAX J1808.4-3658 with RXTE_

_Clarification of orbital elements for SAX J1808.4-3658_

_Broad relativistic iron line observed in SAX J1808.4-3658 by Suzaku_

_Discovery of the shortest recurrence time between thermonuclear X-ray bursts_

_Discovery of burst oscillations near the spin frequency in the intermittent accreting millisecond pulsar HETE J1900.1-2455_
Dear friends, the time has come. Here you can finally check if your name appears in the list below. As my mother, my partner and Michiel often say (Freud might want to comment on this...), I usually say the opposite of what people claim. Therefore this acknowledgments do contain a list of people I’d like to thank, since the original sentence “I will skip the acknowledgments” has been already used and consumed by Nacho and Salvo. So, no more fun.

The first two persons I’d like to thank are of course my bosses Rudy and Michiel for the constant support and for believing in my skills. Since Rudy offered me a job here in Amsterdam I’m afraid I can’t say anything naughty about him... I’ve to thank Michiel for impressing me with his proverbial charisma since my very first day here in The Netherlands. I was just arrived for the interview, and right after that, we went for lunch where I clashed with the world famous dutch delicacies. Michiel took a whole bite of raw herring and drank some karnemelk\(^1\) on top of it. In that moment I saw the light: my roaming was finally over, I had found the beloved country I’d been always looking for. Two angels, fiery in aspect, came on a bike holding a Columbus and brought me in the magic world of Amsterdam. A few weeks later, Rudy and Michiel decided to offer me the Ph.D. position, and I accepted with great joy.

The first year wasn’t that productive, I have to admit. But that was mainly because Nanda kept confusing me with weird plots on the meaning of “magnitude”. After deep searches, and with the help of an anonymous referee, she arrived at the conclusion that a magnitude is 2.5 times the log of flux plus an aubergine. You’ll understand this was too much to handle for a young Ph.D. student. In this first year of deep thoughts I was helped by Peter Curran (still single, at those glorious times...), Martin and the East of Eden crew. It

\(^1\)very typical Dutch drink consisting of cow milk left under the Sun for 4-5 weeks (according to the definition of Simone Migliari).
11. Acknowledgments

took several months to recover, but at the end it did pay off. Thanks to the growth of my knowledge and my spirit, Rudy had the fantastic idea to send me around the world to spread the message. Dubna, Gregynog, Ladek Zdroj. Places as unforgettable as their names... Rudy, I thank you for this!

The fact that those expeditions were tough was finally proved by my office mate prof. dr. drs. mr. miss Diego Altamirano (as he likes to be called by Indians students). He went once in an expedition in India, and came back with unexpected skills: Diego, I thank you for teaching me how a blatant asymmetric object can be somehow-what-kind-symmetric.

I’d like to spend also some nice words on Gemma, former office-mate, that could handle two Italians and a half\(^2\) for about two years. Thanks for that, Gemma. I’m sure that reading the best seller “The Xenophobe’s guide to the Italians” gave you a lot of strength during these hard times.

And how to forget Pg...

...
...

Clearly there are others in the institute I’d like to thank, for the fantastic atmosphere and the myriad gossips I enjoyed with many of you (also for the scientific discussions, of course...). I have to thank therefore Rudy and Yuri for the coffee sessions and especially Yuri for bringing us the “Midget Dance”. Thanks Yuri, also for reading the draft of this thesis in my place. I’ve also to thank Paolino for being so Italian to let us understand why Berlusconi rules undisturbed. Also for the fights with Valeriu about the best technique to clean dishes. Thanks to Lucinda for interesting discussion on the only topic she likes to talk about, Lianne for trying to poisoning me at dinner, Atakan for his Turkish coffee and his sense of humor, Arjan for his conspiracy theories, Eduardo for not paying dinners, Huib for paying me a lunch, Kostadinos for cigarettes, Manu for his pilgrimage in Molfetta and Dave for further clarifications on the concept of magnitude. Special mention goes to Hendrik that explained me that it’s possible to get a free herring the 4th of October in Leiden by filling a form at the municipality. I won’t forget.

I’d like to thank also Marta and Cinzia, especially for the very relaxing moments in Ireland. Marta, I apologize for talking with your boss Gino. I couldn’t believe he wanted really to fire you after the nice chat I had with him!

Thanks goes also to Flavio, that came here in Amsterdam to leave the homophobic and clerical atmosphere one breathes in Italy, and showed me

\(^2\) Controversies exist among ethologists, about the possibility of defining Diego a whole Italian.
that being an atheist in The Netherlands is no fun: the Dutch don’t care about God (which is not enough practical concept) but they do believe in Sinterklaas\(^3\).

Great thanks goes also to Stratos, for gambling my life by locking me out of the room in Montreal, while a furious black giant was looking for me in the streets (to kill me).

Thanks to Claudio Germana’ for his impossible theories and questions that stimulated several non-sense discussions.

A great thanks goes to Nathalie, that helped me with the “samenvatting in het Nederlandse”. Thanks a lot ! Great thanks also to Jake Hartman, nice discussions and lots of fun when we meet and when you are not sleeping in my room on a couch with a beer in a hand (contact me for videos on the episode). Thanks to Anna for all her support and the very nice discussions on theories of neutron stars. I enjoyed a lot !

Thanks goes also to my cousin Nadia, for puking every day (for different reasons) when she came visiting me here. Thanks also to my cousin Claudia and my brother Fabrizio that blamed me constantly for not calling home during these four years. Thanks also for not throwing up every day (differently from Nadia) when visiting me. Thanks to all those friends in Molfetta and many others that I cannot mention ’cause the list is too long.

And finally...thanks to Paola, that always supported me and was willing to fight at the expense of the house integrity, and that still wonders who’s that woman that once told me I’m a too modest guy.

Groetjes !

Alessandro

---

\(^3\)A small (but growing) Dutch sect guided by Peter Polko does also believe in lobsters walking in the city streets

\(^4\)Another (but not growing) sect guided by Gabriele Merolli does also believe in reptiles living at the Earth’s center (empty), where also Atlantis is located, of course.
Bibliography

on the Timing Studies of X-ray Sources, 1–11
on the Timing Studies of X-ray Sources, 13–28
Pulsars, vol. 328 of Astronomical Society of the Pacific Conference Series,
279–289
Society, 657–+
(eds.) The Multicolored Landscape of Compact Objects and Their Explosive
Freire P.C.C., Ransom S.M., et al., Nov. 2007, ArXiv e-prints, 711
   Astronomical Society, 954–+
   363–444, Dordrecht: Kluwer

193

194
Poutanen J., 2006, Advances in Space Research, 38, 2697
Spitzer L., 1978, Physical processes in the interstellar medium, New York
Strohmayer T., Bildsten L., Apr. 2006, New views of thermonuclear bursts,
  113–156, Compact stellar X-ray sources

196
van der Klis M., 2000, ARA&A, 38, 717