Rotational glitches in radio pulsars and magnetars

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Introduction


1 Introduction

Neutron Stars

The idea of a compact object composed of dense neutron matter emerged very soon after the discovery of the neutron particle itself by Chadwick (1932). Baade & Zwicky (1934) were the first to use the term "neutron stars" to describe these celestial bodies, and to suggest they form in supernova explosions as one of the possible outcomes of stellar evolution.

Despite the early theoretical predictions, the existence of neutron stars was not observationally confirmed until 1967. In July of that year, Jocelyn Bell-Burnell discovered the source PSR B1919+21 emitting radio pulsations at a constant period of 1.337 s (Hewish et al. 1968). Soon after this first discovery more sources like this were observed and were named pulsars (Pulsating Radio Stars). Though it was already suggested by Pacini (1967) that a strongly magnetised neutron star might emit in the radio band, the connection of pulsars to neutron stars was established by Gold (1968), who proposed the model of a rotating magnetised neutron star to explain the pulsed emission. The subsequent discovery of the Vela pulsar in the supernova remnant Vela X (Large et al. 1968) and the Crab pulsar in the homonymous nebula (Staelin & Reifenstein 1968) confirmed the identification of pulsars as neutron stars, formed in supernovae.

Neutron stars are the densest and most strongly magnetised objects known in the Universe. Their modelling is one of the most challenging physical problems, since it needs to draw and combine knowledge from almost every field of physics. But it is also one of the most rewarding problems to attack, since neutron stars are the only available laboratory to explore how nature works in such extreme conditions. The study of their rotational dynamics, which is a very powerful tool to probe their properties, is the main subject of this thesis, with the focus on rotational "glitches".
1.1 Formation and structure of isolated neutron stars

At the end of its life, a star, short on fuel, cannot produce enough energy in its interior to provide the pressure that supports it against its own gravity. The subsequent collapse might stop early if the degenerate electron pressure balances gravity, and leaves behind a white dwarf as will happen to our Sun. For heavier stars the collapse continues until the central region gets so dense that nuclei dissolve, forming a conglomerate of nucleons and electrons. The entire stellar core turns to one single nucleus, with huge mass number and a much smaller atomic number. If the abundant degenerate neutrons provide the necessary pressure to oppose gravity the remnant is a neutron star. It is not known whether at even more extreme conditions some other form of pressure is able to stabilise the star against collapse, but it is believed that if the central neutron star pressure is not enough then nothing will stop the collapse, and a black hole will form.

1.1.1 Birth of a neutron star in the aftermath of a supernova explosion

Isolated neutron stars form by the gravitational collapse of progenitor stars with initial masses $M_*$ greater than $\sim 8 M_\odot$, where $M_\odot \approx 2 \times 10^{33}$ g is the Solar mass\(^1\) (Woosley et al. 2002). The initial stages of stellar evolution are common to all stars, however their later evolution and possible end points are quite different. Because the fusion rate increases strongly with stellar mass, heavier stars leave the Main Sequence rather quickly, despite having a larger fuel reservoir than lighter stars. When the initial stellar mass is more than $\sim 2.25 M_\odot$, helium ignites before the core is fully degenerate, without the characteristic helium flash of lighter stars. The fusion of carbon begins around $\sim 0.8 \times 10^9$ K, a temperature that cannot be achieved before the core is fully degenerate if $M_* \leq 8 M_\odot$. Because carbon fusion is highly temperature dependent and releases large amounts of energy, its ignition under degenerate conditions might be unstable. For progenitors with mass greater than $\sim 10 M_\odot$ however, carbon ignites before the degeneration of the core is complete and the evolution continues with the ignition, first in the core, followed by the outer shells, of the products of every previous reaction until iron is formed. Because iron is the stablest nucleus to fusion, the cycle of reactions ends there.

The central iron core has a mass of $\sim 1.5 M_\odot$ and is supported mostly by the degenerate electron pressure. Fusion of silicon in the outer layers increases the iron core mass up to the Chandrasekhar limit, at which point gravitational contraction and heating of the core recommences. Under these conditions neutrinos are produced, which escape freely for densities lower than $\sim 10^{11}$ g cm\(^{-3}\), contributing to the cool-

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\(^1\)Neutron stars can also descend from white dwarfs in multiple systems, which accrete matter from a companion star and collapse when their mass increases above the Chandrasekhar limit.
ing of the star. Any nuclear reactions taking place in the central regions will further reduce the energy because fusion of iron is an endothermic reaction. The loss of thermal energy accelerates the contraction of the core.

When the central temperature reaches a few billion degrees Kelvin, photodissociation of iron nuclei to alpha particles and neutrons begins, according to the reaction $^{26}\text{Fe} + \gamma \rightarrow ^{13}\text{He} + 4n$, further reducing the temperature and accelerating the collapse. In the end even alpha particles photodisintegrate to protons, neutrons and electrons ($^2\text{He} + \gamma \rightarrow 2p + 2n + 2e^-$) and the collapse occurs at nearly free-fall speed. The abrupt increase in density of the electrons, which are by this stage fully degenerate and relativistic, leads to the inverse $\beta$-decay, $e^- + p \rightarrow n + \nu$. In a non-degenerate environment neutrons are unstable, however at the degenerate stellar core they stabilise. This stage, where protons and electrons are converted to neutrons, is called neutronization. The reduction of electron density results in further decrease of the core’s pressure. Once the density reaches $\sim 10^9$ g cm$^{-3}$ electrons become so energetic ($\gtrsim 1.3$ MeV) that they can interact with bounded protons in the atomic nuclei, a process called electron capture. This results in very neutron-rich nuclei, which are also stabilised due to the degeneracy. At even higher densities, beyond $4 \times 10^{11}$ g cm$^{-3}$, nuclei are so close to each other that neutrons are no longer bound and start “dripping” out of the nuclei (neutron drip). Lastly, at densities close to that of nuclear matter, $\rho_0 = 2.8 \times 10^{14}$ g cm$^{-3}$, the stellar core is a very hot fluid of degenerate neutrons and a small percentage of electrons and protons. At this point, if the degenerate neutron pressure can balance gravity the collapse will come to an halt. The outgoing shock wave will be the trigger of a supernova explosion, which ejects parts of the outer layers of the star. The remaining compact object, a proto-neutron star, will have an average density of the order $10^{14}$ g cm$^{-3}$ and typical values for mass and radius of $\sim 1.5 \, M_\odot$ and $\sim 10^6$ cm respectively.

1.1.2 Structure and composition

The first attempt to calculate the equation of state of a neutron star was by Oppenheimer & Volkoff (1939), soon after the prediction of these objects was made. Assuming as a first approximation a star composed purely from non-interacting neutrons, their model predicted a maximum mass of $0.7 \, M_\odot$ (Oppenheimer & Volkoff 1939; Tolman 1939). Close to a century later, a firm equation of state that describes the central regions of a neutron star is still lacking. However modern estimates and recent constraints from observations put the maximum mass limit around 2 to 3 $M_\odot$. The density of nuclear matter at saturation, where the energy per nucleon is minimum, is $\rho_0 = 2.8 \times 10^{14}$ g cm$^{-3}$. In the central regions of neutron stars the density is expected to be much larger, possibly even an order of magnitude greater than $\rho_0$. The behaviour of matter under such extreme conditions pushes the limits of our current
1.1 Formation and structure of isolated neutron stars

In this section we present the science objectives of LOFT starting with dense matter, followed by strong field gravity and ending with the observatory science.

2.1 Supranuclear Density Matter

2.1.1 Introduction

Neutron stars are the densest objects in the Universe, attaining physical conditions of matter that cannot be replicated on Earth. Inside neutron stars, the state of matter ranges from ions (nuclei) embedded in a sea of electrons at low densities in the outer crust, through increasingly neutron-rich ions in the inner crust and outer core, to the supranuclear densities reached in the centre, where particles are squeezed together more tightly than in atomic nuclei, and theory predicts a host of possible exotic states of matter (Figure 2-1). The nature of matter at such extreme densities is one of the great unsolved problems in modern science, and this makes neutron stars unparalleled laboratories for nuclear physics and QCD (quantum chromodynamics).

![Figure 2-1](http://sci.esa.int/loft/53447-loft-yellow-book/)

Figure 1.1: Schematic illustration of the structure and composition of a neutron star. As illustrated here, in the inner crust and especially close to the crust-core boundary, the equilibrium surface shape of the nuclei is no longer expected to be spherical (Chamel & Haensel 2008). Figure credit: LOFT Yellow Book, http://sci.esa.int/loft/53447-loft-yellow-book/

The structure of an isolated neutron star can be separated into 5 regions: the atmosphere, the outer and inner crust and the outer and inner core (Figure 1.1). The atmosphere is only a few metres thick and is mostly composed of H, He, C and O, in abundances that depend (for isolated stars) on the neutron star’s age and the conditions of the progenitor supernova explosion.

In less than a minute after the birth of a neutron star its surface temperature is expected to drop below $10^{10}$ K, mostly via neutrino emission. At such temperatures a crust solidifies, as the nuclei form a body centred cubic lattice (bcc) (Chamel & Haensel 2008). The outer crust is composed of nuclei and electrons. The electrons are degenerate and relativistic everywhere except in an outer layer, a few metres

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$^2$References and additional details on the structure and equation of state of neutron stars can be found in the book of Haensel et al. (2007).
thick. Near the surface the nuclei are mostly iron but as the density gets above $\sim 7 \times 10^6 \text{ g cm}^{-3}$ nuclei richer in neutrons appear. The inner crust begins at the neutron drip point, at density $\rho \sim 4.3 \times 10^{11} \text{ g cm}^{-3}$. In this region the neutron-rich nuclei and the electron gas coexist with free neutrons. These neutrons, as well as the nucleons inside the nuclei, are expected to be in a condensed state at such densities because of their low temperature relative to their Fermi temperature. The free neutrons comprise a superfluid that penetrates the lattice. The percentage of free neutrons increases with increasing density and at the deeper layer of the crust the nuclei dissolve completely. The exact transition density from the inner crust to the core is still unknown, but is thought to be between $\sim (0.5 - 1)\rho_0$ therefore the inner crust has a thickness of one to few kilometres (Chamel & Haensel 2008).

The innermost parts of a neutron star, where the density reaches far beyond $\rho_0$, are not well understood (Lattimer 2012). The maximum central density that can be reached before the star becomes unstable to collapse is also unknown, but is expected to be below $5 \times 10^{15} \text{ g cm}^{-3}$ (Lattimer & Prakash 2007). The composition up to densities $\sim 2\rho_0$, the region called the outer core, is less uncertain. It has a thickness of a few kilometres and for some neutron stars it might extend down to the centre. The outer core consists mostly of neutrons, and a small percentage (probably $\leq 10\%$) of protons, electrons and muons, all of which will be degenerate. The system has to be neutral and stable to $\beta$-decay reactions. Under the local conditions, the electrons and muons can be regarded as an almost ideal fermionic gas, while the neutrons and protons form a fluid of strongly interacting fermions, which are expected to be in a superfluid state.

For stars whose central density exceeds $2\rho_0$ another region, the inner core, must be considered separately since other kinds of fermions and/or boson condensates might appear (see for example Alford et al. 2001, for a discussion of the possibility of crystalline color superconductivity in neutron star cores). Some models predict the formation of hyperons, $\Sigma$, $\Lambda$ and $\Xi$, and $\Delta$ resonances, in abundances that depend on the total neutron star mass. The nucleon-hyperon and hyperon-hyperon interactions are not yet well understood, therefore the equation of state describing the inner core is uncertain (Schaffner-Bielich et al. 2002; Lattimer 2012). The huge pressure in the innermost regions of a neutron star might lead to the decomposition of hadrons to quarks. Such a phase transition must preserve the baryonic and electric charge. Some up and down quarks will convert to strange quarks (which is the lightest quark, with mass $\sim 150 \text{ MeV}$) (Alcock et al. 1986). Neutron stars with strange quarks in their core are often called "hybrid" stars, it is however unknown whether they can be stable (see for example Bonanno & Sedrakian 2012). Because the emergence of

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3A few details on fermionic superfluidity can be found in section 1.4.1.
4Hyperons are baryons that consist of three quarks, of which at least one is a strange quark.
new degrees of freedom is theoretically expected to soften the equation of state, independent measurements of the mass and radius for individual sources, as well as the discoveries of very heavy neutron stars, have the potential of excluding some of the above possibilities.

1.2 Rotationally Powered Pulsars and Magnetars

The first pulsars that were discovered in the radio band triggered a great scientific interest for the detection and study of neutron stars, which can emit radiation across a range of wavelengths. The famous Crab pulsar was the first to be detected in the optical (Cocke et al. 1969; Willstrop 1969) and shows pulsations also in the X and γ-rays (Fritz et al. 1969; Bradt et al. 1969; Albats et al. 1972). To date, more than 2300 pulsars have been identified, mostly in the radio band.

Neutron stars that appear as pulsars rotate extremely fast, with measured spin periods $P$ that range from $\sim$ 12 sec to $\sim$ 1.4 ms (The Australia Telescope National Facility Pulsar Catalogue Manchester et al. 2005). The fastest pulsar known completes 716 full rotations in only one second; at such velocities any gravitationally bound object less dense than a neutron star would have been torn apart (Hessels et al. 2006). The large angular velocity of a neutron star is naturally explained by conservation of angular momentum during the collapse of the progenitor. Millisecond pulsars are most likely further accelerated by mass accretion from a companion donor star (Bhattacharya & van den Heuvel 1991). This scenario is strongly supported by the fact that, while only 1% of normal pulsars belong to a binary system, this percentage increases to 80% for millisecond pulsars. Pulsars lose rotational energy to their environments thus their periods slowly increase with time, at a typical rate of $\dot{P} \sim 10^{-15}$.

The rotating magnetic dipole model provides an adequate first explanation for neutron star pulsations and braking, although it clearly does not describe the full picture. The model assumes a neutron star with magnetic field $B$, which at first approximation can be regarded as a dipole with its axis misaligned with respect to the rotational axis (Figure 1.2).

The exact radio emission mechanism is poorly understood and still one of the major open questions. Strong electric fields, induced by the neutron star’s rotation and magnetic field, accelerate particles off its surface which, via secondary processes like pair creation, create a dense magnetosphere that follows the pulsar’s rotation (Goldreich & Julian 1969). The energetic charged particles of this magnetosphere emit a narrow beam of photons, which is likely collimated along the magnetic axis. Since the latter is typically not aligned with the rotation axis, if it crosses our line of sight the emission appears as pulses of radiation, with a frequency equal to the
neutron star’s spin frequency (Eastlund 1968; Ginzburg & Zaitsev 1969).

The radio luminosities of pulsars are only a tiny fraction of their rotational energy loss rate. The kinetic energy of a neutron star rotating at angular velocity $\Omega = 2\pi/P$ is $E_{\text{rot}} = I\Omega^2/2$ and therefore, assuming a constant moment of inertia $I$, the rate of change of the rotational energy can be estimated from measurements of $P$ and $\dot{P}$ as

$$\dot{E}_{\text{rot}} = I\Omega\dot{\Omega}. \quad (1.1)$$

Denoting by $B_\perp$ the component of the magnetic field perpendicular to the rotational axis, the rate of energy loss due to dipole radiation of a rotator in vacuum is

$$\dot{E}_{\text{dr}} = -\frac{B_\perp^2 R_*^6 \Omega^4}{6c^3}. \quad (1.2)$$

Figure 1.2: Schematic illustration of the misaligned rotating magnetic dipole model. The radio emission beam is thought to be almost aligned with the magnetic field axis and originating from a region above the magnetic poles (Ruderman & Sutherland 1975). On the other hand, X-ray pulsations are usually attributed to an unevenly distributed surface temperature (Haberl 2007).
where $R_*$ is the stellar radius and $c$ the speed of light. This loss comes at the expense of the rotational energy, thus equating (1.1) and (1.2) the rate of magnetic braking can be found

$$\dot{\Omega} = -\frac{B_\perp^2 R_*^6 \Omega^3}{6Ic^3} \tag{1.3}$$

which allows for an estimate of the magnetic field component $B_\perp$.

This result is often generalised assuming that the spin-down rate can be written as

$$\dot{\Omega} \propto \Omega^n \tag{1.4}$$

where the exponent $n$ is called the braking index and can be calculated observationally, since $n = \dot{\Omega}\Omega/\dot{\Omega}^2$. Under the assumption that the braking index remains constant during a pulsar’s lifetime, and a natal spin frequency much greater than the currently observed $\nu = \Omega/2\pi$, Eq. 1.4 can be integrated and leads to a rough estimate of the pulsar’s age (spin-down age) as

$$\tau_{\text{sd}} = \frac{\Omega}{(1-n)\dot{\Omega}} \tag{1.5}$$

There are few pulsars for which a reliable measurement of the long term $n$ is possible, one of which, PSR J1119–6127 is the focus of Chapter 4, which includes a new value of $n$ for this pulsar and a detailed discussion on braking indices. All braking indices that have been determined and reported to date are less than 3, indicating that the simple model of a dipole rotator in vacuum (Eq. 1.3) is not enough to describe the spin-down (Lyne et al. 1993; Lyne et al. 1996; Livingstone et al. 2007; Weltevrede et al. 2011; Espinoza et al. 2011c; Roy et al. 2012). This deviation of $n$ from 3 is usually attributed to the loss of rotational energy in accelerating a wind of charged particles, although other alternatives cannot be ruled out (see for example Blandford & Romani 1988; Chen & Li 2006; Ho & Andersson 2012, and Chapter 4).

A newly formed neutron star is expected to be strongly magnetised because the magnetic flux of its progenitor is carried inwards by the collapsing material. However, conservation of magnetic flux alone is probably not sufficient to explain the population properties of pulsars. Typical inferred surface dipole magnetic fields $B_\perp$ of pulsars are very high, of the order $10^{12}$ G, while for a class of neutron stars called magnetars it can be as high as $10^{15}$ G. Such strong fields are most likely the result of an amplification of the natal magnetic field via dynamo processes (Thompson & Duncan 1993).

Magnetars appear as high energy sources and to date only 3 have been detected in the radio band (Olausen & Kaspi 2014). They typically show thermal surface emission and X-ray pulsations that originate from a hot spot on their surface. Rotation cannot power the energy output of these neutron stars. Their rather stable, quiescent
X-ray luminosities exceed by orders of magnitude their rotational energy losses and are attributed instead to the decay of their strong internal magnetic fields. They undergo energetic X-ray outbursts and more spectacular giant gamma-ray flares, which release over $10^{44}$ erg in just a few minutes (Rea & Esposito 2011; Mereghetti 2013). Though there are many open questions concerning magnetar bursts, the consensus is that this activity is also powered by their very strong magnetic fields (Thompson & Duncan 1995; Thompson & Duncan 1996). A small number of rotationally powered pulsars have inferred magnetic fields of magnetar strength and a somewhat mixed phenomenology (Ng & Kaspi 2011). Such high magnetic field pulsars can be used to study the links between the population of magnetars and typical pulsars (Chapter 4).

1.3 Following the rotation of a neutron star: pulsar timing

Pulsar timing is the process of measuring the arrival time of pulses emitted by a rotating neutron star and therefore tracking its rotation\(^5\). Timing data amount to a vast fraction of the total astronomical observations of pulsars and a wealth of information comes from their analysis. Besides providing estimates for the neutron star’s basic properties, such as the magnetic field strength (Eq. 1.3) and age (Eq. 1.5), the study and understanding of pulsar rotation puts constraints on the equation of state and gives us insights into the magnetosphere and interior of neutron stars.

To measure the pulsar’s frequency $\nu = \Omega/2\pi$, times of arrival (TOAs) assigned to each pulse should correspond to a clear reference point in their profile, which is assumed to represent an emitting area attached to the star’s surface. Individual pulses usually have very different profiles and are too weak to allow unambiguous determination of the TOA. However, folding the observations of many rotational cycles, using a predicted pulsar period, generates a high signal-to-noise template profile which becomes stable when enough pulses are summed. The shape of the standard profile (and the number of pulses needed to reach its required stability) is characteristic of each pulsar. An integrated profile is created from every new set of observations, which is then correlated with the standard template to give an average TOA. This is converted to a Solar System barycentric time, using previous measurements of the pulsar’s proper motion and distance or by fitting for them using a timing model.

A pulsar timing model is used to predict the pulse phase $\phi$. It is usually expressed as a Taylor expansion around an epoch $t_0$:

$$\phi(t) = \phi_0 + \nu_0 (t - t_0) + \frac{1}{2} \dot{\nu}_0 (t - t_0)^2 + \frac{1}{6} \ddot{\nu}_0 (t - t_0)^3 + \ldots \quad (1.6)$$

\(^5\)A detailed presentation of pulsar timing can be found in the book of pulsar astronomy of Lorimer & Kramer (2005).
1.3 Following the rotation of a neutron star: pulsar timing

Figure 1.3: The rotational evolution of the young PSR J0631+1036. **Upper panel:** Frequency residuals with respect to a simple spin-down model as in Equation 1.6, truncated at the \( \dot{\nu}_0 \) term. Large glitches appear as sharp spin-ups in frequency, superimposed on the long-term spin-down which is also evident. **Middle panel:** The evolution of the spin-down rate. Glitch-induced enhancements of \( \dot{\nu} \) and subsequent recoveries are readily identified for the four larger glitches. The effects of timing noise are clearly observed in the scattered \( \dot{\nu} \) measurements, which present variations that greatly exceed their statistical errors shown here. **Lower panel:** The glitch sizes \( \Delta \nu \) and their distribution in time, as well as the observational coverage (bottom row) for this pulsar. Figure courtesy: Cristobal M. Espinoza

where \( \nu_0, \dot{\nu}_0 \) and so on are the spin frequency and its time derivatives at \( t = t_0 \).

The regularly monitored pulsars are typically observed every few days or weeks. Improved rotational parameters are obtained from the new TOAs by minimising the differences between the predicted phase and the observed phase (timing residuals).

Pulsars are famed for their rotational stability as the best clocks of the Universe, however long-term monitoring projects and the increasing precision of timing measurements have revealed significant deviations from the predicted secular spin-down. The irregularities seen in the pulse phase residuals after subtracting a polynomial fit as in Eq. 1.6 are of two sorts: glitches, which are abrupt increases in frequency, and
timing noise, an umbrella term used to describe all other rotational features (Figure 1.3). Both these rotational phenomena are intrinsic to the pulsar and thus of great interest for the physical models of neutron stars and their magnetospheres.

1.3.1 Timing noise

For most pulsars a simple spin-down model is not sufficient to describe the data and the residuals are dominated by timing noise. There is no widely accepted way to characterise timing noise, mainly due to its complexity and the unknown nature of the underlying processes. Several ideas for the origin and cause of timing noise have been investigated. The proposed mechanisms include interaction with the interstellar medium (Scherer et al. 1997), accretion (Qiao et al. 2003; Cordes & Shannon 2008), internal processes due to the superfluid component (Alpar et al. 1986; Jones 1990; Haskell 2011) and their combination with magnetospheric instabilities (Cheng 1987b,a). Despite all these attempts, timing noise has not yet been explained successfully and remains unpredictable.

The level of timing noise varies greatly among pulsars; the rms of the residuals after fitting for a simple spin-down model covers a range of more than 7 orders of magnitude for comparable observational time spans in different sources (Shannon & Cordes 2010). Millisecond pulsars are at the stable extreme, with undetectable noise for most of them, while an excess of timing noise appears in magnetars. Investigation of irregularities for 366 pulsars, the largest sample used so far, supports earlier results that the strength of noise correlates with the magnitude of the spin-down rate, but not with the spin frequency (Hobbs et al. 2010). In many pulsars the underlying power spectrum is "red", indicating a low-frequency noise superimposed on a white component and in a few cases quasi-periodic structures were found in the residuals (Hobbs et al. 2010).

The analysis of Boynton et al. (1972), using the first two years of optical data for the Crab pulsar, revealed a noise component consistent with a random walk in the spin frequency $\nu$. However, later studies that included more data and more pulsars found timing noise to be inconsistent with an idealised random walk process either solely in phase or in one of its first two derivatives - phase, frequency and spin-down noise respectively (Cordes & Downs 1985; Hobbs et al. 2010). A mixture of resolvable jumps in both $\nu$ and $\dot{\nu}$ (and possibly $\ddot{\nu}$), that cannot be explained as accumulations of a random walk process, was found to account for a large part of the timing activity of the Vela pulsar (Cordes et al. 1988). Similar events, often termed "microglitches", are observed in many pulsars and have relative amplitudes of $|\Delta \nu/\nu| \lesssim 10^{-9}$ and $|\Delta \dot{\nu}/\dot{\nu}| \lesssim 10^{-3}$ with no preferential combination of signs (D’Alessandro et al. 1995; Chukwude & Urampa 2010). This component of timing noise, as observed in the Crab pulsar, is presented and discussed further in Chapter 2.
A possibly different class of events, so far identified in the rotation of about ten pulsars, consists of gradual spin-ups that seem to arise from a more abrupt decrease in |\dot{\nu}|. The departure from the standard behaviour usually lasts a few weeks and results in a relative frequency increase of \( \Delta \nu / \nu \sim 10^{-9} \). The prototype of this behaviour is PSR B1822–09 (Shabanova 1998) while more examples can be found in Yuan et al. (2010b). These events are likely associated with magnetospheric changes, as Lyne et al. (2010) reported a strong correlation between pulse profile changes and different spin-down states for some pulsars, including PSR B1822–09. The signature in the timing residuals of these sources could be explained as sudden switches between two or three discrete \( \dot{\nu} \) values, characterised by a different emission signature. This transition between a few distinct pulse profiles is called mode changing and has been observed in several pulsars (Backer 1970; Fowler et al. 1981; Rankin 1986). Sampling at sufficiently short time intervals, and looking for pulse profile changes associated with the timing parameters, might make it possible to account for this kind of rotational irregularities in the pulsar’s timing model.

1.3.2 Rotational Glitches

Large "glitches" were first observed in the rotation of the Vela pulsar as sudden spin-ups (\( \Delta \nu \sim 10 \mu \text{Hz} \)) accompanied by an increase in the spin-down rate of about 1% and followed by a partial recovery to the pre-glitch values on much longer timescales, from days to months (Radhakrishnan & Manchester 1969; Reichley & Downs 1969). The increase in frequency is very fast: for the rise time of the largest Vela glitch observed to date, Dodson et al. (2002) obtained an upper limit of 40 sec. Thus the spin-up is typically unresolved and appears as a very sharp change of slope in the phase residuals and a positive step in the frequency residuals (Figure 1.3).

Although not as widespread as timing noise, glitches have by now been detected in the rotation of more than 150 pulsars. The relative sizes of the change in spin frequency cover a very wide range of \( 10^{-11} \leq \Delta \nu / \nu \leq 10^{-5} \) (Figure 1.4). A jump in the spin-down rate, usually of relative size \( \Delta \dot{\nu} / \dot{\nu} \sim 10^{-4} - 10^{-3} \) and a trend for recovery are also characteristics of a glitch. A few pulsars seem to exhibit Vela-like giant glitches but most of them, like the Crab pulsar, present a variety of glitch behaviour and broad \( \Delta \nu \) size distributions (Espinoza et al. 2011b).

The signature of a glitch in the timing residuals is rather clear for intermediate and large relative jumps (\( \Delta \nu / \nu \gtrsim 5 \times 10^{-8} \)). When a glitch occurs, the parameters previously obtained can no longer predict the TOAs. The pre- and post-glitch sets of TOAs are described by different timing solutions, introducing a jump in \( \nu \) and often \( \dot{\nu} \) at the epoch of the glitch. The parameters \( \Delta \nu, \Delta \dot{\nu} \) used to describe the glitch, measured by extrapolating the pre- and post-glitch timing models to the glitch epoch, can consist of permanent and/or decaying components.
Glitch recoveries present a very diverse phenomenology and range of timescales (Figure 1.5). The detection of any short-term relaxation, with characteristic time of hours to days, requires good observational coverage around and after the glitch. The lack of an observed long-term relaxation can be due either to a true absence of such a recovery or because it happens on much longer timescales and is not detectable ( Chapters 3 and 5). Sometimes investigation of the recovery is prevented by a new glitch that occurs and dominates the residuals.

Whilst for the Vela pulsar a post-glitch relaxation is observed in both the spin frequency $\nu$ and the spin-down rate $\dot{\nu}$, in most glitches seen from the Crab pulsar part of the increase in $|\dot{\nu}|$ is persistent (Wong et al. 2001) and the net result of glitches is a decrease in frequency. Such long-lasting (or permanent) enhancements of the spin-down rate and overshooting in spin frequency are seen in other pulsars too, for example in PSR B2334+61 and PSR B0402+61 respectively (Yuan et al. 2010a,b). By contrast, in many pulsars and even for some giant Vela-like glitches, there is no
1.3 Following the rotation of a neutron star: pulsar timing

Figure 1.5: Schematic illustration of a few commonly observed glitch recoveries. Most of the apparent differences in phenomenology are well accommodated by a unified two-component model (Chapter 3). However, some pulsars and magnetars show glitches with exceptionally unusual recoveries (not shown here), often related to magnetospheric activity, which require an additional mechanism (Chapter 4).

Some magnetars also suffer glitches which are usually of medium or large size, and sometimes present exceptionally strong enhancements of the spin-down rate. Those properties can be explained naturally by the same model as for radio pulsar glitches (Chapter 3). However, magnetar glitches can sometimes be associated with radiative changes such as bursts and pulse profile changes (Dib et al. 2008), while glitches in radio pulsars are almost always radiatively quiet. Moreover, rather unusual recoveries, often related to emission changes, have been observed after glitches in magnetars (Kaspi et al. 2003; Kaspi & Gavriil 2003; Dib et al. 2009) and high-magnetic field radio pulsars like PSR J1846-0258 (Livingstone et al. 2010), J1718-3718 (Manchester & Hobbs 2011), J1119-6127 (Weltevrede et al. 2011, Chapter 4) and J1819-1458 (Lyne et al. 2009a). The possible relation between glitches and magnetospheric phenomena, and the plausible contribution of the latter to post-glitch
1 Introduction

recoveries, is investigated in Chapter 4.

The aforementioned diversity in recoveries is one of the most puzzling aspects of the glitch phenomenon. However, as shown in Chapter 3, a multifluid hydrodynamical model for the interior of a neutron star can explain a variety of glitch phenomenology, even for the same pulsar, without invoking additional mechanisms.

1.4 Glitches as probes of neutron star superfluidity

Soon after the first observations of glitches, several mechanisms were put forward to explain their origin. The lack of any radiative changes associated with most radio pulsar glitches provides evidence for an internal, rather than magnetospheric, origin. Furthermore, the very long post-glitch recovery timescales must arise from the relaxation of the slowly responding superfluid that a neutron star is expected to host (Baym et al. 1969). The most promising glitch models to date invoke the superfluid to explain both the origin of the observed spin-up and its recovery (Anderson & Itoh 1975). Advancing our understanding of the glitch phenomenon therefore has a unique potential of constraining the unknown properties of the inaccessible neutron star interior.

1.4.1 Fermionic pairing and hot condensates in neutron stars

Superconductivity, the property of some materials to show very low to zero resistivity, was discovered in 1911. However it was not until 1957 that a microscopic explanation was given, by Bardeen et al. (1957). Their theory is based on the fact that, under certain conditions, it is energetically favourable for the fermions of a system (for example the electrons of a metal or nucleons inside a nucleus) to create pairs of integral spin which behave as bosons. They will then demonstrate macroscopic properties of a bosonic system, such as superfluidity in low temperatures, a phenomenon well studied in terrestrial liquid helium experiments. Superfluids are neutral and can flow with nearly zero viscosity, while superconducting currents are their charged equivalent.

Soon after the formulation of the BCS theory, Migdal (1959) proposed that the charged particles in neutron star cores might undergo the same second order phase transition as electrons in superconductors. The internal temperature of a neutron star, large as it might be, falls quickly after the formation below the Fermi temperature of the core’s protons - so the system will behave as a superconducting boson condensate even when its temperature is as high as $10^8$ K. For the same reasons, neutrons in the inner crust and core of a neutron star are also expected to form pairs and be in a superfluid state. The extent, type and exact location of the condensates depend on the internal properties of neutron stars as well as the - still poorly understood - nucleon-nucleon interaction (Lattimer & Prakash 2004).
Magnetic fields are usually expelled from a superconductor due to the Meissner effect. In neutron stars however the natal magnetic field is expected to get trapped in the core when the protons condense (Muslimov & Tsygan 1985; Kocharovsky et al. 1996). The outer core of a neutron star likely contains type-II superconducting protons. In this state the magnetic field penetrates the superconductor in the form of proton vortices of quantised magnetic flux, called fluxtubes.

Superfluid flows are inviscid and must also be irrotational. However beyond a maximum rotation rate of the superfluid’s container, a limit which is easily exceeded by the rapidly spinning pulsars, it is energetically favourable for the superfluid to follow the rotation. This is achieved by the formation of vortices of quantised circulation $\kappa$, where the quantum $\kappa = \hbar/(2m_n)$ for neutron stars, $\hbar$ being the Planck constant and $2m_n$ the mass of a neutron pair. In the absence of interactions with the other stellar components, the vortices form an hexagonal Abrikosov lattice configuration and the superfluid mimics solid-body rotation\(^6\). The superfluid angular velocity is determined from the density $n_v$ of vortices that thread it, therefore any changes to its spin rate must involve vortex motion and changes of their density or/and total number. The interaction of the superfluid vortices with the normal and superconducting components of the neutron star can give rise to rich dynamical behaviour, as discussed in the following section. Modelling the rotational dynamics provides insight into these interactions, as well as constraints on the extent of the different neutron superfluid regions, leading to valuable input to our understanding of the strong interaction.

1.4.2 The two-component model for neutron stars

The simple and intuitive idea of a crustquake was among the first mechanisms suggested to explain pulsar glitches (Ruderman 1969; Baym & Pines 1971). Though crustquakes were quickly abandoned as a stand-alone model for glitches, because they cannot reproduce the large and frequent glitches seen in the Vela pulsar, they still remain a valid hypothesis as triggers of glitches (Alpar et al. 1994, 1996; Franco et al. 2000). The crustquake scenario relies on the existence of a solid crust which cannot readjust plastically to the equilibrium shape corresponding to the decreasing angular velocity. Thus as the star spins down stress builds up in the crust, which is expected to be strained for most of a pulsar’s lifetime. When the stress exceeds its critical breaking value a crustquake happens, which releases all or part of the stress, and leaves the star with a less oblate, closer to equilibrium, shape.

The sudden decrease in moment of inertia by $\Delta I$ is responsible for the observed spin-up $\Delta \Omega$. By conservation of angular momentum $I\Omega = (I - \Delta I)(\Omega + \Delta \Omega)$ ⇒

\(^6\)Much of the theory on superfluid vortices in neutron stars draws from the knowledge obtained by experimental studies of Helium II. The book by Donnelly (1991) presents a nice review of this subject.
\[ I \Delta \Omega = \Delta I (\Omega + \Delta \Omega), \] which implies

\[ \frac{\Delta I}{I} = \frac{\Delta \Omega}{\Omega + \Delta \Omega} \approx \frac{\Delta \Omega}{\Omega} = \frac{\Delta \Omega}{\Omega} = \frac{\Delta \nu}{\nu}. \] (1.7)

The external, magnetospheric, torque \( N_{\text{EXT}} \), which defines the angular momentum loss rate \( L = I \dot{\Omega} \), can be assumed constant for the characteristic timescales involved in a glitch (the crustquake and glitch associated \( R_* \) and \( \Omega \) changes have only a negligible effect on \( N_{\text{EXT}} \)). Therefore \( I \dot{\Omega} = (I - \Delta I)(\Omega + \Delta \Omega) \Rightarrow \)

\[ \frac{\Delta I}{I} = \frac{\Delta \dot{\Omega}}{\dot{\Omega} + \Delta \dot{\Omega}} \approx \frac{\Delta \dot{\Omega}}{\dot{\Omega}} = \frac{\Delta \dot{\nu}}{\dot{\nu}}. \] (1.8)

It follows that \( \Delta \nu/\nu \approx \Delta \dot{\nu}/\dot{\nu} \), however \( \Delta \dot{\nu}/\dot{\nu} \gg \Delta \nu/\nu \) is observed for many glitches\(^7\) - thus a crustquake does not suffice to explain the observations. If the observed spin-up is mainly due to a moment of inertia decrease, then \( \Delta \nu/\nu \) constrains the relative change \( \Delta I/I \) while an additional mechanism is necessary to produce the large \( \dot{\nu} \) change and glitch recoveries. These are attributed to the decoupling of the interior superfluid because of the glitch perturbation, as described below.

The solid crust and the charged components of a neutron star (denoted hereafter by a subscript \( c \)) are electromagnetically "locked" together and rotate as a rigid body, spinning down due to \( N_{\text{EXT}} \). In order for the superfluid (subscript \( s \)) to follow this spin-down, vortices need to move outwards reducing the macroscopic superfluid velocity which is proportional to \( n_v \). In the inner crust however, the superfluid coexists with a lattice of nuclei which interacts with the vortices prohibiting their relative motion with respect to the crust. A similar phenomenon might occur in the star's core, due to the interaction of vortices with the superconducting fluxtubes. This "pinning" of vortices leads to the partial decoupling of the superfluid from the charged component and allows for differential rotation of the superfluid and a difference \( v_{sc} = v_s - v_c \) to be built between the components' velocities. The star can then be described as a two-fluid system, with a total angular momentum

\[ L_{\text{tot}}(t) = I_c \Omega_c(t) + \int \Omega_s(r, t) dI_s \] (1.9)

which evolves according to

\[ \dot{L}_{\text{tot}} = N_{\text{EXT}} = I_c \dot{\Omega}_c(t) + \int \dot{\Omega}_s(r, t) dI_s \] (1.10)

where \( N_{\text{EXT}} = -B^2 \Omega^3/6c^3 \) for the standard dipole breaking mechanism.

\(^7\)Typical parameters for a small glitch are \( \Delta \nu/\nu \sim 10^{-9} \) and \( \Delta \dot{\nu}/\dot{\nu} \sim 10^{-3} \).

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The superfluid maintains a higher rotation rate as the star spins down, acting as a reservoir which provides the necessary angular momentum for the observed spin-up glitches of the charged component. Most glitch mechanisms involve a rapid transfer of the excess angular momentum from the superfluid interior to the crust as the result of the sudden unpinning of many vortices (Anderson & Itoh 1975; Alpar et al. 1981, 1984a; Haskell et al. 2012).

The presence of a rotational lag \( \nu_{sc} \neq 0 \) gives rise to a force density exerted on the superfluid, \( f_s = f / \rho_s \), and an opposite force \( f_c = -\rho_c f_s / \rho_c \) that acts on the charged component, where \( \rho_s, \rho_c \) their respective densities. As in terrestrial superfluids, this coupling of the two components can be incorporated in their equations of motion as a mutual friction force (Andersson et al. 2006), of the form

\[
f = \rho_s B \omega_s \times \nu_{sc} + \rho_c B \hat{\omega}_s \times (\omega_s \times \nu_{sc})
\]  

(1.11)

where \( \omega_s = n_s \kappa \hat{\omega}_s \) is the superfluid vorticity, \( \hat{\omega}_s \) is a unit vector aligned with the vortex axis, and the coefficients \( B' \) and \( B \) depend on the local microphysics that describe the strength of the drag force \( f_D \) on vortices. The mutual friction force is a result of the individual forces that act on the vortices (see below) which mediate the interaction of the two components.

Pinned or partially pinned vortices have a relative velocity with respect to the ambient superfluid, which exerts a Magnus (lift) force \( f_M \) on them that tends to unpin them. The Magnus force is proportional to the relative velocity between the two components and increases with time as the crust decelerates and the lag between the two components grows. Completely pinned vortices move with the velocity of the charged component so are not subject to a drag force, and their effect on the superfluid is given only by the Magnus force which is balanced by the static pinning force \( f_P \).

The drag force is mediated only by the fraction of unpinned vortices \( \xi \leq 1 \) which have some relative velocity with respect to the lattice. Thus the mutual friction coefficients \( B \) and \( B' \) in Eq. 1.11 will depend on the factor \( \xi \) (for a slightly more detailed discussion see Chapters 3 and 4). This fraction \( \xi \) is equal to unity in the absence of any pinning, when all vortices are free. In regions where pinning is possible \( (\omega_s < \omega_{cr}) \), vortices are not necessarily completely immobilised. Due to the finite temperature, thermally activated unpinning (Alpar et al. 1984a; Link 2014) contributes to the local \( \dot{\Omega}_s \). When the lag is small, most of the vortices will be pinned and \( \xi(n_v) \to 0 \). On the other hand, if \( v_{sc} \) exceeds a critical value for unpinning (when the Magnus force exceeds \( f_P \), \( \xi(n_v) = 1 \) and vortices can move past several pinning sites before they repin, causing the sudden spin-up of the crust. Thus \( \xi \) is a local quantity that depends on the pinning energy, the temperature \( T \) and the relative velocity of the two components, \( v_{sc} = v_s - v_c \).
When the increasing lag reaches its critical value, a catastrophic unpinning event occurs, leading to an avalanche of many vortices. Glitches triggered this way should happen at more or less canonical time intervals (the time it takes for the critical lag to be reached due to the spin-down) and should have similar sizes. Such behaviour is observed however in only two pulsars: PSR J0835-4510 (the Vela pulsar) and PSR J0537-6910 (McCulloch et al. 1987; Flanagan 1995; Marshall et al. 2004). Most of the frequently glitching neutron stars present a broad range of waiting times between subsequent glitches, and spin-up sizes that span many orders of magnitude (up to 4 for PSR J1740–3015), which indicate a more complex picture. To explain this phenomenology additional unpinning triggers have been examined, such as crustquakes or enhanced coupling of the two components due to heat release in the interior (Link et al. 1992; Link & Epstein 1996; Glampedakis & Andersson 2009; Warszawski & Melatos 2013). The distribution of glitch sizes and inter-glitch waiting times represent diagnostics to discriminate between these competing unpinning triggers (Chapter 2).

The superfluid spin-down rate $\dot{\Omega}_s$ depends on the lag between the two components. Immediately after a glitch this lag, and thus $\dot{\Omega}_s$ and the contribution of the second term in Eq. 1.10, is reduced for most if not all parts of the superfluid. This leads to the post-glitch increase in the crustal spin-down rate. The observed recovery reflects the relaxation of the perturbed superfluid and depends on the vortex drag forces if pinning is negligible (Sidery et al. 2010), and the rate of the thermally activated vortex motion in the regions where pinning dominates. Consequently, modelling of the post-glitch rotational evolution can shed light on many key issues of neutron superfluidity (Chapter 5).
1.4 Glitches as probes of neutron star superfluidity

Epilogue – This thesis

The Golden Jubilee of neutron star observational astrophysics will be celebrated in a few years. Although firm models for some fundamental aspects of neutron stars remain elusive - from the equation of state of the deep interior to the emission mechanism of the radio pulsations that led to their discovery - there has been also substantial progress towards the theoretical understanding of these mysterious compact objects.

Current and future observational projects, adding to the existing plenitude of timing data, will greatly aid the investigation of pulsars’ rotational dynamics. A big part of the underlying motivation for carrying out research on neutron star glitches is their potential as probes into the strong interaction. To fully exploit this however, mutual input and strong connections between the theoretical and observational fronts are required.

The experimental biases and inherent limitations that prohibit glitch detection, especially in the presence of timing noise, are examined in Chapter 2. The results can be used for optimisation of pulsar monitoring schemes, to maximally explore the glitch parameter space. The glitch population of the Crab pulsar is inspected in detail, revealing an unexpected deficit in small glitches that provides insight to the glitch trigger mechanism. Timing analyses have uncovered a rich collection of glitch signatures, with a broad range of amplitudes and timescales for the spin-ups and recoveries. In Chapter 3, a multifluid hydrodynamical approach is used to simulate glitches as large scale vortex unpinning episodes. It is shown how the observed variety of timing behaviour in a single source, as well as the peculiar relaxations of glitches seen in magnetars, can be naturally explained within this framework. New radio observations of PSR J1119–6127, a high magnetic field pulsar with remarkable glitch properties, are presented in Chapter 4. A detailed study of its unusual radio activity, glitch recoveries and their theoretical implications, gives clues as to their connection with magnetospheric phenomena and crust quakes. In addition, a new method is employed to improve the long-term braking index measurement of this source and examine its possible evolution in connection to glitches. Lastly, Chapter 5 considers the description of glitch recoveries with models that include a supplementary signature in the residuals, theoretically predicted to arise from regions where vortex pinning prevails, in addition to the sum of exponential decays traditionally used. A timing analysis of data following a large glitch in the Vela pulsar is carried out for this purpose. Such an exploration constrains the fundamental amplitudes and timescales involved in glitches, which are closely tied to the vortex pinning properties of the inner crust and core. Comparison of the parameters inferred from observations to those expected from microphysical calculations and various glitch models, can yield invaluable crosschecks, and provide powerful discrimination between competing physical mechanisms.
Neutron star glitches have a substantial minimum size

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Abstract

Glitches are sudden spin-up events that punctuate the steady spin down of pulsars and are thought to be due to the presence of a superfluid component within neutron stars. The precise glitch mechanism and its trigger, however, remain unknown. The size of glitches is a key diagnostic for models of the underlying physics. While the largest glitches have long been taken into account by theoretical models, it has always been assumed that the minimum size lay below the detectability limit of the measurements. In this paper we define general glitch detectability limits and use them on 29 years of daily observations of the Crab pulsar, carried out at Jodrell Bank Observatory. We find that all glitches lie well above the detectability limits and by using an automated method to search for small events we are able to uncover the full glitch size distribution, with no biases. Contrary to the prediction of most models, the distribution presents a rapid decrease of the number of glitches below $\sim 0.05 \mu$Hz. This substantial minimum size indicates that a glitch must involve the motion of at least several billion superfluid vortices and provides an extra observable which can greatly help the identification of the trigger mechanism. Our study also shows that glitches are clearly separated from all the other rotation irregularities. This supports the idea that the origin of glitches is different to that of timing noise, which comprises the unmodelled random fluctuations in the rotation rates of pulsars.
2.1 Introduction

Neutron stars are the highly-magnetised and rapidly-rotating remnants of the collapse of the cores of once more-massive stars. Having masses of approximately $1.4 \, M_\odot$ and radii of about 12 km, the high densities of neutron stars indicate a structure of a crystalline-like crust and a superfluid interior (Baym et al. 1969; Haensel et al. 2007). Their large and steady moments of inertia mean that they have extremely stable rotational frequencies, which slowly decrease as energy is lost through electromagnetic radiation and acceleration of particles in their magnetospheres. However, this regular spin-down is occasionally interrupted by sudden spin-up events, known as glitches (Radhakrishnan & Manchester 1969; Espinoza et al. 2011b).

The exact mechanism responsible for glitches is not fully understood but it is thought to involve a sudden transfer of angular momentum from a more rapidly rotating superfluid component to the rest of the star (Anderson & Itoh 1975). This component resides in regions of the interior where neutron vortices, which carry the angular momentum of the superfluid, are impeded in moving by pinning on crustal nuclei or on superconducting vortices in the core (or on both). Since a superfluid in such conditions cannot slow down by outwards motion and expulsion of vortices, the superfluid component will retain a higher rotational frequency as the rest of the star slows down. A glitch occurs when vortices are suddenly unpinned and free to move outwards, allowing for a rapid exchange of angular momentum and the observed spin-up of the crust.

Catastrophic unpinning of vortices is expected once the velocity lag between the two components exceeds a maximum threshold, above which the pinning force can no longer sustain the hydrodynamic lift force exerted on the pinned vortices by the ambient superfluid. It has also been shown (Glampedakis & Andersson 2009; Andersson et al. 2013) that, beyond some critical lag, a two-stream instability might develop and trigger the unpinning. If in such events the lag is completely relaxed (or partially relaxed by a fixed amount) then the interglitch time interval corresponds to the time it takes for the system to reach the critical threshold again, driven by the nearly constant external torque. Models relying on such a build-up and depletion of the superfluid angular momentum reservoir have been successfully used to explain the regular, similar glitches of some young pulsars (Alpar et al. 1993; Pizzochero 2011; Haskell et al. 2012). However this simple picture cannot account for the wide range of glitch sizes and waiting times between glitches seen in most pulsars.

Glitch sizes in rotational frequency can range over four orders of magnitude in individual pulsars and appear to follow a power-law distribution (Melatos et al. 2008). This favours scale-invariant models of the dynamics of individual vortices in the presence of a pinning potential, such as the vortex avalanche model (Warszawski & Melatos 2008) and the coherent noise model (Melatos & Warszawski 2009).
Alternative models involve non-superfluid mechanisms that can act as unpinning triggers before the critical lag is reached, such as crustquakes (Ruderman 1969; Baym & Pines 1971) or heating episodes (Link & Epstein 1996). The crustquake-induced glitch model has been particularly favoured for the Crab pulsar as it may explain the persistent changes in slow-down rate observed after some of its glitches (Gullahorn et al. 1977; Alpar et al. 1994) and could possibly lead to a power-law distribution of event sizes, similar to earthquakes.

A second type of irregularity is often seen in the rotational behaviour of pulsars, namely timing noise. Thought to be partially caused by torque variations driven by two or more magnetospheric states (Lyne et al. 2010), it manifests as a continuous and erratic wandering of the rotation rate around the predictions of a simple slow-down model. While glitches are rapid and sporadic events in rotation rate, timing noise appears as a slow and continuous process.

Owing to observational limitations such as infrequent and irregular sampling and the presence of timing noise, the detection of glitches is an uncertain process. Moreover, the signature of timing noise in the data can be confused with glitches, so that the lower end of a glitch-size distribution is possibly contaminated by spurious detections. Knowledge of this distribution is essential for any glitch theory. The largest glitches are easily detected and can be used to constrain the minimum superfluid moment of inertia that can act as an angular momentum reservoir (Andersson et al. 2012; Chamel 2013a). The biases involved and the question of whether there is a minimum glitch size have not been addressed; so far, the smallest possible glitch has been assumed to lie below our detection limits.

In this paper we study the glitch detection capabilities of the current detection methods and define limits depending upon the intrinsic pulsar rotational stability, observing cadence and sensitivity. We apply these definitions to an extensive set of observations of the Crab pulsar and, by using an automated glitch detector, uncover the full glitch size distribution and show that there is a minimum glitch size.

2.2 Limits on glitch detection

To assess the level of completeness of the existing glitch samples, we quantify simple observational limits on glitch detection, applicable for a given pulsar and observing setup. The first step towards this is establishing a working definition of what constitutes a glitch. Traditionally, glitches are identified by visual inspection of the pulsar’s timing residuals, which are defined as the phase differences between measurements and the predictions of a model for the rotation. To put this on a more formal footing, we define a glitch as an event characterised by a sudden, discrete positive change in rotational frequency ($\Delta \nu$) and a discrete negative or null change in frequency spin-
2.2 Limits on glitch detection

Figure 2.1: Example of a glitch signature in the timing residuals. The residuals are with respect to a model which describes well the rotation before the glitch ($t < 0$ on the plot). This is a simulated glitch in the Crab pulsar’s rotation, with $\nu = 0.01 \, \mu\text{Hz}$ and $\dot{\nu} = -3.0 \times 10^{-15} \, \text{Hz s}^{-1}$.

down rate ($\dot{\nu}$). These two sudden changes together make glitches distinguishable from timing noise (Espinoza et al. 2011b; Lyne et al. 2010).

The timing residuals will be flat if the model describes the rotation of the pulsar well. For such a model, the timing residuals after a glitch at $t = t_g$ will follow a quadratic signature given by

$$\phi_g = -\Delta \nu (t - t_g) - \frac{\Delta \nu (t - t_g)^2}{2}; \quad (t > t_g).$$  \hspace{1cm} (2.1)

The frequency change $\Delta \nu > 0$ produces a linear drift of the post-glitch residuals towards negative values, with the slope being the magnitude of the frequency step. The effect of a change $\Delta \nu < 0$ is a parabolic signature which lifts the residuals towards positive values. Therefore a glitch with a large, negative change in spin-down rate will produce positive residuals rising quadratically soon after the glitch (Fig. 2.1).
Glitches have a minimum size

Based upon these facts, we can define simple limits that describe our ability to detect glitches in the timing residuals. If the observing cadence is not very frequent, it is possible that no observations occur before the post-glitch residuals rise above the extrapolation of the line defined by the pre-glitch ones (Fig. 2.1). This effect will primarily mask glitches with small $\Delta \nu$ and large $|\dot{\Delta} \nu|$. Requiring at least one observation before the rise of the residuals defines a minimum $\Delta \nu$ that can be detected (Eq. 2.2). If $\Delta \nu$ became smaller, the dip would become shallower and in the case that it is undetectably small, the event is unlikely to be recognised as a glitch and might appear as timing noise. To ensure detection, the maximum negative departure of the residuals ought to be larger than both the root mean square (RMS) of the timing residuals prior to the glitch and the typical error of the TOAs. Therefore, a detectable glitch is a rapid event in which the effects of $\Delta \nu$ are recognisable over the effects of $\Delta \dot{\nu}$ and the limiting detectable value of $\Delta \nu$ depends on the observation cadence (one observation every $\Delta T$ days) and the largest of either the sensitivity of the observations or the typical dispersion of the timing residuals in rotational phase, $\sigma_\phi$, as

$$\Delta \nu_{\text{lim}} = \max \begin{cases} \frac{\Delta T|\Delta \dot{\nu}|}{2} & \text{(for } \Delta \dot{\nu} < 0) \end{cases} \sqrt{2\sigma_\phi|\Delta \nu|}$$

(2.2)

For simplicity and because of our particular focus on small events, any exponential recovery of the frequency, often observed after glitches, is not considered here. Nonetheless, we constructed several detectability curves, with decaying components and timescales similar to those observed in the Crab pulsar, and verified that our conclusions are not altered if exponential recoveries are present.

These limits are consistent with the glitch samples of several pulsars, hence we believe they offer a realistic way to assess glitch detectability as it is commonly carried out.

2.3 The Crab pulsar glitches

The Crab pulsar (PSR B0531+21; PSR J0534+2200) is the central source of the Crab Nebula and a young neutron star widely studied since it was first observed in 1968. The rotation of the Crab pulsar has been monitored almost every day for the last 29 years with the 42-ft radio telescope operating at 610 MHz at the Jodrell Bank Observatory (JBO) in the UK (Lyne et al. 1988, 1993). This offers an ideal dataset to test the completeness of the glitch sample because of its rapid cadence, good sensitivity and low dispersion of the timing residuals.
2.3 The Crab pulsar glitches

2.3.1 Observations

The product of each observation was the time of arrival (TOA) of one pulse at the observatory, corrected to the solar system barycenter. The dataset comprises 8862 TOAs starting in January 1984. There is one TOA per day in general and two TOAs per day during some periods of time. In addition, towards the beginning of the dataset, there are some isolated cases in which groups of TOAs are separated by up to 5 days. Finally, there are also a few gaps with no observations, generally no larger than ~ 20 days, when the telescope or observing hardware were unavailable due to maintenance.

The TOAs generally have errors of less than 0.001 rotation, with more than 75% having uncertainties less than 0.0004 rotation. For groups of 20 TOAs, which cover 20 days on average, the timing residuals with respect to a simple slow-down model with two frequency derivatives typically give a dispersion similar to the TOA uncertainties (hence $\sigma_{\phi} \sim 0.0004$ rotations).

2.3.2 Detection limits and the sample of detected glitches

To study the glitch size distribution of the Crab pulsar we need a complete list of glitches for the time interval defined by the 42-ft dataset, and their main parameters $(\Delta\nu, \dot{\Delta}\nu)$. As described above, we classify events as glitches based on the assumption that a glitch is a sudden, unresolved change in spin frequency, implying clearly defined features in the timing residuals.

We use the events included in the JBO online glitch catalogue\(^1\), which correspond to all the events published by Espinoza et al. (2011b) plus one new glitch that occurred on MJD 55875.5 (Espinoza et al. 2011a). The event on MJD ~ 50489, originally reported by Wong et al. (2001), was rejected because of its anomalous characteristics, already described by them. No other glitches have been reported for this time-span by other authors (e.g. Wong et al. 2001; Wang et al. 2012) and we confirmed this by visually inspecting the timing residuals for all our dataset. Our final list contains 20 glitches, with parameters covering the ranges $0.05 \leq (\Delta\nu/\mu\text{Hz}) \leq 6.37$ and $45 \leq (|\dot{\Delta}\nu|/10^{-15}\text{Hz s}^{-1}) \leq 2302$. Here we use the glitch sizes reported by Espinoza et al. (2011b,a).

We note the clear presence of four other glitches prior to the start of the 42-ft observations. However, the available data for that period is highly inhomogeneous and contains large gaps with no observations, making it difficult to define single detectability limits and complicating the use of the glitch detector (see below). Hence, in order to work with a set of glitches that we know is statistically complete, we have not included them in our sample.

\(^{1}\text{http://www.jb.man.ac.uk/pulsar/glitches.html}\)
2. Glitches have a minimum size

Using $\Delta T = 1$ day and $\sigma_{\phi} = 0.0004$ in Eq. 2.2 we find that all glitches (including the four early ones) show a clear separation from the detectability limits (Fig. 2.2). Thus, at least for intermediate and large glitches, we are uncovering the true $\Delta \nu - \Delta \dot{\nu}$ distribution, with no biases.

2.4 The glitch detector

To confirm that we have identified every glitch in the data, especially small ones which may be missed by standard techniques, we developed an automated glitch detector to find and measure every timing signature that might be regarded as a glitch. The detector assumes that a glitch occurred after every observation and attempts to measure its size, producing an output of glitch candidates (GCs) whenever $\Delta \nu > 0$ and $\Delta \dot{\nu} \leq 0$ are detected.

2.4.1 Method

The detector’s technique is based on the fact that timing residuals, in the presence of timing noise or glitches, quickly depart from the best fit model of previous data, resulting in deviations from a mean of zero as newer observations are included and the model is not updated. Below we describe the method step by step, optimised for the JBO dataset for the Crab pulsar, described above. Different parameters should be used for different datasets.

A fit for $\nu$, $\dot{\nu}$ and $\ddot{\nu}$ is performed over a set of 20 TOAs using the timing software `psrtime` and `tempo2` (Hobbs et al. 2006), following standard techniques. To test for a glitch occurring after the last TOA in a set, the timing residuals of the following 10 TOAs, relative to that model, are fitted with a quadratic function of the form of Eq. (2.1) and separately with just the linear term in that equation. The latter is to test the case $\dot{\nu} = 0$. The fit with the smallest reduced $\chi^2$ is selected. When the quadratic fit is selected, an event is characterised as a GC only if the reduced $\chi^2$ is less than 15, the quadratic part of the fit is negative ($\Delta \dot{\nu} < 0$) and if the minimum of the fitted curve is at least 2.5 times the dispersion of the timing residuals of the 20 TOAs below zero. This last condition ensures a positive $\Delta \nu$ and a solid detection of its magnitude. If the linear fit is selected, a new GC is created only if the reduced $\chi^2$ is less than 15 and the slope of the fit is negative, indicating $\Delta \nu > 0$. In this case the GC has a null or undetectable $\Delta \dot{\nu}$. The conclusions of our analysis are not dependent on the choice of the maximum allowed reduced $\chi^2$ threshold\(^2\). The chosen value of 15 is high enough to avoid missing signatures that one might regard as a glitch.

\(^2\)Changing this threshold to 20, for example, we obtained 12 new GCs, homogeneously distributed across the frequency range of GCs. There are no effects on the statistical results described in later sections.
The next step is to move the analysis forward by one TOA to define a new set of 20 TOAs and test for a glitch occurring after this new TOA. By doing this over the whole dataset, the dataset is explored for glitches after every single observation (with the exception of the first 19 TOAs and the last 10 TOAs).

This method, however, causes some events to be detected multiple times. This happens because the effects of $\Delta \nu$ and $\Delta \dot{\nu}$ may be detectable not only in the set of TOAs starting immediately after the event but also in some of the neighbouring trials. Close inspection of the results shows that detections typically cluster in groups of 2–5 trials, separated by no more than 2 days, and that clusters are typically 20 to 30 days apart. To remove the repeated detections and produce a final list of GCs we select from each cluster of candidates the detection with the largest $\Delta \nu$ value. This is a conservative choice which makes the final list of GCs a representation of the maximum possible activity present in the data. Also, this choice follows the experience gained from the detection of previously known glitches (section 2.5.1).

## 2.5 Results

We ran the detector over the 42-ft dataset, using the data from January 1984 to February 2013. The detector found all but one of the known glitches in this time-span as well as a large number of GCs.

### 2.5.1 The output of the glitch detector

The only previously known glitch that was not found by the detector occurred on MJD $\sim 52146.8$, only 63 days after the previous glitch. It was not labelled as a possible glitch because none of the fits, neither the quadratic nor the linear, gave a reduced $\chi^2$ less than 15, one of the conditions to create a GC. The smallest reduced $\chi^2$ among the fits around this glitch was 18. We attribute these poor fits to the influence of the recovery from the previous glitch.

The sizes $\Delta \nu$ and $\Delta \dot{\nu}$ that the detector measured for the known glitches are in good agreement with the values published by Espinoza et al. (2011b). Nevertheless, some differences can be found among the $\Delta \nu$ measurements. As discussed above, because of the way the detector works, the effects of every glitch were detected in more than one set of TOAs. The $\Delta \nu$ value coming from the set of TOAs offering the best fit (smallest reduced $\chi^2$) is always smaller than the published value, which is obtained by standard timing techniques. In addition, this set of TOAs is normally the one starting one to three TOAs after the glitch epoch. On the other hand, the glitch sizes obtained when testing at the correct glitch epoch are typically the largest and the most similar to the published values, though the fits have larger reduced $\chi^2$ values. These effects are likely caused by unmodelled rapid exponential recoveries and were taken
Figure 2.2: Previously known glitches (diamonds, from Espinoza et al. (2011b,a)), glitch candidates (GCs) and anti-glitch candidates (AGCs). The top panel shows their distribution in the $|\Delta v| - \Delta v$ plane. The straight line with the smallest slope represents the detection limit expected (Eq. 2.2) for a cadence of $\Delta T = 1$ day. The one with the largest slope represents the detection limit expected for residuals of $\sigma_\nu = 0.0004$. Our observations are not sensitive to glitches in the shaded areas above these lines. The middle panel shows the $|\Delta v|$ values of those candidates with undetectable $|\Delta v|$. The lower panel shows histograms for the $|\Delta v|$ values of the known glitches (filled grey), GCs (thick black) and AGCs (thin black). The inset shows a zoom in the region $|\Delta v| > 0.02 \mu$Hz.
2.5 Results

Figure 2.3: Time sequence of glitches, glitch candidates (GCs) and anti-glitch candidates (AGCs). Horizontal lines indicate the detection limits (Eq. 2.2) for $|\Delta \nu| = 1 \times 10^{-15} \text{Hz s}^{-1}$ (long dashed) and $10 \times 10^{-15} \text{Hz s}^{-1}$ (short dashed). The low number of detections between the years 1985 and 1990 is caused by the presence of short gaps with no observations.

into account when selecting one candidate from a group of several candidates in the overall search. The uncertainties of the GC sizes are the square root of the variances of the parameters, given by the Levenberg-Marquardt algorithm used to fit the data, multiplied by the square root of the reduced $\chi^2$ of the fit.

We reviewed the output of the glitch detector with the aim of producing a clean list of GCs. First, we removed from the original list all those GCs related to known glitches. Then we kept only one candidate (the one having the largest $\Delta \nu$) per event, as mentioned in the description of the method (section 2.4.1). Next, we visually inspected the timing residuals for all GCs having $|\Delta \nu| > 0.02 \mu\text{Hz}$ and eliminated three which involved large data gaps or with timing residuals clearly contaminated by glitch recoveries. We also examined the possibility that some GCs in this $\Delta \nu$ range could be caused by rapid changes in the electron density towards the Crab pulsar (Lyne et al. 1993), which strongly affects the travel time of the pulsar emission at these low frequencies, introducing signatures in the data which can mimic a glitch. To do so, we used observations taken at higher frequencies (mostly at 1400 MHz, with the Lovell telescope) and removed a further three GCs that were clearly caused by this effect. However, the cadence of the Lovell observations is not as rapid as that of the 42-ft observations and we were unable to confirm some other possible cases of such non-achromatic events. We inspected the timing residuals of the largest remaining GCs ($|\Delta \nu| \geq 0.02 \mu\text{Hz}$) and found their signatures to be indistinguishable from timing...
Glitches have a minimum size, though we acknowledge that discrimination between small glitches and timing noise is difficult. Nonetheless, in many cases no sharp transitions, typical of the known glitches, are observed at the GC epochs and the residuals are consistent with a smooth connection with the pre-GC-epoch residuals.

Our final list contains 381 GCs. They are homogeneously distributed over the entire time-span and are clustered as a population in $\Delta \nu - |\Delta \nu|$ space (Figs. 2.2, 2.3). The vast majority of them exhibit $\Delta \nu$ steps that are smaller than all previously detected glitches, leaving a gap between the $\Delta \nu$ distributions of real glitches and GCs which would be hard to populate with undetected events.

2.5.2 Search for anti-glitches

Given the distinct properties of the GC population, it is possible that the glitch-like signatures found by the detector are a component of the Crab pulsar’s timing noise. To test this idea and explore the noise nature of these irregularities, we performed a search for events with the opposite signature to a glitch, i.e. anti-glitch candidates (AGCs) with $\Delta \nu < 0$ and $\Delta \dot{\nu} \geq 0$, which are subject to the equivalent detection constraints as the normal glitches. After removing repeated detections and 10 events caused by glitch recoveries, gaps with no data and non-achromatic events (see above), we obtain 383 AGCs. They are also separate from the glitch population and show very similar characteristics to the GCs (Figs. 2.2, 2.3).

2.6 Discussion

2.6.1 GCs and AGCs: glitches or timing noise?

Using the Kolmogorov–Smirnov (K–S) test we can compare the $|\Delta \nu|$ distributions for GCs and AGCs, which are found to be statistically consistent with coming from the same parental distribution (with a K–S statistic of $D = 0.037$ and $p_{KS}(D) = 0.96$, thus a probability of only $\sim 4\%$ for a false null hypothesis). The $|\Delta \nu|$ distributions for GCs and AGCs can be well described by lognormal distributions, with probability density function (PDF) of the form

$$p(|\Delta \nu|) = \frac{1}{\sqrt{2\pi\sigma(|\Delta \nu| - \theta)}} \exp\left[-\frac{(\ln(|\Delta \nu| - \theta) - \mu)^2}{4\sigma^2}\right]$$

for $|\Delta \nu| > \theta$, which gives $p_{KS} = 0.85$ and 0.92 respectively. However, this result is only indicative since the lower ends of these distributions are not well probed by the observations (Figs. 2.2, 2.3).

We also compared the $|\Delta \nu| - |\Delta \dot{\nu}|$ distributions of GCs and AGCs using a 2-dimensional K–S test (Press et al. 1992). The test gives $D_{2D} = 0.084$, implying a probability
of ~ 40% that they come from the same distribution. This relatively low probability is likely to be produced by differences in the $|\Delta \dot{\nu}|$ distributions between GCs and AGCs, since a K–S test over these two gives $p_{KS} = 0.55$, considerably smaller than the one for $|\Delta \nu|$.

Neither a power-law nor a lognormal distribution can describe well the joint $\Delta \nu$ distribution of the 20 glitches plus all the GCs, with $p_{KS} < 10^{-4}$. A power-law with a lower cut-off at $\Delta \nu \sim 0.01 \mu$Hz, to account for the incompleteness of the sample at small sizes, gave a similarly poor fit.

Although it is possible that some of the GCs correspond to real glitches, we interpret all the above results as confirmation that the GCs and AGCs are generated by a symmetric noise process and that no new glitches have been found. This timing noise component produces a continuous departure from a simple slow-down trend with variations that can be characterised by changes of $|\Delta \nu| \leq 0.03 \mu$Hz and $|\Delta \dot{\nu}| \leq 200 \times 10^{-15} \text{Hz s}^{-1}$.

### 2.6.2 The glitch size distribution

Having established that the 20 glitches form the complete sample of glitches the Crab pulsar has had in the last 29 years, we can address their statistical properties.

To determine the best-fit exponent $\alpha$ for a power-law PDF of the form

$$p(\Delta \nu) = C \Delta \nu^{-\alpha},$$

with $\Delta \nu_{\min} \leq \Delta \nu \leq \Delta \nu_{\max}$ and $C = (1 - \alpha)(\Delta \nu_{\max}^{1-\alpha} - \Delta \nu_{\min}^{1-\alpha})^{-1}$, we use the maximum-likelihood estimator method. Setting $\Delta \nu_{\min} = 0.05 \mu$Hz and $\Delta \nu_{\max} = 6.37 \mu$Hz, the values for the smallest and largest glitches observed respectively, we obtain $\alpha = 1.36 (+0.15, -0.14)$ (Fig. 2.4). The value of the exponent does not depend strongly on the choice of limits, as long as these are a few times smaller or larger than the observed ones. To assess the goodness of the fit, we calculate the K–S statistic, $D = 0.1$, and its probability value $p_{KS}(D) = 0.9$, which corresponds to a 10% probability that our null hypothesis (that the data follow the PDF described by Eq. (2.4)) is false. Thus our results confirm that the Crab glitch $\Delta \nu$ distribution is consistent with a power-law, a description motivated by theoretical models.

If this power-law continued below $\Delta \nu_{\min}$, we would expect to have detected more than 10 glitches with $0.02 \mu$Hz < $\Delta \nu$ < $\Delta \nu_{\min}$ in the searched data, and the gap between glitches and GCs (in Figs. 2.2 and 2.3) should have been populated. Thus we observe a rapid fall-off of the power-law for $\Delta \nu < \Delta \nu_{\min}$.

However, the small sample size makes it impossible to exclude other distributions. For example, the same K–S probability is obtained for a lognormal distribution (Eq. 2.3) with parameters $\mu = -1.79$, $\sigma = 1.9$ and $\theta = 0.049 \mu$Hz, whose probability
Glitches have a minimum size

![Graph showing the cumulative distribution function of glitch sizes and the corresponding power-law fit.](image)

**Figure 2.4:** The cumulative distribution function of the observed glitch sizes, $s$, and the corresponding power-law fit (solid line) given by Eq. 2.4, with $0.05 \leq \Delta \nu (\mu Hz) \leq 6.37$ and $\alpha = 1.36$. The dashed line corresponds to a lognormal fit (Eq. 2.3) with $\mu = 1.79$, $\sigma = 1.9$ and $\theta = 0.049 \mu Hz$.

The density function also quickly vanishes for $\Delta \nu < \Delta \nu_{\text{min}}$. The same conclusions hold if the four glitches from before the start of this dataset are included in the sample.

Further confirmation of the rare occurrence of small glitches comes from the study of the $|\Delta \nu| - |\Delta \dot{\nu}|$ distribution. Having shown that the latter is not affected by observational biases, the correlation between $|\Delta \nu|$ and $|\Delta \dot{\nu}|$ (apparent in Fig. 2.2) is confirmed to be a robust feature. While $\Delta \nu$ measurements are very accurate, the acquired values of $\Delta \dot{\nu}$ are less certain and depend upon the method used to determine them, leading sometimes to large discrepancies. For this work we consistently calculated the glitch parameters for all 20 glitches, using the technique described in Espinoza et al. (2011b). Using those measurements, the Spearman’s rank correlation coefficient between $\Delta \nu$ and $|\Delta \dot{\nu}|$ is $rs = 0.776$ with $p(rs) = 6 \times 10^{-5}$, which indicates a strong correlation. We note that the correlation becomes stronger if the four early glitches are included.

Given this relationship, any additional glitches would occupy a region of the $|\Delta \nu|$...
2.7 Implications for theoretical models

Such a limit for the smallest glitch size is challenging to our current understanding of glitches and has the potential to constrain the proposed mechanisms.

Some simple considerations can be used to get a rough order of magnitude estimate for the number of neutron superfluid vortices that need to unpin to produce the smallest Crab glitch. Each superfluid vortex carries a quantum of circulation \( \kappa = \frac{\hbar}{2m_n} \approx 2 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1} \). Neglecting differential rotation of the superfluid (and entrainment), its total circulation at distance \( r \) from the rotational axis will be

\[
\Gamma = \oint v_s \cdot dl = N_v(r) \kappa = 2\Omega_s(r)A,
\]

where \( N_v(r) \) is the number of vortices in the enclosed area \( A \) and \( \Omega_s \) is the superfluid angular velocity. Using \( r = 10^6 \text{ cm} \), the total number of vortices for the Crab pulsar is of the order \( N_v \approx 6 \times 10^{17} \). Conservation of the total angular momentum implies that the angular velocity change of the superfluid, \( \delta \Omega_s \), relates to the observed glitch size \( \Delta \nu \) by

\[
\delta \Omega_s = 2\pi \Delta \nu I_c/I_s,
\]

where \( I_c \) is the moment of inertia of the coupled component and \( I_s \) is the superfluid moment of inertia that participates in the glitch. Using a typical value of \( I_c/I_s \approx 10^2 \) and \( \Delta \nu = \Delta \nu_{\text{min}} = 0.05 \mu\text{Hz} \), the total number of vortices must be reduced by \( N_v' \approx 10^{11} \).

The actual change in the superfluid angular momentum \( L_s \) depends on the number of vortices that unpinned, the location and size of the region where this happened and the distance travelled by those vortices before they repin. For a more rigorous estimate, the change in \( L_s \) can be approximated by

\[
\Delta L_s = \hat{\rho} \xi R^3 \delta N_v,
\]

where \( \hat{\rho} \) is the average density of the region involved, \( R \) is the stellar radius and \( \xi \) is the fraction of \( R \) that unpinned vortices travel (Warszawski & Melatos 2013). For a typical value of \( I_c \approx 10^{45} \text{ g cm}^2 \) for the moment of inertia of the coupled component, the smallest glitch observed translates to an angular momentum change of \( \Delta L_s \approx \hat{\rho} \xi R^3 \Delta \nu_{\text{min}} = 3 \times 10^{38} \text{ g cm}^2 \text{ s}^{-1} \). Conservation of angular momentum leads to \( \xi \delta N_v \approx 1.5 \times 10^9 \) if one assumes typical values for the base of the crust, like \( R = 10 \text{ km} \) and \( \rho = 10^{14} \text{ g cm}^{-3} \). Vortices are expected to repin after encountering a few available pinning sites, however as a conservative order of magnitude estimate we assume they cover a distance comparable to the thickness of the crust (1.5 km) and take \( \xi \leq 0.15 \), which means that at least \( 10^{10} \) vortices must unpin in a glitch with \( \Delta \nu = \Delta \nu_{\text{min}} \). Therefore the observed minimum glitch size, which is well above that expected for single-vortex unpinning events, implies the existence of a smaller characteristic length-scale which sets the lower cut-off for the range of the scale-invariant behaviour.
Glitches have a minimum size

The vortex avalanche model is based on the notion of self-organised criticality (SOC, Bak et al. 1987), applications of which can be found for example in earthquake dynamics (Hergarten 2002) or superconducting flux-tube avalanches (Wijngaarden et al. 2006). SOC occurs without the need of fine tuning of parameters, in several dynamical systems consisting of many interacting elements (the superfluid vortices in the case of a neutron star) which, under the act of an external slow driving force (the spin-down of the star), self-organise in a critical stationary state with no characteristic spatiotemporal scale. A small perturbation in such systems can trigger an avalanche of any size. Thus in the glitch avalanche model of Warszawski & Melatos (2008) vortex density is assumed to be greatly inhomogeneous and many metastable reservoirs of pinned vortices are formed, which relax independently giving rise to the observed spin-ups. Since such a system has no preferred scale the resulting glitch magnitudes follow a power-law distribution. This behaviour should however continue down to events involving the unpinning of only a few vortices, which is orders of magnitude below the observed cut-off.

The coherent noise model (Newman & Sneppen 1996) is a different, non critical mechanism which produces scale-free dynamics, even in the absence of interaction between the system’s elements. In such systems a global stress is imposed to all elements coherently, to which they respond if it exceeds their individual unpinning threshold, giving rise to avalanches of various sizes. Both threshold levels (for each element) and stress strength are randomly chosen from respective probability distribution functions. The elements with thresholds smaller than the applied stress will participate in an avalanche and then be re-assigned new threshold values. New thresholds must always be assigned to a few elements, even when no avalanche is triggered, otherwise such a system will stagnate. A possible mechanism for this process in superfluids is the thermally activated unpinning of vortices (Melatos & Warszawski 2009), while the global Magnus force acts as the coherent stress. The model predicts a minimum for the glitch magnitude, which represents the thermal creep only events, present even if all thresholds lie above the applied stress strength. But it also predicts an excess (with respect to the resulting power-law) of such small glitches, in contradiction to what we observe for the Crab pulsar. The lack of this overabundance of small events requires a broad distribution for the pinning potentials. Melatos & Warszawski (2009) studied the top-hat distribution and applied their model to the Crab pulsar. They found that the half-width of the distribution should be comparable to the mean pinning strength. Even when such a broad distribution of pinning energies is introduced, independent unpinning of vortices as a random Poisson process of variable rate proves insufficient to produce scale-invariant glitches (Warszawski & Melatos 2013), indicating that the interaction between vortices and collective unpinning (a domino-like process) must be taken into account. The most prominent
2.8 Conclusions

mechanism for collective unpinning is the proximity effect, in which a moving vortex triggers the unpinning of its neighbours. However such a mechanism requires extreme fine tuning, since power-law size distributions occur only if this effect is neither too weak (where thermal creep dominates) nor too strong (which always leads to large, system-spanning, avalanches) (Warszawski & Melatos 2013).

Another process which could lead to scale-invariant glitches are crustquakes (Morley & Schmidt 1996). Stresses develop in the solid crust of a neutron star because of the change in its equilibrium oblateness as the spin decreases, but also due to the interaction of the crustal lattice with the magnetic field and superfluid vortices in the interior. If the crust cannot readjust plastically it will do so abruptly when the breaking strain \( \epsilon_{ct} = \sigma_{ct}/\mu \) is exceeded (where \( \sigma_{ct} \) is the critical stress and \( \mu \) the mean modulus). This will result in both a spin-up (due to the moment of inertia decrease) and in a reaction of the superfluid (Alpar et al. 1996; Ruderman et al. 1998), which is evident in the post-glitch relaxation. The maximum fractional moment of inertia change associated with the \( \Delta \nu_{\text{min}} \) glitch is \( |\Delta I|/I \leq 10^{-9} \); we note here that the glitch size can be significantly boosted by the crustquake induced unpinning of vortices (Larson & Link 1999; Eichler & Shaisultanov 2010). Elastic stress on the crust due to change of the equilibrium oblateness builds up because of the almost-constant secular \( \dot{\nu} \). Therefore the critical stress \( \sigma_{ct} \) will be reached in regular time intervals if all stress is relieved in each crustquake, and the total energy released will be \( \Delta E_{\text{el}} \propto \epsilon_{ct}^2 \). If the stress is only partially relaxed then the energy released will depend on the stress drop \( \Delta \sigma \), and the time interval to the next crustquake will depend on the size of the preceding one. The latter correlation is observed for the glitches in PSR J0537-6910, which have been interpreted as crustquakes (Middleditch et al. 2006). For the Crab pulsar however, the lack of any such trends in our glitch sample indicates a more complicated picture.

2.8 Conclusions

We have quantified our current glitch detection capabilities and, after a meticulous search for small glitches, we have shown that in the case of the Crab pulsar all glitches in this dataset have already been detected. The full glitch size distribution exhibits an under-abundance of small glitches and implies a lower cut-off at \( \Delta \nu \sim 0.05 \mu \text{Hz} \). The existence of such a minimum glitch size implies a threshold-dominated process as their trigger, which still needs to be identified.

Besides the occasional glitches, we have detected a continuous presence of timing noise having a well defined maximum amplitude, which can be described by step changes \( |\Delta \nu| \leq 0.03 \mu \text{Hz} \) and \( |\dot{\nu}| \leq 200 \times 10^{-15} \text{Hz s}^{-1} \). The distinct properties of this noise component compared to the glitches imply that timing noise cannot be
2. Glitches have a minimum size

attributed solely to unresolved small glitches produced by the exact same mechanism.

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3 Glitch recoveries in radio-pulsars and magnetars

B. D. Haskell & D. Antonopoulou

Abstract

Pulsar glitches are sudden increases in the spin frequency of an otherwise steadily spinning down neutron star. These events are thought to represent a direct probe of the dynamics of the superfluid interior of the star. However glitches can differ significantly from one another, not only in size and frequency, but also in the post-glitch response of the star. Some appear as simple steps in frequency, while others also display an increase in spin-down rate after the glitch. Others still show several exponentially relaxing components in the post-glitch recovery. We show that if glitches are indeed due to large scale unpinning of superfluid vortices, the different regions in which this occurs and respective timescales on which they recouple can lead to the various observed signatures. Furthermore we show that this framework naturally accounts for the peculiar relaxations of glitches in Anomalous X-ray Pulsars.
3.1 Introduction

Neutron Stars (NSs) are some of the few objects that allow us to study the physics of matter at extreme densities and in strong gravity. Not only do the central densities of these objects exceed nuclear saturation density, but the core temperature of a mature NS is also expected to be below the superfluid transition temperature (Baym et al. 1969; Page et al. 2011; Shternin et al. 2011). The presence of a superfluid component significantly modifies the dynamics, allowing for the superfluid in the star to flow relative to the "normal" component and act as a reservoir of angular momentum. A direct manifestation of this is given by pulsar glitches, i.e. sudden increases in the frequency of an otherwise steadily spinning down NS.

Glitches are generally attributed to a large scale superfluid component in the interior of the star that is only loosely coupled to the crust and the magnetosphere. The sudden recoupling of such a component would then lead to the rapid transfer of angular momentum and to a glitch (Anderson & Itoh 1975). In particular, a superfluid rotates by forming an array of quantised vortices, which are being expelled as it slows down. In a NS however, vortices can "pin" to the ions in the crustal lattice or to magnetic flux tubes in the outer core (Alpar 1977; Link 2003). If this is the case vortices cannot move out, the superfluid does not follow the spin-down of the crust and lags behind, storing an excess of angular momentum. Once a sizable lag builds up, hydrodynamical lift forces (Magnus forces) acting on the vortices will unpin them, giving rise to sudden vortex motion and the glitch. Recent work has shown that this mechanism can successfully account for the distribution in glitch sizes and waiting times (Melatos et al. 2008; Melatos & Warszawski 2009) and model giant glitches in the Vela and other pulsars (Pizzochero 2011; Haskell et al. 2012; Seveso et al. 2012).

In general glitches appear as an abrupt increase in the spin frequency, typically accompanied by an increase in spin-down rate. Glitch sizes can span several orders of magnitude even in the same object and the post-glitch recovery can be quite different from glitch to glitch. Some events, such as the large glitches of the Vela pulsar, show an exponential relaxation, on timescales from minutes to months, towards the previous spin-down rate (Cordes et al. 1988; McCulloch et al. 1990). More often glitches are associated with a permanent increase in the spin down rate. Other glitches still appear as a simple step in frequency that does not recover at all. Furthermore the different kinds of glitch recovery can appear in the same object (Espinoza et al. 2011b; Yu et al. 2013). Glitches have also been observed in several Anomalous X-ray Pulsars (AXPs), which are thought to be strongly magnetised NSs, magnetars. In terms of absolute sizes AXP glitches do not differ much from those observed in regular radio pulsars, however they are often associated with radiative events (Israel et al. 2007; Gavriil et al. 2011), and the post-glitch recoveries are also remarkable in many ways, with many featuring recoveries with large fractional increases in the
spin-down rate over long timescales (weeks or months) (Dall’Osso et al. 2004; Dib et al. 2008).

In this paper we address two issues. First of all we use numerical simulations to show that, as also discussed by Alpar & Baykal (2006), the vortex unpinning paradigm can explain the different kinds of relaxation, depending on the coupling timescale of the region that unpins. Secondly we will show that the same mechanism giving rise to glitches in radio pulsars can naturally produce smaller glitches in hotter stars such as magnetars and young pulsars like the Crab, but also lead to a strong relaxation in magnetars.

We model the NS as a two fluid system of superfluid neutrons and a charged component, composed of ions, protons, electrons and all components tightly coupled to them (Andersson & Comer 2006). The two components are coupled by an interaction known as Mutual Friction (MF), which is mediated by the quantised vortices of the neutron superfluid (Andersson et al. 2006). Following Haskell et al. (2012) we can write the equations of motion for the angular frequency of the two components ($\Omega_p$)
for the charged component, $\Omega_n$ for the superfluid neutrons) in the form:

$$\dot{\Omega}_n(\tilde{r}) = \frac{Q(\tilde{r})}{\rho_n} - f(\epsilon_n)\frac{\mathcal{A}}{I_p} \Omega_p^3 \quad (3.1)$$

$$\dot{\Omega}_p(\tilde{r}) = \frac{Q(\tilde{r})}{\rho_p} - g(\epsilon_n)\frac{\mathcal{A}}{I_p} \Omega_p^3 \quad (3.2)$$

where $\tilde{r}$ is the cylindrical radius and $I_p$ the moment of inertia of the charged component. We have defined $Q(\tilde{r}) = \rho_n k n_m B (\Omega_p - \Omega_n)/(1 - \epsilon_n - \epsilon_p)$ with $k$ the quantum of circulation, $n_m$ the density of "free" vortices (which contribute to MF), and $\mathcal{A} = B^2 \sin^2 \theta / 6c^3$ with $B$ the surface magnetic field strength, $\theta$ the inclination angle between the field and the rotation axis and $R$ the stellar radius. The coefficient $B$ is the dissipative MF coefficient, which quantifies the strength of the interaction between vortices and the charged component. The entrainment coefficients are $\epsilon_n$ and $\epsilon_p$, and describe the non-dissipative coupling between the fluids (Andersson & Comer 2006). The functions $f(\epsilon_n)$ and $g(\epsilon_n)$ are such that in the limit of vanishing entrainment $g(\epsilon_n) = 1$ and $f(\epsilon_n) = 0$ (Andersson et al. 2012). Strong entrainment, however, also reduces the maximum lag that can be built up between the two fluids, thus providing a strong constraint for glitch models that only involve the NS crust (Andersson et al. 2012; Chamel 2013a). This constraint is not so severe for models such as those presented here in which the larger glitches involve the decoupling of part of the core (Haskell et al. in preparation, Hooker et al. (2013)). In the presence of a pinning force vortices are not free to move unless the lag between the two components exceeds a threshold. We shall use the realistic profiles obtained by Pizzochero (2011) for the critical lag $\Omega_p - \Omega_n$. Below this threshold vortices are pinned and we set $B = 0$ in equation (3.1). Once vortices have unpinned they are free to move out, but in the crust many will repin and only a fraction will be free to move and contribute to the drag force. Following Jahan-Miri (2006) we assume that the instantaneous number density of free vortices $n_m$ is given by $n_m = \xi n$, where $n$ is the total number density of vortices, and $\xi$ is the fraction of unpinned vortices at a given time. By averaging over time we can obtain an effective drag parameter $\bar{B} = \xi B$.

The mechanisms that give rise to MF, and thus determine $B$, are different in different regions of the star and can vary by several orders of magnitude. In the core MF is expected to be due to electron scattering of magnetised vortex cores (Alpar et al. 1984c) with a value $B_{co} \approx 10^{-4}$ (Andersson et al. 2006). In the crust on the other hand the main dissipative mechanism is the excitation of sound waves in the lattice, which leads to a weak drag $B_{cr} \approx 10^{-6}$ (Jones 1992). If the relative velocity between vortices and ions is high ($\approx 10^2$ cm/s) it becomes possible to excite Kelvin waves of the vortices, leading to fast coupling and $B_k \approx 10^{-3}$ (Epstein & Baym 1992; Jones 1993). Given the uncertainties we will take constant values of the drag parameters (one for the core and one for the crust) and average them over the length of a vortex,
as described in Haskell et al. (2012). We follow the setup of Haskell et al. (2012) and consider a 1.4 $M_\odot$ NS, with a radius $R = 12$ km. The equation of state is an $n = 1$ polytrope and the crust/core transition is assumed to take place at $\rho = 1.6 \times 10^{14}$ g/cm$^3$. The proton fraction is taken to be $x_p = 5 \times 10^{-12}$. 

Note that entrainment enters into the problem in three ways: in (3.1) via the functions $f$ and $g$, as a rescaling of the parameter $B$ in the definition of $Q(\tilde{R})$ and indirectly in the microphysical calculations that lead to the value of $B_{co}$ (Alpar et al. 1984c). For simplicity we will neglect any explicit dependence on entrainment by setting $f(\varepsilon_n) = 0$ and $g(\varepsilon_n) = 1$, as these impact mainly on the lag that can be built up before the glitch. We then study how the solutions vary as we vary the value of $B_{co}$.

Let us now examine how our setup could give rise to different classes of glitches. The general picture is the following: vortices are pinned in certain regions of the star (possibly the crust). In these regions vortex motion is impeded, leading to a significant lag building up. A glitch will occur when vortices are rapidly unpinned in such a region and the superfluid neutrons can then exchange angular momentum with the normal component. We argue that the signature of the glitch can depend strongly on how fast the superfluid can react to the unpinning event, and where it takes place.

We can obtain a good approximation to the local coupling timescale in each region of the star (Haskell et al. 2012) by neglecting differential rotation and taking a constant proton fraction $x_p = \rho_p/(\rho_p + \rho_n)$ in equations (3.1) and (3.2). Given an initial lag $D_0$, this leads to a solution of the form $\Omega_p - \Omega_n \approx D_0 \exp(-t/\tau)$, where the coupling timescale is:

$$\tau \approx \frac{x_p}{2\Omega_n B}$$ (3.3)

In figure 3.1 we schematically illustrate two possible scenarios by considering how the coupling timescale in (3.3) varies throughout the star. We assume that the pinned region is in the crust, but the argument is qualitatively identical if it is in the outer core. We assume that vortices close to the rotation axis cross through the core and are only weakly pinned in the crust, while MF in the core leads to short coupling timescales between the superfluid and the rest of the star. As one moves towards the equatorial region the coupling timescale increases and becomes long for those regions where most vortices are pinned. At the moment of the glitch a number of vortices unpin, leading to an increased fraction of free vortices $\xi$ and thus to a larger MF parameter $\tilde{B}$ and a reduced coupling timescale, leading to an observable spin-up of the star. In the middle panel of figure 3.1 we can see the case in which the coupling timescale in the region that gives rise to the glitch becomes shorter than the timescale on which the outer regions of the core are coupled. These regions cannot follow the glitch and decouple, giving rise to a strong relaxation as they recouple on the local timescale. In the right panel we can instead see the case in which one simply assumes that the increase of $\xi$ in a region gives rise to a coupling timescale that is
short compared to that of the pinned region, but still longer than that of most of the core. In this case only the pinned region will decouple at the glitch, and the recovery will proceed on the much slower timescale associated with it and thus appear as a "permanent" step in the spin down rate of the star. In the presence of a wide range of pinning potentials the response of the crust alone can give rise to a variety of post-glitch behaviours, as discussed in Alpar et al. (1984a), such models however cannot account for large (> 0.1%) fractional changes of the spin-down rate and do not allow for prompt relaxation in old, cool pulsars.

3.2 Relaxing glitches

We now focus on the glitches that show a significant relaxation. The prototype system for this kind of glitch is the Vela pulsar, which shows relatively large glitches followed by an exponential recovery. The frequency $\nu$ after the glitch takes the form $\nu(t) = \nu_0(t) + \Delta\nu + \Delta\nu t + \sum_i \Delta_i \exp(-t/\tau_i)$, where $\nu_0(t)$ is the pre-glitch spin-down solution. In recent glitches up to five decaying terms have been fit, with timescales $\tau_i$ that range from minutes to months. The shorter timescales are associated with strong increases in the spin down rate, which can be comparable to or larger than the pre-glitch rate (Dodson et al. 2007). Relaxing glitches in AXPs appear to be a "scaled-down" version of Vela giant glitches, in as much as they are smaller (Vela glitches have steps of the order of $\Delta \nu \approx 10^{-5}$Hz, compared to $\Delta \nu \approx 10^{-8}$Hz typically in AXPs) and show the kind of strong relaxation that Vela glitches show on timescales of minutes to hours, but on much longer timescales of days to weeks (Dib et al. 2008). It is thus interesting to ask whether the mechanisms that is giving rise to them is the same as in the Vela.

The assumption is that such glitches are triggered once the maximum lag that the pinning force can sustain is exceeded. Then vortices will move out rapidly, exciting Kelvin waves. This leads to a glitch rise timescale shorter than the timescale on which the outer regions of the core are coupled. The outer core thus decouples and will subsequently recouple on the local timescale given in (3.3), giving rise to the observed relaxation. Such a mechanism was studied in detail by Haskell et al. (2012), who were able to predict the correct size and relaxation for the Vela giant glitches. Let us now investigate whether relaxing glitches in magnetars can be due to the same mechanism. We start by noting that the strong increase in the spin-down rate observed on timescales of months in AXPs occurs naturally if one assumes that the relaxation is given by regions of the outer core recoupling on the local coupling timescale given by superfluid MF. If we consider the timescale in (3.3) it is clear that the same process will be much faster in a pulsar such as the Vela, with $\nu \approx 10$ Hz, than in a magnetar rotating at $\nu \approx 0.1$ Hz. One would thus expect that given that in the Vela 2000
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Figure 3.2: In the top left panel we show the effect of changing the size of the unpinning region from 5 m to 80 m at the base of the crust. We switch from a background $B = 5 \times 10^{-9}$ to $B = 8 \times 10^{-7}$ during the glitch. The size of the region has a strong impact on the size of the glitch, but little impact on the change in spin-down rate. In the top right panel we show how the increase in the spin-down rate becomes larger if one increases the value of $B$ during the glitch, in an 80 m region, with a background $B = 5 \times 10^{-9}$. As $B$ increases (and the rise time decreases) $\Delta \nu/\nu$ passes from 0.9% for $B = 8 \times 10^{-7}$, to 1.3% for $B = 9.5 \times 10^{-7}$, and finally for $B = 2 \times 10^{-6}$ one has a clear exponential relaxation. Finally in the bottom panel we have a glitch due to the increase in $B$ from $5 \times 10^{-12}$ (which mimics perfect pinning) to $5 \times 10^{-7}$ over 8 m. The glitch appears as a step in frequency with no significant recovery. In all cases we consider a star rotating at a frequency of $\nu = 0.11$ Hz, and the pre-glitch spin-down has been removed. The low spin rate leads to long rises of $\approx 1$ day, which would be much shorter in a faster pulsar ($< 1$ hour for the Crab)

and 2004 glitches there are increases in the spin-down rate of the order $\Delta \nu/\nu \approx 0.5$ associated with decay timescales of $\tau \approx 0.5$ days (Dodson et al. 2007), such an increase would be associated with a decay timescale of $\approx 50$ days for a magnetar, as is observed for example in the AXP 1RXS J17084 (Dall’Osso et al. 2004; Dib et al. 2008).

The other main difference between Vela giant glitches and relaxing glitches in magnetars is the size of the step in frequency, which is smaller for magnetars ($\Delta \nu \approx$
3.3 Slow "step like" glitches

Let us now turn our attention to glitches that are associated with "permanent" increases in the spin down rate but with no appreciable relaxation. Our assumption is that this kind of glitch is not due to the vortices that have accumulated close to the maximum of the critical lag, but rather involves regions far from the maximum, in which vortices are "creeping" out. While the sudden release of vortices close to the maximum of the critical lag could excite Kelvin waves and lead to short coupling

### Table 3.1: Size of the glitch and magnitude of the increase in spin-down rate after 1 and 50 days for a (magnetar-like) star spinning at 0.011 Hz, for varying MF parameters in the crust ($\tilde{B}_{cr}$) and core ($\tilde{B}_{co}$).

Weaker MF parameters in the core, such as those predicted if one has a type II superconductor in the outer core (Link 2012) would result in larger glitches and a strong increase in the spin-down rate after the glitch.

$\nu_0 \approx 10^{-8}$Hz) than for the Vela and other radio pulsars that exhibit giant glitches ($\Delta \nu \approx 10^{-5}$Hz). Several models predict smaller glitches for younger, hotter objects (Alpar et al. 1984a; Glampedakis & Andersson 2009; Haskell et al. 2012) and it is thus plausible that the same mechanism giving rise to the giant glitches in Vela will produce smaller glitches in hotter stars, such as magnetars and young pulsars.

To test this hypothesis we use the code developed by Haskell et al. (2012), and solve the equations (3.1-3.2) as in the models considered for the Vela, but using a spin frequency typical for magnetars and a higher background MF parameter $\tilde{B}_{cr}$ in the crust. The latter is to account for a larger vortex creep rate, due to the magnetar’s higher temperature. This leads to a smaller region of the crust that has not relaxed and can participate in the glitch. We use $\tilde{B}_{gl} = 10^{-3}$ for the rise. The results can be seen in table 3.1, in which we show the size of the glitch and the step in frequency derivative after 1 day and after 50 days. As we can see the results are consistent with the kind of glitches seen in magnetars. Furthermore we predict that one should be able to observe stronger increases in spin-down rate on short timescales, if the MF in the outer core is weak, as could be the case if protons are in a type II superconducting state (Link 2012). Better coverage of AXP glitches thus has the potential to constrain the MF parameters and determine the nature of the pairing in the NS interior.

### 3.3 Slow "step like" glitches

Let us now turn our attention to glitches that are associated with "permanent" increases in the spin down rate but with no appreciable relaxation. Our assumption is that this kind of glitch is not due to the vortices that have accumulated close to the maximum of the critical lag, but rather involves regions far from the maximum, in which vortices are "creeping" out. While the sudden release of vortices close to the maximum of the critical lag could excite Kelvin waves and lead to short coupling
timescales, in the case we consider now the coupling timescale of the region giving rise to the glitch would decrease compared to that of the crust, but still be longer than the coupling timescale of the core, as shown in the right panel of figure 3.1. The core will thus not decouple and give rise to a visible relaxation, but rather the crust will be decoupled on long timescales, leading to what will appear as a permanent increase in the spin-down rate.

We will study this problem by once again using the code of Haskell et al. (2012) to solve the equations in (3.1-3.2). First of all we allow for the crust to reach a steady state, in which the two fluids are spinning down together with a lag between the two of $\Delta \Omega \approx \frac{\Omega}{2 \Omega_{cr}}$. In this background configuration we use $\bar{B}_{cr} = 5 \times 10^{-9}$. We now take a region of varying thickness at the base of the crust and switch the MF parameter to $\bar{B}_{gl} = B_{gl} \approx 10^{-6}$. This will be the situation if, for example, a sudden event, such as a crust quake or a vortex avalanche (Ruderman 1969; Melatos & Warszawski 2009) frees the vortices in this region, leading to $\xi \approx 1$.

The results can be seen in figure (3.2). In the left panel we see that the size of the region over which vortices are freed (i.e. the extent of the avalanche or quake) has a strong influence on the size of the glitch, but little influence on the increase in spin-down rate. This can be understood if one considers the equations of motion in (3.1-3.2) locally in the crust, neglecting differential rotation and electromagnetic spin down. From angular momentum conservation we see that the size of the glitch $\Delta \Omega_G$ will depend on the lag $\Delta \Omega$ between the superfluid and the charged component via $\Delta \Omega_G = \frac{I_g}{I_c} \Delta \Omega$, where $I_g$ is the moment of inertia of the regions in which vortices unpin, while $I_c$ is the moment of inertia of the region that is coupled fast enough to follow the rise of the glitch. The size of the region in which vortices unpin thus determines $I_g$ and is crucial for the step size. The moment of inertia coupled during the glitch, $I_c$, is essentially the moment of inertia of the whole core. The rest of the crust will be coupled on a longer timescale compared to that of the rise, given that $\bar{B}_{cr} \ll \bar{B}_{gl}$, and will thus decouple and only recouple slowly on a timescale given by equation (3.3). If this timescale is long compared to the timescale on which the post-glitch relaxation is observed, the crust will be decoupled during the whole period and the increase in the spin down rate will appear permanent. The star will spin down faster by a fraction $\approx I_{cr}/I_c$, with $I_c$ the moment of inertia of the crust. For a slow enough rise this fraction is approximately constant. This can be seen in the middle panel of figure (3.2) in which we show that the quantity $\Delta \nu / \nu$ varies little with increasing $\bar{B}$, until one gets to a value large enough that the rise is sufficiently fast to involve part of the core, giving rise to a visible relaxation. Note that in all figures we have assumed a ‘magnetar’ spin rate of $\nu = 0.11$ Hz, to illustrate that in this case the rise may be slow enough ($\approx 1$ day) to be detected (see e.g. Woods et al. (2004) for a possible detection of a slow rise). For a faster pulsar the rise would be much faster.
and difficult to detect even in this "slow" case (e.g. < 1 hour for the Crab).

Finally, let us note that a small number of glitches show no observable increase in the spin down rate and are consistent with being pure steps in frequency (Dib et al. 2008; Espinoza et al. 2011b). A sudden release of vortices in a region in which vortices are creeping out will lead to decoupling of the rest of the crust and to an increase in the spin-down rate after the glitch. If the increase in mobility were, however, to take place while most of the superfluid in the crust is still decoupled from the observed charged component (e.g. because it is strongly pinned), one would still have a step in frequency but would there be no change in the effective moment of inertia and thus in the spin down rate. The region that is now coupled on a short timescale will relax rapidly and give rise to the glitch, while the other crustal superfluid regions were decoupled before the glitch and remain decoupled after. The effective moment of inertia thus remains unaffected. We show an example of a glitch in this setup in the right panel of figure (3.2) in which we assume a sudden release of vortices while the superfluid in the crust is decoupled. The result is a step in frequency, with no visible increase in the spin-down rate.

### 3.4 Conclusions

We have presented an analysis of different kinds of glitching behaviour in radio pulsars, and shown that glitches followed by a strong relaxation may have the same origin as the giant glitches of the Vela pulsar, but naturally scaled down in hotter, younger systems, such as the Crab or Magnetars. We have also shown that the slower hydrodynamical response to (possibly smaller) events due to random unpinning or crust quakes in the crust will lead to glitches that do not appear to relax, but appear as steps in frequency and frequency derivative. If the event takes place in a strongly pinned region it will appear only as a step in frequency.

We also argue that the same mechanisms that are at work in radio pulsars could naturally give rise to the sizes and recoveries observed in magnetar glitches. In particular, the longer periods of magnetars would naturally lead to a long-term strong increase in the spin down rate, with no need for an additional mechanism to be involved. The slower timescales associated with AXPs and SGRs make such events especially interesting as they would lead to long timescales for the rise, possibly of days, that could be observed if one were to have better coverage of these objects. This would allow us to understand to what extent these events do, indeed, have the same origin as radio pulsar glitches or to which extent they could allow us to probe the physics of the magnetosphere, which is likely to play an important role (Lyutikov 2013) and could be the cause of phenomena such as "anti-glitches" (Archibald et al. 2013).
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The unusual glitch recoveries of
the high magnetic field pulsar J1119–6127

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Abstract

Providing a link between magnetars and radio pulsars, high magnetic field neutron stars are ideal targets to investigate how bursting/magnetospheric activity and braking torque variations are connected to rotational glitches. The last spin-up glitch of the highly magnetised pulsar J1119–6127 back in 2007 was the first glitch in a rotationally powered radio pulsar to be accompanied by radiative changes. Moreover, it was followed by an uncommon glitch relaxation that resulted in a smaller spin-down rate relative to the prediction of the pre-glitch timing model. Here, we present 4 years of new radio timing observations and analyse the total of 16 years of timing data for this source. The new data uncover an ongoing evolution of the spin-down rate, thereby allowing us to exclude permanent changes in the external or internal torque as the main cause of the maximum peculiar glitch recovery. Furthermore, no additional variations of the radio pulse profile are detected, strengthening the association of the previously observed transient emission features with the glitching activity. The braking index has been measured in a self-consistent way, resulting in a value of $n \approx 2.7$, which is more robust compared to previous published values. Under this braking index, PSR J1119–6127 is evolving more towards the magnetar region of the $\dot{P}-P$ plane than expected from canonical pulsar spin-down. The potential evolutionary link between this highly magnetised pulsar and the magnetars might be strengthened by the fact that the long-term effect of the glitch on the braking index is a, possibly permanent, reduction of $\sim 15\%$. 


4.1 Introduction

Studying the rotational dynamics of radio pulsars is of key importance for the advance of our global understanding of neutron stars (NSs). Isolated NSs slowly brake as they lose energy through electromagnetic torques in their magnetospheres. In the simplest approximation, the spin-down is described by the dipole radiation of a misaligned rotator in vacuum, \( \dot{\nu} \propto \nu^3 \), where \( \nu \) is the NS spin frequency and dots represent time derivatives. However, the less understood contribution from magnetospheric currents can be significant or even dominant. It is conventionally presumed that a generalised spin-down law

\[
\dot{\nu} = -C \nu^n
\]

(4.1)

governs the rotation which, under the assumptions that \( n \) and the positive factor \( C \) are time-independent, leads to the observable

\[
n = \nu \dot{\nu} / \nu^2,
\]

(4.2)

the so-called braking index. According to the prediction of the dipole in vacuum braking mechanism, \( n = 3 \) if \( C \) can be considered constant for most of the pulsar’s life. In this case \( C \) depends on the magnetic dipole moment and the moment of inertia of the stellar component that follows the spin-down. Measurements of \( n \) can probe the contribution of other braking mechanisms in the spin-down rate, as well as the validity of the generalised spin-down law and the underlying assumptions for \( C \) and \( n \). There are only a few pulsars for which a reliable measurement of \( \dot{\nu} \) is possible, and in all of them deviations from the rudimentary vacuum dipole braking are observed, with \( n < 3 \) (see for example Table 1 in Lyne et al. (2014) and references therein).

Another generic feature of NS rotation is timing noise. It manifests as small fluctuations of the star’s frequency with respect to a simple spin-down model, on time scales of days to years. The nature of timing noise remains unknown, though recent work relates it to magnetospheric phenomena such as pulse profile changes (Lyne et al. 2010). Finally, occasional abrupt spin-ups called glitches have been observed in \( \approx 160 \) pulsars. The increase in spin frequency \( \nu \) during a glitch happens very rapidly and typically lies in the range \( 10^{-5} - 100 \) \( \mu \text{Hz} \). Glitches are usually accompanied by an increase in spin-down rate \( |\dot{\nu}| \) of the order of \( 10^{-19} - 10^{-11} \) \( \text{Hz s}^{-1} \), often followed by a slow relaxation towards the pre-glitch rotational parameters (Espinoza et al. 2011b; Yu et al. 2013).

Large glitches in rotationally powered pulsars (RPPs) have not been connected to observed radiative changes in either intensity, polarisation or pulse profile shape, which supports an internal, rather than magnetospheric, origin. Our understanding of pulsar glitches is far from complete, however a two-component model for the NS’s interior has been successfully implemented to describe their main attributes.
Glitches in PSR J1119–6127

(Anderson & Itoh 1975; Alpar et al. 1984a; Haskell et al. 2012). In such models, a neutron superfluid component, usually assumed to be the inner crust’s superfluid, is at least partially decoupled from the rest of the star and rotates faster than the normal component. The latter encompasses the solid outer and inner crust and all fluid and superfluid components that are strongly coupled to it, and is spinning down under the external torques. When angular momentum is rapidly exchanged between the two components, a glitch occurs. The spin-up brings the two components closer to corotation, which weakens their coupling. As a result the external torque acts on a reduced effective moment of inertia \( I_{\text{eff}} \), leading to an enhanced spin-down rate following the glitch. The recoupling process of the superfluid is reflected in the post-glitch relaxation, which can often be described as exponential with long characteristic timescales, from days to months (Shemar & Lyne 1996).

Though most common in young radio pulsars, glitches appear in other RPPs like millisecond pulsars and old, slower pulsars too (Cognard & Backer 2004; Espinoza et al. 2011b). Glitches have also been observed in the rotation of magnetars, NSs which present X-ray luminosities that exceed their rotational energy losses and bursting activity such as very energetic \( \gamma \)-ray flares or X-ray outbursts. Magnetars are thought to be highly magnetised NSs, powered by the decay of their strong magnetic fields. Contrary to ordinary radio pulsars, glitches in magnetars often (but not always) coincide with bursts or smaller radiative changes (Dib et al. 2008; Dib & Kaspi 2014). Furthermore, magnetars show a larger variety of post-glitch recoveries as well as other spin-down rate fluctuations, sometimes accompanying a radiative event but without an apparent glitch association (see for example Woods et al. 1999; Gavriil et al. 2009; Archibald et al. 2013; Dib & Kaspi 2014). PSR J1846–0258, which is normally powered by rotation, exhibited magnetar-like activity (Gavriil et al. 2008) together with a large glitch which was followed by an atypical \( \dot{\nu} \) evolution, indicative of a possibly permanent decrease in the braking index (Kuiper & Hermsen 2009; Livingstone et al. 2010, 2011).

PSR J1119–6127 is a high magnetic field pulsar, a small class of RPPs with spin parameters and inferred magnetic field strengths close to those of magnetars. It was first discovered in the radio during the Parkes Multibeam Pulsar Survey and has a period of \( P = 408 \) ms and period derivative \( \dot{P} = 4 \times 10^{-12} \) (Camilo et al. 2000). Its surface dipole magnetic field \( B_d \) strength is estimated, assuming conventional dipole braking, as \( B_d \approx 4.1 \times 10^{13} \) G, very close to the quantum electrodynamics (QED) limit \( B_{QED} = 4.41 \times 10^{13} \) G above which phenomena like spontaneous pair creation and suppression of pair cascades due to photon splitting must be taken into account (Baring & Harding 2001).

The characteristic age \( \tau_{sd} = P/2\dot{P} \sim 1.6 - 1.9 \) kyr of PSR J1119–6127 suggests it is a young pulsar, and indeed it has been associated with the supernova remnant
SNR G292.2-0.5 (Crawford et al. 2001). It is one of the youngest radio pulsars with detected thermal emission, as it was discovered in X-ray observations by Pivovaroff et al. (2001) and since then observed by several missions like ASCA, ROSAT, XMM and Chandra (Gonzalez & Safi-Harb 2003; Gonzalez et al. 2005; Ng et al. 2012). Pulsations were also recently detected by Fermi, as expected from its high rotational energy loss rate ($\dot{E} = 2.3 \times 10^{36}$ erg s$^{-1}$), making PSR J1119–6127 the source with the highest inferred $B_d$ among $\gamma$-ray pulsars (Parent et al. 2011). The relatively stable spin-down and long-term monitoring with the Parkes radio telescope allowed the accurate measurement of its braking index, which was found to be $n = 2.684 \pm 0.002$ (Weltevrede et al. 2011, hereafter WJE11).

In 2004, PSR J1119–6127 suffered a rather common glitch of magnitude $\Delta \nu_g \approx 0.7 \mu$Hz (where an index $g$ indicates values extrapolated at the glitch epoch), accompanied by a change in spin-down of $\Delta \dot{\nu}_g \approx -9 \times 10^{-14}$ Hz s$^{-1}$ which recovered with a characteristic timescale of $\sim 3$ months. However the glitch recovery was unusual, in that the post-glitch spin-down rate appeared to settle at a smaller value than the one extrapolated from the pre-glitch spin parameters. This is very atypical of glitches, where either the pre-glitch spin-down rate resumes once the post-glitch relaxation is over, or the end result is a larger spin-down rate which does not appear to recover completely. The evolution of this anomalous $\dot{\nu}$ recovery was interrupted in 2007 by another glitch. The second glitch had a much larger magnitude, with $\Delta \nu_g > 10 \mu$Hz and $-10^{-10} \leq \Delta \dot{\nu}_g \leq -10^{-11}$ Hz/s inferred at the glitch epoch, showing a relaxation on two timescales ($\sim 10$ days and $\sim 6$ months). As in the first glitch, the post-glitch spin-down rate eventually decreased below the value extrapolated from the pre-glitch timing model, showing a maximum departure of $\Delta \dot{\nu}_{\text{max}} \approx 3.5 \times 10^{-14}$ Hz/s. Even more surprisingly, the first radio observation after this glitch showed a very different pulse profile and erratic pulse components. The magnetospheric activity was present only after the glitch and lasted no more than $\sim 3$ months, a strong indication that the two phenomena might be related (WJE11). This remarkable behaviour offers a unique opportunity to explore the connection of internal and external processes during glitches.

In this paper we present the latest timing observations of PSR J1119–6127 and a self-consistent method to analyse its braking index. The new data show that the magnitude of the spin-down rate has been relatively increasing for the last $\sim 5$ years, meaning that the difference with the pre-glitch predicted $\dot{\nu}$ is reducing. We discuss the follow up of the puzzling $\dot{\nu}$ evolution after the 2007 glitch in view of current theories of glitches and pulsar spin-down.
4.2 Observations and timing analysis

For this work we analyse 16 years of timing observations of PSR J1119–6127. This is the data analysed in WJE11, supplemented with four years of new data obtained from the ongoing timing observations with the 64-m Parkes radio telescope in Australia. In this timing program (Weltevrede et al. 2010) each pulsar is typically observed once per month at a wavelength of 20 cm and twice per year at 10 and 50 cm. The timing analysis detailed in WJE11 was repeated for this longer data set. This process is summarised below while for details we refer the reader to the aforementioned papers.

The individual observations were summed resulting in a high signal-to-noise "standard" profile (see also Weltevrede & Johnston 2008). The time-of-arrival (TOA) of each observation was determined by cross-correlation of their pulse profile with this standard. These TOAs were projected to the solar system barycenter using the TEMPO2 timing package (Hobbs et al. 2006). Timing analysis of these corrected TOAs was performed using custom software (see WJE11).

The basic timing model used to describe the effect of spin-down on the rotational phase of the pulsar as a function of time $\phi(t)$ is a truncated Taylor series:

$$\phi(t) = \phi_0 + \nu_0 \cdot (t - t_0) + \frac{\dot{\nu}_0}{2} \cdot (t - t_0)^2 + \frac{\ddot{\nu}_0}{6} \cdot (t - t_0)^3.$$  \hspace{1cm} (4.3)

Here $\phi_0$, $\nu_0$, $\dot{\nu}_0$ and $\ddot{\nu}_0$ are the reference phase, spin frequency and its first two time derivatives defined at epoch $t_0$. Each glitch was modelled by including an additional function $\phi_g(t)$ to the timing model after the glitch epoch $t_g$, which is parameterised as

$$\phi_g(t) = \Delta \phi + \Delta \nu_p \cdot (t - t_g) + \frac{\Delta \ddot{\nu}_p}{2} \cdot (t - t_g)^2 + \frac{\Delta \ddot{\nu}_p}{6} \cdot (t - t_g)^3$$

$$- \left( \sum_i \Delta \nu_d^{(i)} r_d^{(i)} e^{-(t-t_g)/\tau_d^{(i)}} \right),$$  \hspace{1cm} (4.4)

where $\Delta \phi$ is a phase offset arising from the fact that the glitch epoch is not accurately known. The parameters labelled with an index $p$ correspond to permanent changes while those with an index $d$ refer to decaying components of the spin evolution. The full set of parameters in Eq. 4.4 is degenerate for our dataset, so each glitch was modelled with a subset of these terms as described in the next subsection. The $\Delta \ddot{\nu}_p$ term was not required in the description of the shorter data-span presented in WJE11 and was set to zero in their timing model.
4.2 Observations and timing analysis

Figure 4.1: Top-left panel: The measured rotational frequency of PSR J1119–6127 as a function of time (error bars are smaller than the points). The frequency is steadily decreasing, as is expected from the measured $v_0$ and $\dot{v}_0$ in Table 4.1 (dotted line). Middle-left panel: The effect of a constant spin-down rate is subtracted from the rotational frequency. Especially the second glitch can clearly be seen as a deviation from the parabola-shape. The latter indicates a significant and stable braking-index and the prediction according to the measured $\ddot{v}_0$ is shown as a dotted line. Bottom-left panel: The difference between the measured spin frequency and the contributions from $v_0$, $\dot{v}_0$ and $\ddot{v}_0$. The solid lines show the prediction according to the glitch model A. Top-right panel: The measured spin-frequency derivative as a function of time compared to the $\dot{v}_0$ and $\ddot{v}_0$ contribution (dotted line). Middle-right panel: The difference between the data and the dotted line of the top-right panel. The solid line indicates the prediction according to the glitch model A. Bottom-right panel: This plot is identical to the middle-right panel, but using a slightly more constrained vertical range. The timing model from WJE11 is indicated as the grey dashed line. Although these plots show theoretical curves according to predictions of model A, both models in Table 4.1 are virtually identical over the time span covered by observations.
4.2.1 Spin evolution

The rotational evolution (the spin frequency and the spin-down rate) of PSR J1119–6127 is shown in Fig. 4.1, which is essentially an update of Fig. 10 in WJE11. These parameters were measured for illustrative purposes by fitting a timing model to short stretches of data. For each TOA we defined a stretch of data that included all TOAs separated by less than 75 days from the central TOA. Stretches of data containing less than four TOAs were excluded from the analysis.

The rotation of PSR J1119–6127 is slowing down over time with an approximately constant rate, parameterised by \( \dot{\nu}_0 \). This results in the steep gradient in the spin frequency against time as observed in the top left panel of Fig. 4.1. However, as a result of the significant and stable \( \ddot{\nu} \) (WJE11), the shape is better described by a parabola (parameterised by \( \ddot{\nu}_0 \)). This is revealed after subtracting the effect of \( \dot{\nu}_0 \) (middle-left panel). The main deviations from an otherwise almost perfect parabola are the two glitches and their recoveries. These deviations are more pronounced after subtracting the effect of the long-term spin evolution, parametrised by \( \dot{\nu}_0 \) and \( \ddot{\nu}_0 \) (bottom-left panel).

The two unambiguous glitches at MJD \( \sim 53290 \) and \( \sim 54240 \) are characterised by a sudden spin-up, followed by a gradual recovery which can be parameterised in terms of a model for the evolution in \( \dot{\nu} \) (top-right panel of Fig. 4.1). This evolution is dominated by a linear increase as a result of the stable \( \nu_0 \) (dotted line). The deviations (\( \Delta \dot{\nu} \)) caused by the two glitches and their recoveries are presented (zoomed-in) in the middle and bottom right panels. Here \( \Delta \dot{\nu} \) is the difference of \( \dot{\nu} \) from what we will refer to as the "projected spin-down rate", corresponding to the dotted line in the top-right panel (i.e. the expected \( \dot{\nu} \) evolution for a constant \( \dot{\nu} \)).

Both glitches presented a sudden increase in spin-down rate followed by an exponential recovery, which is a common feature of glitching behaviour. Based on data up to MJD 55364, WJE11 reported that the recoveries were such that \( \dot{\nu} \) overshoots the projected pre-glitch spin-down rate, resulting eventually in a slower spin-down rate. Thus the initially negative \( \Delta \dot{\nu} \) evolves to a positive, persisting \( \Delta \dot{\nu} > 0 \), about \( \sim 0.5 \) and \( \sim 1.5 \) years after the 1st and 2nd glitch respectively (see bottom-right panel). As discussed in WJE11, this evolution is not normal among the rest of the glitch population.

The new data (after MJD 55364) reveal that the described picture is incomplete for the second, larger glitch. The end of the data-span used in WJE11 happens to correspond roughly to the date at which \( \Delta \dot{\nu} \) peaks in the bottom-right panel of Fig. 4.1. Clearly the post-glitch recovery is not converging to a permanent and constant positive value of \( \Delta \dot{\nu} \). Instead, the currently positive \( \Delta \dot{\nu} \) is slowly decreasing, and

---

1However, see section 4.2.2 for the slightly different, but self-consistent, way to model the phase in terms of the generalised power law spin-down.
the spin-down rate evolves towards the projected pre-glitch \( \dot{\nu} \). This newly discovered long-term evolution cannot be modelled with the set of parameters of the timing solution presented by WJE11, hence an additional term is required. The functional form of this term is not a priori known. Here we explore two different parameterisations to model the long-term recovery of the second glitch.

In the timing solution presented by WJE11, the 2007 glitch recovery is parameterised by instantaneous changes in both \( \nu \) and \( \dot{\nu} \): the permanent \( \Delta \nu_p \) and \( \Delta \dot{\nu}_p \) steps (but without a \( \ddot{\nu}_p \) term) and two exponentially decaying terms which describe the relaxation. In a way, the most natural extension of this model would be to add a third exponential recovery term with an amplitude \( \Delta \nu_d^{(3)} \) and a long associated timescale \( \tau_d^{(3)} \). We refer to this as model A (see Table 4.1). There is no need for a permanent jump in \( \dot{\nu} \) in this model, because it can be absorbed in the other fit parameters. Hence model A has effectively one extra free parameter compared to the timing solution presented by WJE11. Note that \( \Delta \nu_d^{(3)} < 0 \), opposite to the other two decaying terms. Since the associated timescale \( \tau_d^{(3)} \) is very large (about 6 years according to model A), it is currently impossible to distinguish between an exponential or linear long-term recovery in \( \dot{\nu} \). The latter possibility is further explored in what we refer to as model B. We model a linear evolution by replacing the third exponential term which was included in model A with the \( \Delta \nu_p \) and \( \Delta \dot{\nu}_p \) terms. Therefore both models have the same number of free parameters.

The set of rotational parameters defined in Eqs. 4.3 and 4.4 were determined simultaneously for the entire dataset by applying the timing model, including both glitches, directly to the TOAs. The values are optimised by minimising the RMS (root-mean-square) deviation from zero of the timing residuals (the difference of the measured TOA from the one expected from the timing model, as shown in Fig. 4.2). For details of the fitting procedure, see WJE11.

The results are presented in Table 4.1. As reflected on the errors of the fast decaying term \( \Delta \nu_d^{(2)} \), the initial part of the 2007 glitch relaxation is poorly constrained. This is due to the observing cadence. For either model however, the instantaneous spin-up at the glitch epoch appears to be very large, \( \Delta \nu_g \sim 80 \mu \text{Hz} \) \( (\Delta \nu_g / \nu \sim 3 \times 10^{-5}) \), placing this glitch among the largest ones ever observed. The inferred spin-down change at the glitch epoch is also very large, comparable to those seen in magnetars, and might be as high as \( \Delta \dot{\nu}_g / \dot{\nu} \sim 4 \).

There are clear structures in the timing residuals (Fig. 4.2), indicating that there is significant timing noise present which is not included in our timing model. Although the RMS of the timing residuals is slightly better for model A (see also Table 4.1), this is not significant within the systematic uncertainties resulting from the timing noise. Observationally it is therefore currently impossible to distinguish between the two models. This is illustrated in Fig. 4.3, which shows the difference between the
Table 4.1: Rotational parameters for PSR J1119–6127 according to two different models describing the 2007 glitch and its recovery. Model A includes three exponential recovery terms and a permanent change only in $\nu$, while in model B the longest timescale exponential recovery is replaced with a permanent change in $\dot{\nu}$ and $\ddot{\nu}$. Although all model parameters were optimised simultaneously for the whole dataset, only the parameters describing the 2007 glitch are significantly different for these two models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoch (MJD)</td>
<td>54000</td>
<td>54000</td>
</tr>
<tr>
<td>$\nu_0$ (Hz)</td>
<td>2.447266543(6)</td>
<td>2.447266540(6)</td>
</tr>
<tr>
<td>$\dot{\nu}_0$ (10^{-15} Hz s^{-1})</td>
<td>-24050.94(7)</td>
<td>-24050.97(8)</td>
</tr>
<tr>
<td>$\ddot{\nu}_0$ (10^{-24} Hz s^{-2})</td>
<td>637.4(4)</td>
<td>637.2(5)</td>
</tr>
<tr>
<td>DM (cm^{-3} pc)</td>
<td>713</td>
<td>713</td>
</tr>
<tr>
<td>$n$</td>
<td>2.677(2)^†</td>
<td>2.677(2)^†</td>
</tr>
<tr>
<td>MJD range</td>
<td>50850 – 56794</td>
<td>50850 – 56794</td>
</tr>
<tr>
<td>RMS residuals (ms)</td>
<td>68.6</td>
<td>74.8</td>
</tr>
</tbody>
</table>

2004 glitch parameters

| Glitch epoch | 53290 | 53290 |
| $\Delta \nu_p \ (\mu$ Hz) | -0.05(2) | -0.04(2) |
| $\Delta \dot{\nu}_p \ (10^{-15} Hz s^{-1})$ | 7.5(4) | 7.3(4) |
| $\Delta \nu_d \ (\mu$ Hz) | 0.70(5) | 0.71(6) |
| $\tau_d \ (days)$ | 86(11) | 80(11) |

2007 glitch parameters

| Glitch epoch | 54240 | 54240 |
| $\Delta \nu_p \ (\mu$ Hz) | 6.5(2) | -0.91(2) |
| $\Delta \dot{\nu}_p \ (10^{-15} Hz s^{-1})$ | – | 32.4(4) |
| $\Delta \nu_d^{(1)} \ (\mu$ Hz) | 5.80(4) | 5.66(5) |
| $\tau_d^{(1)} \ (days)$ | 194(2) | 184(2) |
| $\Delta \nu_p^{(2)} \ (\mu$ Hz) | 75(200) | 80(200) |
| $\tau_d^{(2)} \ (days)$ | 10(5) | 9(7) |
| $\Delta \nu_d^{(3)} \ (\mu$ Hz) | -7.7(2) | – |
| $\tau_d^{(3)} \ (days)$ | 2324(60) | – |

† The quoted measurements for the braking index do not follow directly from the spin parameters of the two timing solutions, but from a self-consistent analysis of pre-glitch data only, as explained in section 4.2.2.
observed and predicted values of $\dot{\nu}$ for the two models.

Nevertheless, the extrapolated spin evolution for the two models is quite different. By extrapolating model B, it can be predicted that the projected $\dot{\nu}$ ($\Delta \dot{\nu} = 0$) according to the timing solution prior to the 2004 glitch, will be reached again at MJD $\sim$59178; 13.5 years after the occurrence of the second glitch. On the other hand, according to model A this value is never reached and $\Delta \dot{\nu}$ tends to $7.5 \times 10^{-15}$ Hz s$^{-1}$, in accordance with the prediction of the post-2004 glitch model. We note that the permanent change in $\dot{\nu}$ included in model B implies a permanent change in the braking index, a possibility which is explored in more detail in section 4.2.2.

Only two possible functional forms of the glitch recovery are explored here. In reality the recovery might be quite different and it could be, for example, a damped oscillation. As can be seen in Fig. 4.3, the unmodeled features in the post-glitch $\dot{\nu}$ evolution do not appear to be strictly periodic, similarly to what is observed in the timing residuals (Fig. 4.2). However, a simple Fourier transform indicates some excess power at a period of $\sim$ 400 days, which can perhaps be confirmed with future observations. Notably, Yuan et al. (2010a) reported a significant post-glitch oscillation in the residuals of PSR J2337+6151 with a similar period of 364 days.

It remains to be seen if the long-term glitch recovery for PSR J1119–6127 can be determined experimentally, as this evolution might be disrupted by another glitch. It is also not clear if the 2004 glitch presents a similar long-term evolution: the data are too sparse after MJD $\sim$54000 and up to the second glitch epoch to determine whether $\dot{\nu}$ evolves to a constant $\Delta \dot{\nu} > 0$ or back towards $\Delta \dot{\nu} = 0$.

### 4.2.2 The braking index

The long-term $\dot{\nu}$ is typically unmeasurably small for old pulsars while for young ones it is difficult to determine reliably because the spin-down evolution is dominated by timing noise and glitch recoveries. PSR J1119–6127 is one of the only eight pulsars to date for which a long-term, stable $\dot{\nu}$ and a braking index have been determined (Lyne et al. 2014). In this section we want to measure the long-term $n$, and explore its possible change as a consequence of the 2007 glitch. Previously, the braking index has been found to be $n = 2.91 \pm 0.05$ by Camilo et al. (2000), and $n = 2.684 \pm 0.002$ by WJE11 who analysed a longer dataset. Both these measurements were derived by Taylor expanding the long-term spin-down around a pre-glitch epoch. As in Eq. 4.3, the free parameters were $\phi_0$, $\nu_0$, $\dot{\nu}_0$ and $\ddot{\nu}_0$ while higher order time derivatives were set to zero. The braking index was obtained from the resulting fit parameters by applying Eq. 4.2. For the dataset analysed by WJE11 additional terms to account for the glitch effects were included in the timing model, as described in section 4.2.1.

However, a fit of the spin-down as in Eq. 4.3 to a long dataset is not strictly appropriate for the purpose of measuring the long-term $n$. This is because Eq. 4.1
implies non-zero higher-order frequency derivatives, which are assumed to be negligible in the truncated Taylor approximation (Eq. 4.3). This affects the measurement of the braking index if the timespan covered by the observations is large enough to invalidate the approximation $\dot{\nu}_0 (t - t_0) \ll \dot{\nu}_0$. Consequently, when the third and higher order time derivatives (which depend on $t_0$) are ignored, the resulting value of the braking index depends on the arbitrary choice of the reference epoch $t_0$. To demonstrate this effect, $t_0$ was varied between MJD 50850 and 53290 (i.e. the timespan covered by the data before the 2004 glitch) while fitting for $\nu_0$, $\dot{\nu}_0$ and $\ddot{\nu}_0$ in this range. The derived braking index varied between $n = 2.66$ and 2.69, a variation far greater than the very small error-bar derived via standard error propagation. When $t_0$ is set at the end of the total data-span, $n = 2.72$.

To overcome this disadvantage we employed a different, self-consistent method to estimate the underlying braking index. The measurement of a braking index as in Eq. 4.2 is meaningful if the inter-glitch data are indeed described reasonably well
4.2 Observations and timing analysis

Figure 4.3: The difference between the measured spin-down rate of PSR J1119−6127 and the timing models of Table 4.1. Black points are used for model A and grey points for model B. Some points directly after the two glitches fall below the plotted range. The glitch epochs are indicated by the dashed lines.

by a power law with constant \( n \) and \( C \). Under these assumptions, Eq. 4.1 can be integrated twice (from \( t_0 \) to \( t \)) and results, for \( n \neq \{1, 2\} \) and using \( C = -\ddot{\nu}_0 / \nu_0^n \), to the following description of the rotational phase of the star:

\[
\phi(t) = \frac{\nu_0^2}{\ddot{\nu}_0 (2 - n)} \left[ 1 + \frac{\dot{\nu}_0}{\nu_0} (1 - n)(t - t_0) \right]^{\frac{1}{2n}} - 1 + \phi_0. \tag{4.5}
\]

This equation is fully consistent with a spin-down that can be described by the generalised power-law (Eq. 4.1). Since in that case Eq. 4.5 is more precise than a truncated Taylor series, it should provide a more sensitive and robust measurement of the braking index. We therefore replaced the long-term timing model (Eq. 4.3) by Eq. 4.5 in our custom timing analysis software, and performed a fit of the data before the 2004 glitch. These fits use the braking index \( n \) as a free parameter instead of the long-term \( \ddot{\nu}_0 \), but have otherwise the same number of fitted parameters and result
The braking index $n(t)$ of PSR J1119–6127 obtained from a timing model with a long term power-law spin-down of constant $n$ (short-dashed line) and the two glitches parametrised as in model A (continuous line) and B (long-dashed line). The very high values of $n$ immediately after the glitches are excluded from the plot for clarity. The points are measurements of $n(t)$ from fits of Eq. 4.3 to small subsets of data as described in the text. Note that these measurements are highly contaminated by timing noise, not reflected in their statistical error bars plotted here which are much smaller than the overall variations.

Measurements of the higher-order frequency derivatives can provide insights to the braking mechanism. The generalised power-law (Eq. 4.1) for constant $C$ and $n$ implies a non-zero $\ddot{\nu}$ for all positive $n \neq 0.5$, which should be

$$\ddot{\nu} = n(2n - 1)\nu^3\nu^{-2}. \quad (4.6)$$

Inserting the measured braking index into this equation and the spin parameters of
4.2 Observations and timing analysis

Table 4.1 leads to the prediction\(^2\) that \( \dot{\nu}_0 \sim -2.7 \times 10^{-32} \text{ Hz/s}^3 \). By including the higher order \( \ddot{\nu}_0 \) term in Eq. 4.3, and fitting for the pre-glitch data only, leads to \( \ddot{\nu}_0 = (-4.5 \pm 0.1) \times 10^{-31} \text{ Hz/s}^3 \), a measurement which is most likely highly contaminated by unmodelled effects such as timing noise, which are not taken fully into account in the quoted statistical error bars. Since this measurement is an order of magnitude larger than the prediction, it does not provide independent support to the generalised power-law braking hypothesis.

Although timing noise hinders the determination of the braking index, it should be stressed that the top right panel of Fig. 4.1 clearly demonstrates a linearly evolving \( \dot{\nu} \), displaying a stable \( \ddot{\nu} \). This stability is further illustrated in Fig. 4.4, which shows the instantaneous value \( n(t) \) of the braking index as function of time. These measurements show the resulting braking indices of fits of Eq. 4.5 to short stretches of data. The fitting process is similar to that employed to produce Fig. 4.1, except that each data subset included all TOAs separated by less than half a year from the central TOA. Stretches of data containing less than six TOAs were excluded from the analysis. The derived \( n(t) \) after Eq. 4.2, from fits of Eq. 4.3 to the same stretches of data (not shown), are virtually identical. We stress here that the quoted statistical error on the braking index in Table 4.1 does not take fully into account the much larger systematic errors caused by timing noise (see Livingstone et al. 2011) which, as seen from the scatter of the instantaneous measurements in Fig. 4.4, is significant. However, though clearly affected by timing noise, the measurements of the instantaneous value of \( n(t) \) vary around a relatively well defined average. There is no evidence for a longer-term systematic evolution of the braking index before the first glitch.

The glitches disrupt the gradual spin-down. The effect of the change in \( \ddot{\nu} \) after the 2007 glitch on the measured instantaneous braking index \( n(t) \), a decrease below its pre-glitch value as predicted by both model A and B, is clear in Figure 4.4. The measurements from fits in smaller subsets also indicate a similar decrease, despite their scatter due to timing noise. According to model A this apparent decrease in \( n(t) \) is transient and the instantaneous braking index is currently slowly recovering to its pre-glitch value. The permanent changes in model B however (see Table 4.1), predict that the braking index will settle at a lower value, corresponding to a decrease in \( n \) of about \( \sim 15\% \) after the 2007 glitch. We stress that it is unclear if the glitch resulted in a permanent decrease of \( n \). For example model A, which describes the data equally well, predicts that \( n \) will recover to its original long-term value.

We cannot know for certain in what way the braking index is evolving in the long-term, however it is clear that the 2007 glitch resulted in a (possibly permanent) decrease in \( n \). As discussed further in section 4.3 the timescales involved in the

\(^2\)This calculation is dependent on the choice of the reference epoch \( t_0 \), but not at a level relevant for the arguments made here.
response of the superfluid stellar component can be very long. It is therefore rather likely that, at least in some pulsars, spin-down equilibrium is never reached between glitches. If that is the case then $I_{\text{eff}}$ and the observed spin-down rate vary in a way that is not captured by the generalised Eq. 4.1. A time-dependent observed instantaneous braking index (as for example predicted by model A) is a strong indication of such an incomplete glitch recovery. The evolution prior to the observed glitches (to which the previously reported measurements of $n$ and the one quoted in Table 4.1 correspond) could also be "contaminated" by the recovery of a glitch that occurred before our first observations. If such a glitch resulted also in a long-lasting decrease of the apparent braking index, we cannot rule out the possibility that the pre-glitch value of $n$ in Table 4.1 is underestimated. Potentially, the underlying braking index could even be consistent with $n = 3$, the canonical value expected for the vacuum dipole braking mechanism.

4.2.3 Limits on pulse profile shape variations

The pulse profile of PSR J1119–6127 is usually single peaked. Remarkably, the first observation after the 2007 glitch shows a double-peaked pulse profile. A double-peaked profile was not observed in any other observations analysed by WJE11 or in the subsequent data analysed in this paper, making this event extremely rare. With the additional 77 new observations analysed (in total 5.1 hours of data) the double-peaked profile is observed in only 0.09% of the total amount of data. The fact that this event is so rare suggests that it is not a coincidence that the first observation after the 2007 glitch showed a double-peaked profile, but rather that the two events are linked.

We also looked for more subtle shape variations than the above discussed profile change. This was done by measuring the profile width at 10% ($W_{10}$) and 50% ($W_{50}$) of the maximum intensity, by fitting two von Mises functions to the individual observed profiles. No evidence of significant variations was found. Moreover, to quantify any lower level profile shape changes throughout the long-timescale recovery after the 2007 glitch, we summed the first and second half of the post-glitch data separately before measuring their widths. No significant evolution could be identified within a precision of 4% in $W_{10}$ nor $W_{50}$. Measuring the width from the summed data before and after the 2007 glitch (excluding the observation with the double-peaked profile) did not reveal any significant profile shape difference either.

It is known that timing noise, in particular $\dot{\nu}$ changes, can be related to profile shape changes (Lyne et al. 2010). We therefore tested for the presence of a correlation between the variations seen in Fig. 4.3 and both $W_{10}$ and $W_{50}$, but no such correlation

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3Formulating a formal uncertainty in this case is complicated, since the epoch of the glitch is close to the change from an analogue to a digital filter bank backend and a slight change in centre frequency.
was found.

In addition to the change in the radio profile after the 2007 glitch, WJE11 report the appearance of abnormal erratic emission shortly after the glitch at rotational phases where normally no radio emission is observed. Since WJE11, only two 3 minute long observations (at a wavelength of 20 cm) were recorded for which the individual pulses were stored. These observations (from April and May 2012) do not show any evidence for similar erratic radio emission, thereby strengthening the claim that the previously identified erratic behaviour is linked to the 2007 glitch.

### 4.3 Physical interpretation of the post-glitch spin-down evolution

Let us now explore the various physical mechanisms possibly involved in the “over-recovery” of the spin-down rate after the 2007 glitch and its subsequent evolution. Ordinary glitches are attributed to the loose interaction of the NS’s normal matter with the frictionless and irrotational neutron superfluid. The rotation of the latter is supported by neutron vortices, which have quantised circulation \( \kappa = \frac{h}{2m_n} \), where \( h \) is the Planck constant and \( 2m_n \) is the mass of a neutron pair. Vortices carry the superfluid’s angular momentum \( L_c \) and are expected to be very dense in pulsars since their number density \( n_v \) is proportional to the superfluid’s spin frequency. When free and in equilibrium, vortices arrange themselves in an array, mimicking solid body circulation for the superfluid. The superfluid follows the rotation of its container by creation/expulsion of vortices and adjustments of their density.

In the NS’s interior the superfluid is immersed in normal matter and possibly coincides with a proton superconducting condensate in parts or the entire core. Thus the required adjustments of \( n_v \) might not always be possible, allowing differential rotation of the superfluid which alters the system’s dynamics. The braking is then described by

\[
\frac{dL}{dt} = \int \dot{\Omega}_s(r, t) \, dI_s + I_c \, \dot{\Omega}_c \tag{4.7}
\]

which can be re-written as:

\[
N_{\text{EXT}} = \left( \int \frac{\dot{\Omega}_s(r, t) \, dI_s}{\dot{\Omega}_c} + I_c \right) \dot{\Omega}_c = I_{\text{eff}} \, \dot{\Omega}_c . \tag{4.8}
\]

Here \( N_{\text{EXT}} \) is the sum of all external torques, \( L \) the angular momentum, \( I \) denotes moment of inertia, \( \Omega \) the angular velocity, indices ’s’ and ’c’ refer to the superfluid and the crust respectively and we have defined a new effective moment of inertia \( I_{\text{eff}} \). The component \( I_c \) includes the charged particles in the core and the magnetosphere,
Glitches in PSR J1119–6127

which are magnetically locked to the crust, and rotates at the pulsar’s frequency \( \nu = \Omega_c / 2\pi \).

The crystalline matter of the inner crust provides an inhomogeneous interacting potential for the vortices, as they are attracted-or repulsed-by the lattice nuclei. In the NS core, protons might form a type-II superconductor, in which case neutron vortices will interact with the lattice of proton flux-tubes. Consequently NS vortices might not be free to move outwards in order to track the decreasing spin frequency of the star. If the interaction is strong vortices will be completely immobilised, “pinned”, and \( \dot{\Omega}_s \) will be zero in this region. Smaller interaction energy \( E_p \) will result in only partial restriction of the vortex flow, \( |\dot{\Omega}_s| < |\dot{\Omega}_c| \). In both situations an excess of vorticity builds up locally, which translates as a rotational lag between the two components, \( \omega(r, t) = \Omega_i(r, t) - \Omega_s(t) \). In equilibrium, \( \omega(r, t) = \omega_{eq}(r, t) \) and the two fluids spin-down together \( (\Omega_i = \Omega_s) \).

The differential rotation between a pinned vortex and the superfluid induces a lift (Magnus) force \( F_m \) which counteracts pinning. In the axisymmetric case, \( F_m \propto \rho_s r \omega \), where \( \rho_s \) is the superfluid density and \( r \) the cylindrical radius. If vortex outflow is restricted, the increasing lag \( \omega \) will eventually reach a critical value \( \omega_{cr} \), defined by the maximum \( F_m \) that can be sustained by the pinning force, unfreezing suddenly all excess vorticity. Such vortex avalanches might also be triggered locally by perturbations caused for example by fluid instabilities or a crustquake, and could lead to glitches of various sizes (Warszawski & Melatos 2013).

In regions where pinning is not very strong, the thermal energy of vortices allows them to slowly escape outwards, hopping from one pinning site to another, which results in a temperature-dependent \( \dot{\Omega}_s \). This idea of vortex “creep” in NSs was firstly put forward by Alpar et al. (1984a), who argued a temperature dependence of the form

\[
\dot{\Omega}_s \propto \exp \left[ -\frac{E_p}{kT} \left( 1 - \frac{\omega}{\omega_{cr}} \right) \right] - \exp \left[ -\frac{E_p}{kT} \left( 1 + \frac{\omega}{\omega_{cr}} \right) \right]
\]

for the creep rate in the case of thermally activated unpinning. Therefore the contribution of these regions to \( I_{\text{eff}} \) can be highly sensitive to temperature changes. For very cool NSs, vortex unpinning by quantum tunnelling dominates the creep rate, which becomes almost independent of temperature (Baym et al. 1992).

Vortices interact with both the superfluid and the normal component, via the Magnus force and a drag force respectively, exchanging angular momentum between the two. As in terrestrial superfluids, this coupling of the two components can be described by the mutual friction (MF) (Andersson & Comer 2006), which incorporates the effects of the vorticity and has a force density:

\[
f = \rho_s \mathcal{B} \mathbf{w}_s \times (\mathbf{v}_s - \mathbf{v}_c) + \rho_i \mathcal{B} \mathbf{w}_i \times (\mathbf{v}_s \times (\mathbf{v}_s - \mathbf{v}_c)) .
\]
We have introduced the superfluid vorticity \( \mathbf{w}_s = n_\kappa \mathbf{\hat{w}}_s \), where \( \mathbf{\hat{w}}_s \) is a unit vector aligned with the vortex axis, and the MF coefficients \( \mathcal{B}' \) and \( \mathcal{B} \).

While all vortices, either pinned or free, contribute to the superfluid’s velocity \( \mathbf{v}_s \), only unpinned vortices move with respect to the normal fluid and hence feel the drag force. Therefore the above coefficients \( \mathcal{B}' \) and \( \mathcal{B} \) depend not only on the strength of the coupling mechanism but also on the fraction \( \xi \) of unpinned vortices, which is a function of the temperature \( T \) and the lag \( \omega \) relatively to \( \omega_{\text{cr}} \). This fraction will be proportional to the probability for unpinning from an energy barrier of \( \Delta E = E_p(\omega_{\text{cr}} - \omega)/\omega_{\text{cr}} \), therefore for thermal unpinning \( \xi \propto \exp(-\Delta E/kT) \).

If a significant increase in coupling strength happens quickly, due to catastrophic unpinning (vortex avalanche, \( \xi \rightarrow 1 \)) and fast dissipation, the superfluid accelerates the crust causing a glitch. Immediately after the spin-up of the crust the lag decreases, leading to temporarily decoupling of the superfluid, a decrease in \( I_{\text{eff}} \) and the observed post-glitch recovery.

Permanent changes in spin and spin-down rate that are sometimes seen after glitches (Yu et al. 2013) have been attributed to magnetic axis re-orientation or/and moment of inertia changes, due to NS crustquakes (Link et al. 1992; Alpar et al. 1994; Ruderman et al. 1998). Crustquakes are expected if the solid crust does not plastically adjust to the less oblate equilibrium shape required by the pulsar’s spin-down. In this case the most natural outcome of the readjustment is a decrease in moment of inertia\(^4 \) \( \Delta I < 0 \), which results in permanent changes in rotation \( \Delta \nu_p/\nu = \Delta \dot{\nu}_p/\dot{\nu} = -\Delta I/I \). Such abrupt events could trigger the unpinning of vortices and have been connected to both glitches and outbursts (e.g., Link & Epstein (1996); Pons & Rea (2012)).

We will adopt the above picture of an abrupt unpinning event (either supplemented by a crustquake episode or not) for the origin of both PSR J1119–6127 glitches. This is motivated by their striking similarity, in both the initial glitch parameters, \( \Delta \nu \) and \( \Delta \dot{\nu} \), as well as the first part of the relaxation process, to regular radio pulsar glitches. However this mechanism alone cannot explain the “overshooting” of the pre-glitch \( \dot{\nu} \) (\( \Delta \dot{\nu} > 0 \)). Moreover, it might result in persistent \( \Delta \dot{\nu} < 0 \) because of the long recoupling timescales of the crustal superfluid (Link 2014; Haskell & Antonopoulou 2014). Consequently the subsequent \( \dot{\nu} \) evolution must be governed by a different process.

The regular glitch relaxation (with \( \Delta \dot{\nu} < 0 \)) masks the behaviour of the “irregular” term for the first few years post-glitch, therefore it is not possible to obtain a clear description for this period. Nonetheless, the maximum observed positive \( \dot{\nu} \) deviation \( \Delta \dot{\nu}_{\text{max}} \approx 3.5 \times 10^{-14} \) Hz/s provides a lower limit for the amplitude of the decrease in spin-down rate, which we use to examine the plausibility of various physical mechanisms for the post-glitch variation.

\(^4\)This however might not be the case for a magnetically strained crust.
We first discuss internal (superfluid) mechanisms that could be responsible for the strange long-term recovery of $\dot{\nu}$, in section 4.3.1. The peculiar radio emission features observed after the 2007 glitch recovered on a rather fast timescale ($\lesssim 3$ months) compared to the timescales characterising the long-term $\dot{\nu}$ evolution. Nevertheless, it indicates magnetospheric activity related to the glitch so changes in $N_{\text{EXT}}$ cannot be excluded. This possibility is further explored in section 4.3.2. As we will show, small changes in magnetospheric conditions suffice to explain the observed variation in $|\dot{\nu}|$. The consequences of such changes in the radio emission could have been small enough to be missed entirely, if it were not for the one observation of the double-peaked profile right after the glitch.

### 4.3.1 Internal, superfluid mechanisms

We can think of two ways in which the superfluid might be responsible for the observed $\dot{\nu}$ evolution. First, it could be that the spin-up glitch was accompanied by an "anti-glitch", that is, a fast spin-down event. In this scenario some vortices move inwards rather than outwards during the glitch, causing a spin-up of the respective superfluid region $I_{\text{in}}$ and an increase $\Delta \omega(I_{\text{in}}) = \Delta \omega_{\text{in}} > 0$ of the local lag. For simplicity we will consider the case where this region does not coincide with the superfluid region that drives the glitch. The lag of the region $I_{\text{in}}$ would decrease by $\Delta \omega_{\text{g}} \sim -2\pi \Delta \nu_{\text{g}}$ because of the glitch, in the absence of a vortex flow. The net effect on the lag, and therefore the coupling strength of this region with the normal component and its contribution to $I_{\text{eff}}$, will depend on the ratio $|\Delta \omega_{\text{in}}|/|\Delta \omega_{\text{g}}|$. Thus such a picture can lead to a large variety of features in the timing residuals, depending on the above ratio and the size and local coupling timescale of the region where vortex inflow took place.

However an inwards vortex motion is not favoured for a spinning-down NS, thus an additional mechanism to the ones discussed above must be invoked. Alpar et al. (1996) proposed the formation of a new pinning region (vortex trap) as a possible explanation for a similar, peculiar glitch feature observed in the Crab pulsar (Lyne et al. 1992), where a gradual increase in $\Delta \nu$ (relative decrease of $|\dot{\nu}|$) followed a normal (sudden) glitch. If in the creep regime the inward vortex motion is not largely suppressed, some vortices will end up pinned in this "trap" at a distance closer to the rotational axis than they were before. The formation of a new "trap" would leave a permanent imprint in the spin-down rate $\Delta \nu_{\text{p}} < 0$, since $I_{\text{eff}}$ reduces. This is indeed observed in the Crab, but is not in accordance to the $\dot{\nu}$ evolution seen so far in PSR J1119–6127.

It could be that vortices were instead forced to move inwards because of their interaction with the crustal lattice or the core’s flux-tubes, to a region where pinning properties remained unchanged. In this case, if $\Delta \omega_{\text{in}}/\Delta \omega_{\text{g}} > 1$ then this region contributes more to $I_{\text{eff}}$ after the glitch (reflected in the post-glitch relaxation as a relaxing
component with $\Delta \dot{\nu} > 0$, as in model A). According to the parameters derived from the data (see Table 4.1), the region where this happens must have a very long coupling timescale, of the order of years as indicated by $\tau_d^{(3)}$. This scenario is examined in detail by Akbal et al. (submitted to MNRAS), who extend the phenomenological creep model of Alpar et al. (1984a) to account for a region with vortex inflow at the time of the glitch. Because of the non-linear creep term included in their 9-parameter fits, the derived parameters do not correspond directly to the ones of model A. However, a permanent change in the external torque of similar magnitude to our calculations (see sections 4.2 and 4.3.2) is also required by their model.

There is a different way in which the superfluid could be responsible for the post-glitch $\dot{\nu}$ evolution, that does not require inwards vortex motion. Rather, the temporal enhancement of the coupling strength between parts of the superfluid and the normal component can be attributed to local temperature variations following the glitch, instead of an increase of their lag. From Eq. 4.9 and in the axisymmetric case, the superfluid spin-down rate $\dot{\Omega}_s(r,t)$ in Eqs. 4.7 and 4.8 can be written as (Sidery et al. 2010)

$$\dot{\Omega}_s(r,t) = B(r)|w_s(r,t)|(\Omega_c(t) - \Omega_s(r,t))$$ (4.10)

where for simplicity we ignore the effects of entrainment. Increased vortex mobility, expressed as a higher value $\xi$ and therefore larger $B$, leads to stronger coupling between the superfluid and normal component for a given lag $\omega$ and to a temporal increase of $I_{\text{eff}}$ (Eq. 4.8). This mechanism offers a promising possibility for explaining the observed long lasting $\dot{\nu}$ evolution, since both the NS cooling and the superfluid response are relatively slow processes. Furthermore, such changes in coupling strength have been shown to lead to observable effects on $\dot{\nu}$ very similar to the irregular feature of the PSR J1119–6127 glitches (Haskell & Antonopoulou 2014).

During the glitch, energy is dissipated due to vortex motion, while crustquakes and radiative bursts can release elastic and magnetic energy stored in the strained crust and magnetic field lines. Therefore some heat input in the inner crust coincidental with the glitch is to be expected, which can be as large as $10^{40} - 10^{44}$ erg if additional processes such as a crustquake are involved (Perna & Pons 2011). Part of this energy is transferred to the magnetosphere on fast timescales, and it could be responsible for the activation of the radio emission mechanism in a region of previously inactive magnetic field lines, producing the observed changes in the pulse profile. Depending on the temperature and physical properties of the region where the energy was injected, part of it will be lost quickly via neutrino emission and in the highly conductive core, while the rest will cause local heating, eventually being radiated as thermal emission from the surface on much longer timescales (see for example Aguilera et al. 2008). A strong toroidal magnetic field around the crust-core boundary, as inferred for PSR J1119–6127 (Ng et al. 2012), might help the confine-
ment of heat in this region.

The energy release can be regarded as "instantaneous" since the injection timescale for most plausible mechanisms (glitch dissipation, crustquake or magnetic reconnection) is much smaller than the timescales of interest (for the $\dot{\nu}$ behaviour). In the vicinity of the energy source the temperature will start growing and the extent of the heated region $I_H(t)$ will increase with time. The effect of the reduced lag $\omega_H$ of this region because of the glitch might be stronger than the change in coupling due to a slightly higher temperature. Thus initially the net effect can be a negative change in $\dot{\nu}$. As the superfluid re-couples though, the relaxation will be towards the new pseudo-equilibrium lag $\omega_{eq,H}(t)$ of the heated area, which will remain smaller than the pre-glitch one during the cooling back to the pre-glitch temperature profile. As a consequence, the contribution to the effective moment of inertia $I_{\text{eff}}$ of this region will be larger after the glitch than it would have been had the temperature remained constant. This is reflected in the positive $\Delta \dot{\nu}$. The observed deviation $\Delta \dot{\nu}_{\text{max}}$ provides a lower limit for the extra contribution of the heated region in $I_{\text{eff}}$. The equivalent change in $B$ depends on the size $I_H(t)$ of the affected region, as well as on its properties (as depth, pinning/drag strength and initial temperature) which define $\omega_{eq}$. Note that for the above mechanism to work, the temperature increase must happen over a region that was partially decoupled ($\dot{\nu}_s, \dot{\nu}_c$) before the glitch.

In this scenario the glitch will not be accompanied by a sudden spin-down (as parameterised in model A), unlike the previously discussed mechanism. Instead, $I_{\text{eff}}$ (and thus $\Delta \dot{\nu}$) will gradually reach a maximum followed by a decrease at a rate defined by the post-glitch cooling rate. This decrease might be reflected in the $\Delta \dot{\nu}_{p}$ term of model B and the corresponding change in the braking index. Near the end of the cooling phase however, a turn in $\Delta \dot{\nu}$ is expected, which will either fully recover to the projected pre-glitch $\dot{\nu} (\Delta \dot{\nu} = 0)$ or, in the presence of a change in the external torque, to a new stable state. The functional form of the relaxation is defined by the global response of the superfluid to the glitch, the temperature evolution $T(t)$ as well as the exact form of the dependence of the dissipative mutual friction coefficient $B$ on $T$, which is unknown. If such a mechanism is at work, glitches where this signature is clear in the timing residuals can be very useful probes of $B(T)$. Since the neutron superfluid forms very soon after the NS birth, the dependence of $\dot{\Omega}_s$ on $T$ could also have an impact on the early rotational dynamics of pulsars, during the fast cooling phase.

### 4.3.2 Magnetospheric mechanisms

We will now focus on external, magnetospheric mechanisms that could lead to the peculiar spin-down rate evolution. Such mechanisms have observational corollaries and can potentially be tested by future high-energy observations.
In the simplest approximation for pulsar spin-down, the vacuum dipole model, the magnetospheric torque on the star is given by

\[ N_{vd} = -\frac{R_*^6 \Omega^3}{6c^3} B_*^2 \sin^2 \alpha \]  

(4.11)

where \( B_* \) is the surface magnetic field at the pole, \( R_* \) the stellar radius and \( \alpha \) the inclination angle.

Pulsars are not surrounded by vacuum as rotation induces an electric field \( E = -(\Omega \times r) \times B \), which accelerates charges off the neutron star surface. Secondary pair creation from those charges and cascades are expected to fill the magnetosphere with plasma, which will screen the electric field along the magnetic field lines (hereafter denoted \( E_{||} \)). The currents that flow in the magnetosphere and close under the star’s surface provide an additional braking torque which is comparable to the one of Eq. 4.11 (Harding et al. 1999) and therefore should not be neglected.

The plasma density required to completely screen \( E_{||} \) is the Goldreich-Julian density, \( \rho_{GJ} = -B_* \nu/c \) (Goldreich & Julian 1969). If plasma is abundant, the magnetosphere can be considered in the force-free regime, where \( E_{||} = 0 \) everywhere except in the acceleration zones above the polar caps and regions where plasma flow is required, like the equatorial current sheet. The drift velocity of the magnetospheric plasma has a rotational component \( \Omega_F \), which represents the angular velocity of the magnetic field lines. For an aligned rotator, which is the approximation we will mostly use for numerical calculations in the following, \( \Omega_F = \Omega_F \hat{e}_z \). The behaviour of the magnetosphere will be qualitatively the same for inclination angles as the ones inferred for PSR J1119–6127 (\( \alpha \sim 17^\circ - 30^\circ \), WJE11). In the open field line region \( \Omega_F < \Omega_* \) and is determined by the potential drop along the magnetic field lines, which depends on the poorly understood microphysics of the cascade zone. Different \( \Omega_F \) and poloidal current density distributions lead to different global magnetospheric structure and Poynting flux to infinity. The energy losses, and thus the additional braking \( \dot{\nu} \), depend in general on \( \Omega_F \Omega_* \) and the size of the open field line region (Contopoulos 2005; Contopoulos & Spitkovsky 2006; Timokhin 2006).

The spin-down rate for a force-free magnetosphere was shown to be \( \geq 3 \) times larger than that for a misaligned rotator in vacuum. In this regime simulations by Spitkovsky (2006) show a dependence of the torque on the inclination angle of the form:

\[ N_{ff} \approx -\frac{B^2 R_*^6 \Omega^3}{c^3} (1 + \sin^2 \alpha) \]. 

(4.12)

If the relative decrease in \( |\dot{\nu}| \) of PSR J1119–6127 was due to a glitch-induced change in the surface magnetic field, then it would imply, for a rotator in vacuum, a decrease in the dipole component \( B_d \) of the order \( \Delta B_d \sim 3 \times 10^{10} \text{G} \). An actual decrease of the surface magnetic field strength would have released energy ~
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\[ \Delta B \times B/8\pi \approx 5 \times 10^{22} \text{ erg, however a more likely explanation for a decrease of } B_d \]

is a change in the inclination angle \( \alpha \). Such a geometric readjustment is possible if the glitch was accompanied by crust failure. In order for the pulsar to become more spherical at a crustquake platelets should move towards the rotational axis (see section 4.3.1). The magnetic field lines, anchored at the highly conductive crust, follow this motion, thus \( \alpha \) can decrease. The frequency jump at the glitch places an upper limit in the respective decrease in moment of inertia \( I < I_c \Delta \hat{\nu}/\nu \) and has a small impact on the spin-down rate \( (\Delta \hat{\nu}/\nu)_{\text{quake}} \lesssim 3 \times 10^{-5} \). Therefore the overall effect can be a decrease in spin-down rate, as observed.

Such structural changes will result in a permanent \( \dot{\hat{\nu}} \) change instead of a decaying \( \Delta \hat{\nu} > 0 \) (as in model A). Nevertheless, they cannot be ruled out and could potentially explain the permanent \( \Delta \hat{\nu}_p \) required by model B, or the smaller residual \( \Delta \hat{\nu} \approx 23\% \Delta \hat{\nu}_p \) to which the solution of model A tends asymptotically. For a small change \( \Delta \alpha \) of the inclination angle, the relative change in torque is \( \Delta N_{vd}/N_{vd} \equiv 2\Delta \alpha/\tan \alpha \) for the vacuum dipole model, or according to Eq. 4.12

\[ \Delta N_{ff}/N_{ff} \equiv \Delta \alpha \sin 2\alpha/(1 + \sin^2 \alpha). \]

From model B, \( \Delta N_{\text{EXT}}/N_{\text{EXT}} = \Delta \hat{\nu}_p/\nu = -13.5 \times 10^{-4} \). Using the inferred possible range of inclination angles for PSR J1119–6127, the equivalent decrease in inclination angle is found \( \Delta \alpha \approx -(3.9 - 2) \times 10^{-4} \) or \( \Delta \alpha \approx -(2.6 - 1.9) \times 10^{-3} \) degrees according to Eqs. 4.11 and 4.12 respectively. The effects of such a change of \( \alpha \) on the pulse profile cannot be excluded by the observations (see section 4.2.3).

The appearance of the extremely rare double peaked radio profile in PSR J1119–6127 indicates some perturbation of the pre-glitch magnetospheric state, as a direct or indirect consequence of the glitch. The first post-glitch observations suggest global magnetospheric changes which reflect on the radio emission (WJE11). There is growing evidence for similar changes, as well as a correlation between radio emission characteristics and spin-down rate, in magnetospheric active pulsars like mode changing and intermittent pulsars (Kramer et al. 2006; Lyne et al. 2010; Camilo et al. 2012; Lorimer et al. 2012; Young et al. 2013). Thus it appears that while some pulsars have relatively stable magnetospheric states, others can switch between metastable states, possibly within a few spin periods (Kramer et al. 2006). This magnetospheric state switching could be induced by a glitch, as suggested not only by PSR J1119–6127, but also by the observations of the mode changing PSR J0742–2822. For this pulsar, Keith et al. (2013) reported a stronger correlation between radio emission mode and \( \dot{\hat{\nu}} \) after a glitch.

The observed \( |\Delta \hat{\nu}|_{\text{max}} \) of the PSR J1119–6127 glitch recovery requires small changes (compared to those observed in intermittent pulsars for example) around its pre-glitch magnetospheric state, which appears stable overall. It could be that
4.3 Physical interpretation of the post-glitch spin-down evolution

PSR J1119–6127 is delicately balanced in this state and the glitch triggered a short-lived transition to a different one, or the permanent transition to another quasi-steady state of similar characteristics. The change of the magnetospheric torque when the profile was double peaked could have been more dramatic, but \( \dot{\nu} \) was still dominated by the glitch relaxation at that time. A similar mechanism might be at work in other glitching pulsars too but, due to (1) smaller glitch sizes, (2) geometric effects or (3) more stable magnetospheric states and faster relaxation timescales, we would not necessarily see prominent pulse profile changes (Timokhin 2010) and the magnetosphere-related decreases in spin-down rate might be masked by the post-glitch relaxations.

The glitch (and/or a simultaneous crustquake) could have triggered a plasma deficiency in the magnetosphere, for example by altering temporarily the properties in the cascade zone. As coherent radio emission is expected to be sensitive to those properties, such a change might explain the short-lived erratic components and the double-peaked profile seen right after the glitch, as well as the transient decrease in spin-down rate. After the glitch, the plasma supply is restored and the magnetosphere recovers to another stable state, on a short (model B) or long timescale (model A). In the first case (model B), the magnetospheric changes are reflected in the permanent terms \( \Delta \dot{\nu}_p \) and \( \Delta \dot{\varphi}_p \). Even though intuitively the transition is expected to be achieved in the rather fast timescales that characterise the magnetosphere (of the order of \( \mu \)sec for the cascade zone and up to \( \sim 1 \) year for Ohmic decay (Ruderman & Sutherland 1975; Beloborodov & Thompson 2007)), non-linear processes involving global magnetospheric adjustment (from the polar cap to the rest of the magnetosphere and the current sheet) can lead to much longer timescales. Thus model A cannot be excluded, as recovery to a new stable state might be slow. Alternatively, possible glitch-induced reconnection of magnetic field lines at distances \( \sim R_{LC} \), where \( R_{LC} \) is the light cylinder radius\(^5\), which alters the return current and the magnetospheric structure, might lead to similar results.

A simple estimate for the decrease in \( |\dot{\nu}| \) due to plasma shortage can be obtained if we assume that the positive \( \Delta \dot{\nu}_{\text{max}} \) is achieved when the charge density \( \rho = 0 \) (vacuum regime, Eq. 4.11), while in the pre-glitch state there is an additional braking torque, proportional to the current through the polar cap. Then the plasma density in the pre-glitch state should have been of the order (Harding et al. 1999)

\[
\rho = \frac{3I \Delta \dot{\nu}}{R_{\text{PC}}^3 B_*} = \frac{3c^2 I \Delta \dot{\nu}}{4\pi^2 R_{\text{PC}}^6 B_* \nu^2} \sim 10 \text{ statC cm}^{-3} \tag{4.13}
\]

where we used \( R_{\text{PC}} \approx (2\pi R_*^3 \nu/c)^{1/2} \) as an approximation for the polar cap radius, \( R_* \) is the surface of the neutron star, and the magnetic field there is considerably weaker than at the surface.

\(^{5}\)
Glitches in PSR J1119–6127

\[ \Delta \dot{v} = \Delta \dot{v}_{\text{max}}, \quad R_* \sim 10^6 \text{ cm and } I \sim 10^{45} \text{ g cm}^2. \]

This is a very small fraction of \( \rho_{GJ} \) (less than 1%). It is more natural to assume that the seemingly stable pre-glitch state is closer to the force-free regime (which seems energetically favourable and will have a plasma density comparable to \( \rho_{GJ} \)) and that the magnetosphere does not deplete completely from plasma in the low \(|\dot{v}|\) state. In this case the observed evolution of \( \dot{v} \) reflects the build-up of plasma density, the effect of which on spin-down is investigated (indirectly, by varying the conductivity of the magnetosphere) in Li et al. (2012).

Another way to understand this is that when charges are not copiously available, the magnetosphere is unable to support a poloidal electric current as large as before. Therefore a state with smaller open field regions and different current and toroidal \( B_\theta \) configurations is energetically favourable (Contopoulos 2005; Contopoulos & Spitkovsky 2006). In this state the Poynting flux to infinity is smaller, and so is the magnetospheric torque and \( |\dot{v}| \).

In the case of PSR J1119–6127, this new magnetospheric state might have been achieved for example via reconnection of open field lines close to the light cylinder quickly after the glitch, and would appear as a positive \( \Delta \dot{v} \). Once the plasma supply is restored and the poloidal current increases again, the magnetosphere will evolve to a state with larger open field line region, which does not necessarily coincide with the pre-glitch state. The timescale of this recovery will depend amongst other things on the rate in which energy is stored in the magnetosphere (and will thus vary from pulsar to pulsar).

For an aligned rotator, Contopoulos (2005) investigated the above effects of charge deficiency on \( \Omega_F \), and the subsequent decrease of electric currents, on the magnetospheric structure and spin-down rate. The difference in magnetospheric energy contained within a cylindrical radius \( r \), for two different \( \Omega_F \) of the open field line regions \( \Omega_{F,1} \) and \( \Omega_{F,2} \), is (see Eq. 24 in Contopoulos (2005))

\[ \Delta E_{\text{EM}} \sim B^2 R_*^3 \left( \frac{R_*}{R_{\text{LC}}} \right)^3 \left( \frac{r}{R_{\text{LC}}} \right) \left( \Omega_{F,2}^2 - \Omega_{F,1}^2 \right). \] (4.14)

The ratio of the respective spin down rates \( \dot{v}_1 \) and \( \dot{v}_2 \) of those states (with \( \Omega_1 = \Omega_2 = \Omega_c \)) is

\[ \frac{\dot{v}_1}{\dot{v}_2} = \frac{\Omega_1^3 \Omega_{F,1}}{\Omega_2^3 \Omega_{F,2}} \approx \frac{\Omega_{F,1}}{\Omega_{F,2}}. \] (4.15)

We can make a crude approximation by assuming an initial state with \( \Omega_{F,1} = 0.803 \Omega_c \) and using \( \dot{v}_2 = \dot{v}_1 + \Delta \dot{v}_{\text{max}} \). This leads to \( \Omega_{F,2} = 0.802 \Omega_c \) for a NS with mass \( 1M_\odot \) and \( R_* = 10^6 \text{ cm} \). The released energy is then

\[ \Delta E \sim 10^{44} \left( \frac{R_*}{R_{\text{LC}}} \right)^3 \left( \frac{r}{R_{\text{LC}}} \right) \text{ ergs}, \] (4.16)
thus for $r \approx R_{L\phi}$, $\Delta E \sim 10^{34}$ ergs. Note however that this quantitative approach is just indicative, since, according to the spin-down luminosity calculated by Contopoulos (2005), the condition $0 < \Omega_F \leq \Omega_c$ implies a large and rather unlikely underestimation of the real magnetic field of PSR J1119–6127, which should be more than an order of magnitude higher than the characteristic magnetic field inferred from Eq. 4.11.

4.4 Discussion and conclusions

Continuous monitoring of PSR J1119–6127 for 7 years after its 2007 glitch, which was followed by an abnormal post-glitch recovery and glitch-induced radio emission changes, reveals an ongoing evolution of the residual post-glitch $\Delta \dot{\nu}$. This allows us to exclude structural changes as the main cause of the peculiar post-glitch feature in spin-down rate. Such changes, like a change in inclination angle $\alpha$ or the weakening of vortex "pinning" in some crustal region due to lattice failure, are expected to result in a permanent shift in $\dot{\nu}$.

While the thermal output of some internal mechanisms (section 4.3.1) might be below detectable levels, the external processes discussed in section 4.3.2 could have a significant imprint on the emission. A positive $\Delta \dot{\nu}$ was observed in both the large and moderate glitch of this pulsar, so it might be a common feature of its post-glitch spin-down behaviour. Broadband regular monitoring, from radio to $\gamma$-rays, and pointed X-ray observations as soon as possible after its next glitch to follow any thermal evolution, could allow us to clarify whether internal or external (magnetospheric) mechanisms dominate its complicated post-glitch spin-down.

Glitch associated radiative changes and rotational features like the ones discussed here are observed mostly in magnetars. Similar behaviour might be present in typical NSs although not always detectable. For example, in the case of the Crab pulsar the relaxation after some of its glitches seems to be better described when a timing model with an exponentially decaying $\Delta \dot{\nu} > 0$ term is included (Lyne et al. 2014). Nevertheless, in the Crab pulsar this feature is weaker than in PSR J1119–6127 so that $|\dot{\nu}|$ stays always larger than the pre-glitch projected value.

Some of these differences in glitch phenomenology are possibly related to the magnetic field strength and structure. The X-ray pulse profile of PSR J1119–6127 is indicative of a strong toroidal component in the interior (Ng et al. 2012) which might be still evolving in the crust (Viganò et al. 2013), as is also suspected to be the case in magnetars. Analysis of its braking index, employing a self-consistent and phase coherent technique, indicates a long-lasting apparent decrease after the 2007 glitch. The extrapolated evolution in the $P - \dot{P}$ plane according to the pre-glitch braking index would bring PSR J1119–6127 to a region populated by long-period RRATs.
and magnetars. If the glitch resulted in a new, lower long-term braking index then the inferred magnetic field would appear to be growing at a faster rate than before the glitch, and the $P - \dot{P}$ track would turn more towards the magnetar region.

A post-glitch evolution characterised by a decrease in braking index has been observed before in only one other rotationally powered pulsar, the X-ray pulsar PSR J1846–0258, which might also be evolving towards the magnetar population. This pulsar shares some interesting properties with PSR J1119–6127, as it is also a very young ($\sim 0.8$ kyr) pulsar with a high inferred magnetic field of $\sim 5 \times 10^{13}$ G. Moreover, like the second glitch of PSR J1119–6127, the 2006 glitch of PSR J1846–0258 was relatively large in magnitude for such a young pulsar, and accompanied by radiative changes, with a X-ray flux increase and several X-ray bursts, an episode similar to magnetar outbursts (Gavriil et al. 2008). Following this glitch, a decrease in the braking index of $18\% \pm 5\%$ was observed (Livingstone et al. 2011). However the phenomenology of the glitch recoveries for these two pulsars appears different, possibly due to differences in the characteristic timescales of the underlying physical mechanism(s) and the amplitudes of the glitch-induced changes. In the case of PSR J1846–0258 the post-glitch evolution is dominated by a very large increase in $|\dot{\nu}|$, which results in an "over-recovery" in frequency, parameterised as a large ($\sim -10^{-4}$ Hz) permanent decrease in $\nu$ (Livingstone et al. 2010). In both pulsars the part of the glitch recovery modelled as exponential relaxations on short and intermediate timescales is over after about two years. However, the post-glitch $|\dot{\nu}|$ of PSR J1846–0258 remains larger than the pre-glitch projected value for at least four years (Livingstone et al. 2011) and is dominated by the glitch-induced enhancement of the spin-down rate (as in the case of the Crab pulsar mentioned above). This persistent faster spin-down is rather common for glitches in general but is in clear contrast with the remarkable "over-recovery" of $\dot{\nu}$ observed in PSR J1119–6127. Nonetheless, in both cases there appears to be a prolonged decrease in $\dot{\nu}$, which becomes detectable once the short-term recovery is no longer dominant, and is the main factor responsible for the apparent decrease in the braking index.

The RRAT PSR J1819–1458 is the only other radio rotationally-powered pulsar with a post-glitch $\dot{\nu}$ evolution that results in a relative decrease in $|\dot{\nu}|$, and it is also a high magnetic field pulsar, with inferred $B_d \approx 5 \times 10^{13}$ G. The recovery after its second observed glitch demonstrates the peculiar characteristics as in PSR J1119–6127, with a measured persistent change $\Delta \dot{\nu}_p \sim 0.8 \times 10^{-13}$ Hz s$^{-1}$. Interestingly, there are also indications for augmented activity of the radio emission following the glitches in PSR J1819–1458 (Lyne et al. 2009b). However, RRATs are characterised by their irregular radio emission, while PSR J1119–6127 displays a very stable radio emission and pulse profile during all other observations in the 16 years of monitoring. With emission characteristics much closer to those of normal,
steady radio pulsars and timing noise levels considerably lower than in magnetars, timing studies of PSR J1119–6127 and similar high magnetic field pulsars might offer the possibility of disentangling the contribution of the magnetosphere and the neutron superfluid in the glitch-bursting puzzle.

Acknowledgments

We thank the anonymous referee for constructive comments and Onur Akbal for an advance copy of their manuscript. D.A. would also like to thank Ali Alpar and Daniele Viganò for useful discussions. D.A. and A.L.W. acknowledge support from an NWO Vidi Grant (PI Watts). Pulsar research at JBCA is supported by a Consolidated Grant from the UK Science and Technology Facilities Council (STFC). C.M.E. acknowledges support from FONDECYT (postdoctorado 3130512). The Parkes radio telescope is part of the Australia Telescope which is funded by the Commonwealth of Australia for operation as a National Facility managed by CSIRO.
Confronting post-glitch relaxation theory with observations of the Vela pulsar after its 2000 major glitch

D. Antonopoulou, B. Link, A. L. Watts

To be submitted
Abstract

Spin-up glitches in the rotation of pulsars are attributed to the dynamics of their superfluid interiors and provide a unique way to probe the properties of the inner crust and core of neutron stars. The post-glitch response is usually modelled with a multi-component exponential decay, often supplemented with terms that describe permanent changes in the spin and spin-down rates. In addition to exponential recoveries, a different response, non-linear in the glitch spin-up amplitude, is also theoretically possible and expected to arise for a range of plausible stellar properties. Here, we use the high-resolution data from the Vela glitch of 2000, to investigate whether such data are of sufficient quality to distinguish between a model with three exponential terms, and permanent increases in both the spin rate and the spin-down rate, or a model in which one of the exponential terms is replaced by the predicted non-exponential relaxation. We find, consistent with previous analyses of this glitch, exponential decay over two distinct timescales, \( \sim 0.5 \) and \( \sim 3.5 \) days. In the context of vortex slippage theory introduced by Link (2014), such short timescales cannot originate from regions where vortex pinning is dominant, since these are expected to respond in the non-linear regime and recover with a time delay of a few weeks for the glitch examined here. The presence of such a delayed-response term is neither significantly favoured nor disfavoured from our analysis, compared to a long-lived exponential decay or a linear decay in spin-down rate, but we are able to obtain constraints on the theory of superfluid vortex slippage.
5.1 Introduction

Glitches in the rotation of neutron stars are rapid increases of the spin frequency $\nu$, which is otherwise secularly decreasing under the external electromagnetic torque. The changes of the observed angular velocity $\Omega_c = 2\pi \nu$ during a glitch cover a wide range of magnitudes, with fractional sizes $\Delta \nu/\nu \sim 10^{-11} - 10^{-6}$ (Espinoza et al. 2011b; Yu et al. 2013). Enhancements in spin down rate of the order $\dot{\Omega}_c/\Omega_c \sim 10^{-8} - 10^{-2}$ are often observed to accompany such events (Shemar & Lyne 1996; Espinoza et al. 2011b; Yu et al. 2013). The rotational parameters typically recover, at least partially, to their pre-glitch values on long timescales of days to years. These long relaxation timescales are one of the most important features of glitches, since they provide indirect evidence for a superfluid component in the interior of neutron stars (Baym et al. 1969). In most glitch models, the superfluid is also thought to be driving the glitch itself, since it can maintain a higher angular velocity than the rest of the star and thus provide the necessary angular momentum for the spin-up (Anderson & Itoh 1975).

Neutron stars cool soon after their formation below the transition temperature for the onset of proton superconductivity in the core and neutron superfluidity in the core and inner crust. For densities $\rho \gtrsim 3 \times 10^{14}$ g cm$^{-3}$ the superconducting protons will be likely of type I, with the magnetic field confined in small regions without clear structures (Glampedakis et al. 2011). At lower densities (the outer core region) type-II superconductivity is expected (Baym et al. 1969), with the magnetic flux quantised in fluxtubes. Equivalently, the circulation of the superfluid neutrons will be quantised in vortex lines and the superfluid’s angular velocity is defined by the number of vortices threading it. In order for the superfluid to follow the spin-down of the star due to the external torque, the vortex density needs to decrease by outwards vortex motion and expulsion. However, the interaction of vortices with the charged component can lead to vortex “pinning”, for example to the lattice nuclei of the inner crust (Alpar 1977; Epstein & Baym 1988) or the flux-tubes in the outer core (Jones 1991; Chau et al. 1992). Strong pinning can greatly suppress vortex outflow in these regions, in which case the superfluid spins down less effectively and a rotational lag builds up between the charged and superfluid stellar components. Anderson & Itoh (1975) have proposed that glitches represent catastrophic unpinning of many vortices. As the freed vortices move outwards, they impart torque to the charged component, producing the observed spin-up of the crust. Independently of the origin of the spin-up, the glitch perturbation in the velocity of the charged component affects the rate of vortex outflow, which depends on the rotational lag of the two components. This results in a temporal decoupling of the superfluid, and since the external torque acts on a lesser moment of inertia, it gives rise to the initial change in spin-down rate observed to accompany most glitches. The recoupling of this superfluid component is reflected in
the post-glitch recovery. In the following, we briefly review the expected dynamical response of a superfluid to a glitch, in the presence or absence of significant vortex pinning, which this work aims to examine.

5.1.1 Post-glitch recoveries

In the pioneering work of Alpar et al. (1981, 1984a), inspired by the flux-tube creep process observed in terrestrial superconductors, the equivalent thermally-activated process of superfluid vortex creep and the response to a glitch was explored. They concluded that the pinned superfluid in neutron stars could be in one of two different regimes, characterised by a linear or non-linear response to the glitch amplitude, depending on the local temperature and pinning properties (Alpar et al. 1989).

Models with solely exponential decays, the response of the above linear (in glitch size) regime, appear to provide a good description of the data (Downs 1981; Alpar et al. 1993; Lyne et al. 2000; Wong et al. 2001) and typically the post-glitch behaviour is investigated by fitting the data with a permanent frequency change, a permanent change in the spin-down rate, plus one or more such exponential terms (Yu et al. 2013). On the basis of such fits, attempts have been made to deduce the physical properties and dynamics of the neutron star interior. An important question is whether such exponential functions are always favoured by the data, or whether different basis functions, particularly those motivated by physical glitch models, can describe the data equally well or are preferred.

Recent work by Link (2014) predicts that, in regions where pinning is prevalent, thermally activated unpinning and the subsequent vortex motion will always be in the non-linear regime and will result in a distinct signature in the frequency residuals $\delta \Omega_c(t) = 2\pi(\nu(t) - \nu_0(t))$, where $\nu_0(t)$ is the pre-glitch timing solution. This response depends non-linearly on the initial glitch perturbation size $\Delta \nu_g = 2\pi \Delta \nu_g$, giving a non-exponential signature in the recovery. The contribution to the residuals of one such superfluid pinned region, characterised by a moment of inertia $I_{s,p}$ and a relaxation time $t_r$, can be approximated as (Equation 83 of Link 2014; Alpar et al. 1989):

$$\frac{\delta \Omega_c(t)}{\Omega_c} \approx 1 - \frac{I_{s,p}}{t_d} \left[ \frac{t}{t_d} + \frac{t_r}{t_d} \ln \left( \frac{e^{t/t_r} + e^{t/t_r} - 1}{t_d} \right) \right]$$

with $t_d$ being the decoupling time which is given by:

$$t_d = \frac{\delta \Omega_c^{(g)}(t)}{\dot{\nu}} \approx 2 t_{sd} \frac{\Delta \nu_g}{v} \approx 7 \left( \frac{t_{sd}}{10^4 \text{ yr}} \right) \left( \frac{\Delta \nu/\nu}{10^{-6}} \right) \text{ days}$$

In this connection, we note that when two or more exponentials are used, the fits become somewhat degenerate in the fitting parameters, representative of the degeneracy in the choice of basis functions.
where we define the spin-down age \( t_{sd} = \nu/(2|\dot{\nu}|) \).

The signature of Eq. 5.1 is a delayed-recoupling (hereafter denoted DR) recovery, linear in spin frequency for time \( t_d \), followed by an exponential relaxation over a timescale \( t_r \) as illustrated in Fig. 5.1. The superfluid remains decoupled for time \( t_d \), before it is able to recouple over a timescale \( t_r \). Thus the total recovery time is \( \sim t_d + t_r \). In the limit \( t_r \gg t_d \), the last term sets in at an early time, and the response will appear to be exponential. However, for large glitches in mature neutron stars the time delay \( t_d \) is expected to be \( t_d > t_r \). Consequently, any relaxation components associated with timescales shorter than \( t_d \) cannot be due to thermally activated vortex slippage in pinning regions.

Instead, relaxation on such timescales might originate from regions where pinning is absent or very weak and the coupling between the two components is determined by vortex motion against drag forces. Then the predicted response is also exponential (see for example Andersson & Comer 2006; Link 2014), as for a pinning region in the \( t_r \gg t_d \) limit, and can be written as:

\[
\frac{\delta \Omega_c}{\Delta \Omega_c} = 1 - Q(1 - e^{-t/t_r}).
\]

In the above we introduced the glitch recovery fraction \( Q \), which is defined as \( Q = \sum \Delta \nu_d/\Delta \nu_g \), where \( \sum \Delta \nu_d \) is the sum of the amplitudes of all decaying glitch components. The initial spin-up might have contribution from permanent components \( \Delta \nu_p \), so that \( \Delta \nu_g = \Delta \nu_p + \sum \Delta \nu_d \) and \( Q \neq 1 \) and represents the fractional moment of inertia of the regions that relax back to their pre-glitch state.

Disentangling the expected signatures of each region from observational data can constrain the moment of inertia of the pinning regions which are able to act as angular momentum reservoirs. It is usually assumed that the component responsible for glitches and the post-glitch relaxation (on observable timescales) is the inner crust superfluid. Pinning is possible there because the lattice of nuclei provides a strong interacting potential for the neutron vortices. Moreover, large glitch recoveries seem to involve a superfluid moment of inertia of \( I_s \ll I \), where \( I \) is the total moment of inertia, close to the theoretically estimated value for the crust. However, recent work by Chamel (2012) indicates that entrainment of neutrons in the inner crust, that arises from their scattering off the crustal lattice, might limit considerably the crustal superfluid moment of inertia available to act as an angular momentum reservoir (Andersson et al. 2012; Chamel 2013b; Hooker et al. 2013). If that is the case then observations of large glitches can lead to valuable constraints on the neutron star properties and equation of state, but their interpretation relies crucially on whether a portion of the core takes part in the glitch or not.

Though much effort has been devoted to understanding the dynamics of the crustal superfluid (see for example Alpar et al. 1984a, 1993; Link & Epstein 1996; Jahan-
Miri 2005; Pizzochero 2011; Link 2014), the contribution of the core to the glitch phenomenon remains much less explored. For type-I proton superconductivity vortex pinning is negligible (Sedrakian 2005), however in the case of type-II superconducting protons the fluxtube-vortex interaction can lead to pinning in parts of the core (Ruderman et al. 1998; Link 2003). Therefore, modelling glitch recoveries can shed light on the nature of superconductivity in the core of neutron stars and the interaction between neutron vortices and proton flux tubes. A thermally-activated response takes exactly the same form as the drag response in the limit $t_r \gg t_d$, thus when this condition is true the two signatures cannot be discriminated (see Figure 5.1). However, according to Eq. 5.2, the decoupling timescale is expected to be long and potentially $t_d \gg t_r$ for the, typically large, Vela pulsar glitches.

In this work we analyse data for its 2000 glitch and model the post-glitch recovery with a function that takes into account the possible presence of the DR response term. Link (2014) finds that the coupling timescale for the inner crust is years to decades long, suggesting that the crustal superfluid in not responsible for the relatively faster decays observed in the post-glitch response. Moreover, according to Link (2014)
and for the glitch parameters presented here, the presence of strong pinning in the outer core would leave a distinct signature in the residuals $\Delta \Omega(t)$ of linear decay for $t_d \sim 25 \text{ days}$, followed by an exponential relaxation of this component on a timescale of $t_r \sim 3 \text{ days}$. We test directly for the predictions of thermally activated vortex slippage theory in the post-glitch frequency residuals and contrast our results with those obtained from a recovery model with simple exponential decays.

### 5.2 Observations of the 2000 Vela glitch

The Vela pulsar was the first neutron star seen to glitch (Radhakrishnan & Manchester 1969) and suffers large glitches every $\sim 2 - 3 \text{ years}$. Furthermore, observations at the Hobart Radio Observatory (Mount Pleasant) successfully constrained the rise time for one of the largest Vela glitches, which occurred on January 2000 (MJD 51559), to under 40 sec. Because of the good quality of data, high cadence of observations and the regular behaviour of its glitching activity, the Vela pulsar has been used extensively to test theoretical glitch models (Alpar et al. 1984b; Chau et al. 1993; Link & Epstein 1996; Haskell et al. 2012).

We want to test for the presence of a residual $\Delta \nu_t$ relaxing component as predicted by Eq. 5.1, which could indicate regions in the thermally activated slippage regime. For this purpose, we used the data recorded with the 14-m diameter antenna of the University of Tasmania at Mount Pleasant for the 2000 Vela glitch, which is the best sampled post-glitch recovery to date. The times of arrival (TOAs) of the 2 minutes integrated observations from MJD 51559 to MJD 51645 were corrected and barycentered using standard techniques and resulted in 22844 TOAs. We refer the reader to Dodson et al. (2002) for details of the observations and the preliminary analysis of the TOAs.

The basic timing model for the frequency $\nu(t)$ of a neutron star is derived by a Taylor expansion around a reference epoch $t_0$, typically truncated at the second frequency derivative term $\ddot{\nu}$:

$$\nu(t) = \nu_0 + \dot{\nu}_0 (t - t_0) + \frac{1}{2} \ddot{\nu}_0 (t - t_0)^2 \quad (5.4)$$

where $\nu_0$, $\dot{\nu}_0$ and $\ddot{\nu}_0$ are the rotational parameters at $t = t_0$. Glitches and glitch recoveries are usually modelled by permanent offsets $\Delta \nu_p$ and $\Delta \dot{\nu}_p$ and a sum of exponentially decaying terms $\sum \Delta \nu_{d,i}$. These are parameterized by introducing, after a glitch event, an additional term $\Delta \nu(t)$ in Eq. 5.4 as follows:

$$\Delta \nu(t) = \Delta \nu_p + \Delta \dot{\nu}_p t + \sum_i \Delta \nu_{d,i} e^{-t/t_i} \quad (5.5)$$

where we have set the glitch epoch to be $t = 0$. 

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In the framework of thermally activated vortex slippage however, the recovery will be approximated by

$$\Delta \nu(t) = \Delta \nu_{p} + \Delta \nu_{p} t + \sum_{i} \Delta \nu_{d,i} e^{-t/\tau_{i}} + \sum_{j} \Delta \nu_{s,j}$$ (5.6)

where the exponentially decaying terms are associated with drag-dominated regions and the terms $\Delta \nu_{s,j}$ represent the response of regions where pinning prevails. According to Eq. 5.1, the response $\Delta \nu_{s}$ of a given region can be written as:

$$\Delta \nu_{s}(t) \approx \Delta \nu_{g}(1 - Q_{p}) - Q_{p} \Delta \nu_{0} t + Q_{p} |\Delta \nu_{0}| t \left[ e^{\frac{t}{\tau}} + e^{\frac{\Delta \nu_{g}}{Q_{p}} \cdot t} - 1 \right]$$ (5.7)

where

$$Q_{p} = I_{s,p}/I$$ (5.8)

is the fractional moment of inertia of the superfluid region that responds in this way, and $\Delta \nu_{g}$ is the spin-up extrapolated to the glitch epoch $t = 0$. For simplicity, we have assumed that this superfluid component is not the same one that drives the glitch (Link 2014), and we will consider only one region that responds as Eq. 5.7 for the data analysis.

The dataset presented here consists of post-glitch data, which extend from the glitch epoch up to MJD 51645, and was kindly shared with us by Richard Dodson, in the form of corrected TOAs and a pre-glitch timing solution according to Eq.5.4 (Dodson et al. 2002). Since we cannot refine the pre-glitch solution via a global fit to both pre- and post-glitch data that includes glitch recovery models, we extracted the given pre-glitch timing solution (presented in Table 5.1) to find the residuals of the post-glitch data. While the pre-glitch values of the frequency and spin-down rate are rather well defined, the pre-glitch $\Delta \nu_{0}$ value, which is contaminated by timing noise, differs from the one used by Dodson et al. (2002), which was calculated from a global fit to a longer dataset that included pre-glitch TOAs (See table 1 of Dodson et al. 2002). This leads to small discrepancies with their results for the late post-glitch times, when the effects of the different $\Delta \nu_{0}$ are larger (see Section 5.3). The post-glitch phase residuals $\Delta \phi$ were obtained using the timing package TEMPO2 (Hobbs et al. 2006), while calculations of the frequency residuals and the model fits were performed using custom software as described below.

The post-glitch evolution of the frequency residuals with respect to the pre-glitch timing solution is shown in Figure 5.2. These measurements arise from individual fits of a Taylor expansion to non-overlapping subsets of TOAs. The number of TOAs included in each subset was optimised to lead to a robust $\Delta \nu$ measurement. A minimum number of TOAs $N^{\text{min}}_{\text{TOAs}} = 100$, and a maximum time separation of 1 day between the first and last TOA were also imposed for every fitted subset, to ensure the quality
5.3 Model fits and results

Previous models used to describe the recovery of this glitch included permanent changes $\Delta \nu_p$ and $\Delta \dot{\nu}_p$, as well as 4 exponential terms (Dodson et al. 2002). The three longer decay timescales characterising the relaxation were found to be around 0.5, 3 and 19 days, which are similar to the timescales observed in other glitches from this pulsar (Flanagan 1990; Alpar et al. 1993). A term associated with an even faster decay timescale of $\sim$ 1 min has been identified for this glitch, using the available TOAs from 10 sec folded data (Dodson et al. 2002). We do not attempt to fit for the one-minute decay, which cannot be resolved in the dataset used in the present anal-

Figure 5.2: The post-glitch frequency residuals, with respect to the pre-glitch timing solution presented in Table 5.1. The frequency residuals are derived from fits to two different ensembles of TOA subsets (see Section 5.2), with $N_{\min}^{\text{TOAs}} = 80$, leading to 115 frequency measurements (set 1) and $N_{\min}^{\text{TOAs}} = 100$, resulting in 56 frequency measurements (set 2). The statistical error bars (shown here) are smaller than the symbol size for the second set. The long dashed line corresponds to the fit solution of Model A, while the dotted line is the result of the fit for Model B, both for set 2 (see Table 5.1 and the text for details).
Table 5.1: The pre-glitch timing solution used in this analysis, and the inferred glitch parameters for the 2000 glitch of PSR J0835–4510, according to two different models describing the recovery. Model A includes three exponential recovery terms and permanent changes in $\nu$ and $\dot{\nu}$, while in Model B one exponential term is replaced with the DR term of Eq. 5.6. All model parameters were optimised simultaneously, for the two datasets of frequency residuals (see text for details). For the parameters that were fitted, statistical $1 - \sigma$ errors in the last significant digit are presented in brackets. These are indicative of our ability to constrain a given parameter via our model fits, but should otherwise be regarded only as a lower limit to the real errors, since they do not take into account the systematic errors introduced by the method used to derive the frequency residuals (see text) and the presence of timing noise.

<table>
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<th>Set 2</th>
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<td>51559</td>
</tr>
<tr>
<td>$\nu_0$ (Hz)</td>
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<td>11.19461522857</td>
</tr>
<tr>
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<td>-1.5567</td>
</tr>
<tr>
<td>$\dot{\nu}_0$ ($10^{-20}$ Hz s$^{-2}$)</td>
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<td>-2.1463323</td>
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<tr>
<td>MJD range</td>
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<td>Glitch epoch</td>
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Model A

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<tr>
<td>$\Delta \dot{\nu}_p$ ($10^{-14}$ Hz s$^{-1}$)</td>
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<td>25(7)</td>
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<td>$\Delta \nu_{d1}$ ($\mu$ Hz)</td>
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<tr>
<td>$\Delta \nu_{d2}$ ($\mu$ Hz)</td>
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<tr>
<td>$\tau_{d2}$ (days)</td>
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<tr>
<td>$\Delta \nu_{d3}$ ($\mu$ Hz)</td>
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<td>$\tau_{d3}$ (days)</td>
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Model B

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<tr>
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<tr>
<td>$t_r$ (days)</td>
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<td>27(8)</td>
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We also set the glitch epoch $t_g$ to be the one derived by Dodson et al. (2002), using the higher resolution TOAs close to the glitch epoch.

We therefore model the recovery as in Eq. 5.5, with three exponentially relaxing terms (Model A), leaving a total of 8 free parameters to characterise the permanent and decaying changes. To investigate for a function of the form Eq. 5.7, we also tested for a model where we replaced one of the three exponential components with a $\Delta \nu_i$ (DR) term as in Eq. 5.6. This response (Model B) was parameterised as:

$$
\Delta \nu(t) = (1 - Q_p) \left( \Delta \nu_g - \frac{\sum \Delta \nu_d}{1 - Q_p} \right) + (\Delta \dot{\nu}_p - Q_p|\dot{\nu}_o|) t \\
+ \sum_{i=1}^{2} \Delta \nu_{di} e^{-t/\tau_i} + Q_p|\dot{\nu}_o| \tau_r \ln \left[ e^{\frac{\Delta \nu}{Q_p |\dot{\nu}_o| \tau_r}} - 1 \right]
$$

(5.9)

Both fitting functions have the same number of eight free parameters, since we now leave the total glitch magnitude $\Delta \nu_g$ and the relaxation timescale $\tau_r$ and fraction $Q_p$ as free parameters, in the place of the amplitude and timescale of a third exponential term and the permanent term $\Delta \nu_p$. We implemented the Levenberg–Marquardt method for the fitting procedure (see for example Press et al. 1992; Press et al. 2002), whose results are presented in Table 4.1. For direct comparison, we report $\Delta \nu_g = \Delta \nu_p + \sum \Delta \nu_{di}^{(i)}$ for Model A too, instead of $\Delta \nu_p$.

The rms and the $\chi^2$ value are better for Model A than for Model B for both solutions presented in Table 5.1, however this difference is not significant within the uncertainties introduced by timing noise and the chosen method to derive the frequency residuals. This can be seen clearly in Fig. 5.3, where the residuals between the two models and the observations are shown, for both Sets 1 and 2. Derivations using frequency residuals from different ensembles of data subsets present similar residuals and fit parameters, as long as the minimum number of TOAs and the maximum allowed timespan are varied within reasonable values.

The 2000 Vela glitch recovery analysed in this work has the best observational coverage among the glitches reported so far, which allows for relatively small uncertainties in the derived frequency residuals $\Delta \nu$. Even so, there are systematics that enter such calculations, which affect the results of our fits, as reflected in the different best-fit parameters and residuals (see Fig. 5.3) for the two frequency residual $\Delta \nu(t)$ datasets presented here2.

We tested a plethora of sets of $\Delta \nu(t)$ measurements and confirmed that these differences do not alter our conclusions, nor do they significantly affect the best-fit tim-

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2Ideally, a numerical scheme that integrates Eq. 5.9 must be employed, and model-fitting should be performed directly in the phase residuals. Since such a method would exploit all the available information of our TOAs, it could potentially be even more constraining for theoretical glitch models.
Figure 5.3: The post-glitch timing residuals, in $\mu$ Hz, according to Model A (open triangles) and Model B (filled circles). The displayed results are from fits to two different sets of frequency residuals, with 115 frequency measurements ($N_{\text{min}}^{\text{fits}} = 80$, top panel) and 56 frequency measurements ($N_{\text{min}}^{\text{fits}} = 100$, bottom panel), obtained as described in Section 5.2.
ing solutions obtained (see Table 5.1). The main attributes of our best-fit models, the separation of the recovery into three (short, intermediate and long) timescales and, for Model B, the ratio $t_r/t_d$, remain unchanged. Model B was never found to be an improvement over Model A, which appears slightly more favoured when shorter subsets of TOAs are used. Nevertheless, the relative difference in the residuals of the two timing models is in all cases like the one shown in Fig. 5.3. Therefore, although formally Model A describes better the derived frequency residuals, because of the aforementioned uncertainties and the presence of timing noise, it is not possible to exclude Model B for the given dataset. Comparably good fits for those two models were also found in earlier analyses of other Vela pulsar glitches (Alpar et al. 1993).

One must be cautious in comparing results from different analyses, which might have used different model-fitting methods and datasets of unequal length and TOA densities. The best-fit solution of Table 5.1 gives a small positive change in spin-down rate and a larger, but highly uncertain, amplitude and decay timescale for the third exponential (Model A), compared to the results of Dodson et al. (2002). This disparity is due mainly to the different pre-glitch $\dot{\nu}_0$ value used in our analysis, the negative value of which indicates that it is highly contaminated by timing noise. In general, longer datasets than the one presented here are needed in order to robustly infer the persistent change in spin-down rate $\dot{\nu}$ and any long-term exponential decaying terms. For example, with the dataset at hand we are not able to distinguish between a slow exponential or linear decay in $\dot{\nu}$. A model where the third exponential of Model A is replaced by a permanent change in $\dot{\nu}$ fits the data almost equally well\(^3\), and gives $\Delta \dot{\nu}_p = -1.36(3) \times 10^{-13}$ Hz/s, like the previously published estimate for this parameter (Dodson et al. 2002).

It is worth noting however that the amplitudes and shorter timescales $\tau^{(1)}$ and $\tau^{(2)}$ found by our analysis are well in accordance with the results of Dodson et al. (2002), as expected. Similar relaxation timescales to Model A, of $\sim 0.5$, $\sim 3$ and $\sim 30$ days, have been identified before and proved to describe well the recovery of 8 earlier glitches of the Vela pulsar (Alpar et al. 1993), indicating that the same stellar regions are involved in its glitches.

5.4 Discussion and conclusions

We have extended the exponential multicomponent post-glitch recovery of Eq. 5.5 to account for the presence of a non-linear in glitch magnitude (DR) term (Eq. 5.6) in modelling the post glitch relaxation of the largest Vela pulsar glitch. We find that

\(^3\)A model where one of the exponentials of Model B is replaced by a $\Delta \dot{\nu}_p$ term considerably worsens the fits, but proves to be highly sensitive to the choice of starting fit parameters thus cannot be conclusively excluded.
it is not possible to categorically distinguish between these models in the frequency residuals, despite the very high resolution of the available timing data for this glitch. Nevertheless, we can gain some insights by comparison of the best-fit parameters of our models to their physical counterparts within the framework of vortex slippage theory asserted by Link (2014).

According to Model B, $\Delta v_g = 35.7 \mu$Hz which, by use of Eq. 5.2, leads to a decoupling timescale $t_d \approx 26.5$ days. Even though the best-fit values for $t_r$ have large uncertainties (see Table 5.1), we find $t_r \geq t_d$. In this limit the DR response signature $\Delta v_s$ reduces almost to a simple exponential relaxation, as illustrated in Fig. 5.1. However, for large glitches in old, cool neutron stars, like the one examined here, a clear DR response is predicted for the range of pinning energies discussed by Link (2014) if most of the inner crust and outer core superfluid is pinned. This is not supported by the observations since the condition $t_d > t_r$ is not fulfilled and the presence of such a component does not appear to be favoured by the data, which are found to be consistent with simple exponential decays.

Furthermore, if the DR response $\Delta v_s$ is present, it is associated with a recovery fraction $Q_p$, implying a moment of inertia $I_{DR} \approx 2.6\%I$ for this pinned superfluid region. A similar total fractional moment of inertia is inferred for the two exponentially decaying components. This indicates that part of the core contributes to the observed relaxation, since the crustal superfluid is not enough to provide the total implied moment of inertia. According to vortex slippage theory (Equation 95 of Link 2014), strong pinning in the outer core might lead to $t_r$ of a few days, and therefore $t_d > t_r$, in contrast with our results for the DR term. We tried to look explicitly for such a component with relatively short $t_r$ in the residuals, by setting $t_r$ and $t_d$ to their estimated values, but failed to converge to an acceptable fit. Thus if the DR component represents the response of the core, our results imply that $t_r$ must be of the order of 25 to 30 days, as for our best-fit results. However, since this value is suspiciously close to $t_d$, in a regime where the DR term resembles an exponential decay, sound conclusions can only be drawn if in the future the same timescale is detected in the recovery of an even larger Vela glitch, which will have $t_d > t_r$, allowing for the delayed-recoupling response to be clearly observed.

Since $t_d \gg t_r$ for the shorter exponential timescales of $\sim 0.5$ and $\sim 3.5$ days found here, if these were representative of regions with a non-linear dynamical response, then a clear DR response should have been observed. Link (2014) argued that the condition for the exponential-decay limit of the response of pinned regions in neutron stars, is met only when $t_d << t_r$. If so, then these two terms must represent the response of regions where drag forces, instead of pinning, govern the motion of vortices, perhaps in the pasta region of the inner crust or the inner core. The moment of inertia for each such region will be $I_i/I \approx \Delta v_i/\Delta v_g$. Therefore almost $\sim 2\%$ of the
5.4 Discussion and conclusions

total moment of inertia takes part in the fast ($\tau \sim 0.5$ hrs) recovery, and is most likely associated with the core response, as for example in the glitch model of Haskell et al. (2012).

If all the components that seem to recover respond through the motion of vortices against only drag forces (Model A), then the only component that can be identified with a region $I_{s,p}$ of strong pinning, thus in the DR regime, is the one responsible for the long-lasting enhancement of the spin-down rate. Such a component has inferred moment of inertia $I_{s,p}/I \approx |\Delta \nu_p/\nu_0|$ which, for $\Delta \nu_p = -1.36(3) \times 10^{-13}$ Hz/s, implying that approximately 1% of the total moment of inertia has to remain decoupled over very long timescales. The decoupling timescale $t_d$ could be this high if there were vortex motion in this region during the glitch, in which case $t_d$ is

$$t_d = \frac{\delta \omega^{(g)}}{|\Omega_c|}$$ (5.10)

where $\delta \omega^{(g)}$ is the local change in the rotational lag. Since $I_{s,p}$ is the only region that pinning of vortices dominates, it is natural to identify it with the region that drives the glitch, which must comprise most of the crust. Angular momentum conservation then leads to

$$\delta \omega^{(g)} = \frac{I_{s,p}}{I} \Delta \nu_g$$ (5.11)

and a decoupling timescale $t_d \sim 3.5$ years for the 2000 glitch, approximately the inter-glitch time interval seen in the Vela pulsar. However, this interpretation leaves unexplained the component relaxing on a long timescale (either exponentially on a timescale $\sim \tau^{(3)} \sim 50$--100 days or by a DR response on a timescale $t_d + t_r \sim 50$ days).

In the absence of any pinning of the core vortices, this timescale is rather long to be explained by the response of the core under drag forces only, which is expected to lead to fast relaxation (Alpar et al. 1984c; Alpar & Sauls 1988). Unfortunately, the amplitude of this component cannot be robustly inferred from the dataset at hand, preventing us from obtaining very strict constraints. Nevertheless, a total moment of inertia for the pinned superfluid of $I_{DR} + I_{s,p} \gtrsim 3\%$ cannot be accommodated by the crustal superfluid alone, even in the absence of entrainment (Chamel 2013a).

In conclusion, we have tested the physical glitch model of thermally-activated vortex slippage (Link 2014), and found that, although not favoured over a multiple-component exponential response, it can provide an adequate description of the frequency recovery following the 2000 Vela glitch. We can place constraints on pinning properties within this model, since the estimated response of a strongly pinned superfluid in the outer core is not observed. Because of the inherent difficulty in disentangling a DR response in the limit $t_r \gtrsim t_d$ (which seems to be favoured by the data) from simple exponential decay (Fig. 5.1), timing data used for this purpose need to
have high timing resolution, but also a sufficient extent of many months to several years following the glitch, to resolve the long-term evolution which is attributed to the pinned superfluid. This is only possible when this evolution is not disrupted by a following glitch, however a global fit to a longer dataset, containing a series of glitches, can also prove very enlightening.
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Summary

This thesis presents detailed studies of different theoretical and observational aspects related to the rotational dynamics of isolated neutron stars, and in particular their spectacular spin up events known as glitches. Neutron stars play a special role in the advance of our astrophysical and physical understanding. They are the compact descendants of stars much heavier than our Sun, which collapsed under their own gravity and ended their life in a supernova explosion, some of the most energetic events observed in the Universe.

Neutron stars are expected to have a mass greater than that of our Sun, mostly in the form of neutrons as is implied by their name, contained in a region with a radius of only $10^{15}$ kilometres. This is the equivalent of compressing an average human to almost half the volume of a single red blood cell: the inferred density of neutron stars is enormous, making them the densest stars known in the Universe. At their centre, the density can be even higher than ten times the nuclear density, which is the density inside the nuclei of the atoms of which we are made. The behaviour of matter under such extreme physical conditions is hard to grasp because, neutron stars apart, it can be understood only by theoretical considerations of the fundamental forces of nature, and far extrapolation of the knowledge obtained from laboratory studies. The neutron star puzzle gets further complicated by the presence of very strong magnetic fields - some of these tiny objects, a class named magnetars, are the strongest magnets ever detected. Observations of neutron stars therefore offer a unique opportunity to test current physical theories, and inspire their development and improvement.

There are three basic regions, each with very distinct properties, that must be considered when studying the behaviour of neutron stars. The inner parts, where the density is higher than the nuclear density and nucleons (neutrons and a small percentage of protons) are no longer bound inside nuclei, comprise the neutron star core. The composition of the core is still uncertain: it could contain mostly neutrons, but in the deeper regions, where the density can be very high, other components such
as hyperons or deconfined quarks might appear. Modelling of neutron star cores is challenging even when such exotic components are absent, since even the much more familiar nucleonic matter is predicted to be in an extraordinary state. For the inferred range of densities and temperatures in the core, nucleons will be in their ground state, with the lowest possible energy, forming condensates. In this state, exchange of energy with the environment is limited to certain channels and nucleons acquire "super" characteristics. The protons are superconducting, which means that the electric currents generated by their flow lose energy and decay at a very slow, almost zero, rate. Any magnetic field in superconducting regions must be confined in vortex-like structures called flux tubes (superconductivity of type II), or be contained in irregular patches of normal protons (type I superconductivity). On the other hand, the neutral neutrons form a superfluid, which has almost zero viscosity and can support rotation only by forming microscopic vortices. Many millions of billions vortices are expected to penetrate the superfluid in neutron stars. Surrounding the core is an outer shell of matter at sub-nuclear densities. This region is expected to host nuclei in the solid state, which form a crystalline lattice, which is why it is called the crust. With the exception of the, approximately one kilometre thick, outer layer, unbound neutrons in a superfluid state are also present in the crust. Since neutron stars posses a solid crust like the Earth’s (but much much stronger), they can also "break" and slightly change shape in starquake events. Finally, right outside the surface of a neutron star lies its magnetosphere, an extended magnetised region which is probably filled with plasma (electrons, positrons and ions) and is dynamically connected to the star.

Neutron stars are "dead" stars, that is to say, there is no internal nuclear burning to power a large luminosity like in regular stars. However, we observe emission that originates from specific magnetospheric regions and, in some cases, thermal emission from their hot surface. This latter radiation is also often not uniform, indicative of warmer and colder spots on the neutron star crust, attributed to the anisotropic heat conduction due to the interior magnetic field. One of the most important characteristics of neutron stars is that they rotate very rapidly: the fastest amongst them complete several hundreds of revolutions per second. If the emission zones cross our line of sight in every rotation, the received radiation appears in the form of pulses, with arrival frequency equal to the rotational spin frequency. Most neutron stars are discovered by the observations of such pulses in the radio band, which gave birth to the term pulsars (pulsating radio stars), used by now to describe all neutron stars from which pulsed emission is detected. By recording a series of pulses, the spin frequency and rotational evolution of the pulsar can be extracted via a technique called pulsar timing. There is a rapidly increasing amount of pulsar timing data, which contains invaluable information on the physics of these objects because the rotation is...
determined by the dynamical behaviour and interaction of the crust, core and magnetosphere.

The loss of energy from isolated neutron stars, of which typically only a small part reaches us as detectable emission, makes them spin down over time. In general this evolution is very slow and rather steady, rendering possible very precise measurements and predictions of the spin frequency. Some neutron stars though, mostly "adolescent" and young pulsars, sometimes spin up suddenly. These events are referred to as rotational glitches, and usually happen so fast that they cannot be time resolved in the data, but instead appear as step-like increases in the spin rate.

Glitches are thought to represent the transfer of angular momentum from the neutron superfluid to the rest of the star. While all charged stellar components are strongly coupled to each other due to electromagnetic forces, the superfluid neutrons can be partially decoupled and rotate at a different rate. This happens because superfluid vortices can pin in some parts of the crust and the core, which act as vortex traps that contain more vortices than those required to match the spin frequency of the crust. Therefore, the superfluid hidden inside neutron stars might rotate at a faster rate than the spin frequency observed. It is thought that, occasionally, a lot of vortices are released from the pinned regions in an avalanche-like event, and adjust their density so as to bring the superfluid and charged components closer to corotation. This leads to the spin-up of the crust, a glitch whose magnitude scales with the size of the superfluid region and number of vortices involved in the avalanche.

It is not always easy to identify and measure a glitch in the timing data of a pulsar. Other kinds of timing irregularities can mask or be confused with glitch signatures. Moreover, it is not possible to monitor the spin frequency of all pulsars as often as required. Chapter 2 presents the detectability curves for glitches that arise when these effects are taken into account (Section 2.2). We use these limits, together with a new automated technique for detecting and measuring glitch-like variations in timing data (Section 2.4), to explore the statistics of glitches in the pulsar J0534+2200, the famous Crab pulsar. This source is the neutron star lying in the centre of the Crab Nebula, which formed in a supernova explosion recorded on Earth in the year 1054. It suffers rather frequent glitches that present a broad range of sizes and has been observed frequently, nearly daily, for the last 30 years, which makes it easier to study the lower end of the size range for its spin-ups. The smallest of these events could - theoretically - arise when only a few vortices take part in an avalanche, and would then be undetectable in the timing data. Rather unexpectedly, however, our study revealed that vortex avalanches must involve at least a few billion superfluid vortices in order to appear as typical glitches in this pulsar (Chapter 2, Section 2.7). This result will help to unravel the trigger mechanism behind the occasional collective vortex unpinning events, one of the most important unsolved mysteries behind the
glitch phenomenon.

The rotation of pulsars after a glitch depends on the way the different neutron star regions respond to the spin-up. The charged particles of the core and magnetosphere will quickly follow the accelerating crust; on the other hand, the neutron superfluid is less strongly tied to the rest of the star and reacts more slowly, on timescales that can be as large as decades. As a consequence of the glitch-induced decrease in the relative velocity between the superfluid and the other components, the superfluid can decouple from them. The post-glitch secular spin-down of the neutron star proceeds then at a higher rate, since it affects a smaller stellar moment of inertia. In most cases this results in an at least partial recovery of the spin frequency to its pre-glitch lower value. Because observations of this post-glitch relaxation can be used to infer information, otherwise unaccessible to us, about the properties of the neutron superfluid and those of the other internal stellar components with which it interacts, their theoretical modelling is of great physical interest.

Not all glitches seem to recover in the same way, even those of just one particular pulsar. The spin-down rate might not change at all (step-like glitches) or it might show a small increase of less than one percent. In other cases still it might increase dramatically at the glitch, as is often seen in magnetars or after the very large giant glitches. Often, all or part of this initial increase in spin-down rate decays away on timescales that range from few hours to tens of years, in a way that usually appears exponential. Theoretical models in which the neutron star is treated as a hydrodynamical two-fluid system, aiming to describe the coupled dynamics of the loose superfluid component and the rest of the star, offer a very promising framework to explore glitches. It has been shown that such a model, together with realistic input from microphysical calculations, can explain the exponential relaxations seen after giant glitches (Haskell et al. 2012). The work presented in Chapter 3 uses this model to further investigate the superfluid dynamics. We demonstrate that our approach also naturally accounts for the strong recoveries often observed in magnetars, as well as for a range of different post-glitch signatures. Moreover, we consider events more subtle than a catastrophic unpinning of vortices, and show that they can explain step-like glitches (Chapter 3) and can result in unusual post-glitch recoveries (Chapter 4) as well as a rich rotational behaviour in general.

The response of the superfluid component to some other perturbation, like a sudden increase in temperature or a rapid change of the magnetic field, is likely connected to other dynamical events in the life of neutron stars, for example starquakes or the small explosions, bursts of high-energy emission, which are typical of magnetars. These magnetospheric bursts are powered by the strong magnetic fields of magnetars, and in many instances coincide with a glitch. Additional internal or magnetospheric mechanisms might contribute to the following spin evolution, which is often irregu-
lar. In contrast to magnetars, the magnetosphere was thought to be an almost inactive region during glitches in ordinary pulsars, because no changes in radio emission were observed to accompany them. Recently however, in 2007, such changes were seen for the first time following a large glitch of the radio pulsar J1119−6127, in addition to an atypical decrease of the spin-down rate at later times. This very interesting source is the focus of Chapter 4, which presents a thorough timing analysis of this pulsar’s rotational history over the last sixteen years. In this work we also introduce and use a new method to assess the secular spin-down of neutron stars (Section 4.2.2). Our study reveals the details of a very peculiar post-glitch relaxation and allows for a detailed examination of the physical mechanisms that might be at play in the glitches of the pulsar J1119−6127 (Section 4.3). Because this neutron star, like the magnetars, is highly magnetised, these observations provide further evidence that the strong magnetic field might link glitches to magnetospheric phenomena.

While such case-studies of unconventional post-glitch recoveries help to shed light on less explored aspects of the phenomenon, case-studies of typical glitches can provide tight constrains on the basic theories about the glitch mechanism and superfluid response. The exact details of how vortices move about, in regions where they are free or trapped, depends on unknown microphysics. Insights into these can be gained by confronting theoretical predictions with observations of the more common post-glitch relaxations. The Vela pulsar is a good target for this aim, because it undergoes easily detectable giant glitches, recorded in radio data of high timing resolution. In Chapter 5 we analyse three months of such observations, following the largest known glitch of this pulsar, and model the post-glitch recovery according to the different theoretical predictions for the superfluid response for the range of expected strengths of its interaction with the crust and the charged core particles. Our results strongly indicate that most of the neutron superfluid residing in the crust, as well as at least part of that in the core, decoupled at the glitch and contributed to the observed relaxation. However, according to our findings, the core provides a less effective trap for vortices than some theoretical estimates for superconducting core protons of type II predict, a valuable input for the microphysical studies of vortex motion and pinning.
Nederlandse Samenvatting

Dit proefschrift presenteert gedetailleerde studies van verschillende theoretische en observationele aspecten van de rotatie van geïsoleerde neutronensterren, met bijzondere aandacht voor de plotselinge verhogingen in de rotatiesnelheid van een neutronenster die bekend staan als glitches. Neutronensterren zijn de compacte overblijfselen van sterren vele malen zwaarder dan onze zon, die zijn ingestort onder hun eigen zwaartekracht, en hun leven hebben beëindigd in een supernova, een van de meest energetische verschijnselen in het universum. Deze sterren spelen een bijzondere rol in de natuurkunde en astrofysica.

Neutronensterren hebben naar verwachting een massa groter dan die van de zon, samengepakt in een gebied met een straal van slechts 10-15 km. Zoals de naam impliceert bestaat deze massa voornamelijk uit neutronen. De dichtheid van neutronensterren is enorm; vergelijkbaar met het samendrukken van een gemiddeld mens tot ongeveer de helft van het formaat van één enkele rode bloedcel. Dit maakt neutronensterren de meeste compacte sterren die we kennen in het universum. In de kern van een neutronenster kan de dichtheid oplopen tot meer dan tien keer de nucleaire dichtheid, de dichtheid in de kernen van de atomen waar we van gemaakt zijn. Het gedrag van materie onder zulke extreme omstandigheden is moeilijk te begrijpen, want behalve door het bestuderen van neutronensterren kan het alleen begrepen worden uit theoretische studies van de elementaire natuurkrachten, of door verre extrapolatie van de kennis verkregen uit laboratoriumstudies. Het beeld van neutronensterren wordt verder gecompliceerd door de aanwezigheid van zeer sterke magneetvelden. Sommige van deze objecten, een groep die magnetars wordt genoemd, zijn de sterkste bekende magneten. Zodoende bieden observaties van neutronensterren een unieke mogelijkheid om hedendaagse fysische theorieën te testen, en aan te zetten tot ontwikkeling en verbetering van deze theorieën.

Neutronensterren bestaan uit drie lagen met zeer verschillende eigenschappen. Het binnenste deel is de kern. Hier is de dichtheid hoger dan de nucleaire dich-
heid, en hier zijn nucleonen (neutronen en een klein percentage protonen) niet langer gebonden in atoomkernen. De samenstelling van de kern is nog altijd onzeker: hij zou voornamelijk uit neutronen kunnen bestaan, maar in de binnenste en meest dichte gebieden zouden ook andere componenten kunnen voorkomen, zoals hyperonen of vrije quarks. Het modelleren van de kern van een neutronenster is uitdagend zelfs zonder deze exotische componenten, want de nucleonen bevinden zich naar verwachting in een buitengewone toestand: de grondtoestand. Dit is de toestand met de laagst mogelijke energie, waardoor de nucleonen condensaten vormen. In deze staat kunnen nucleonen maar op beperkte manieren energie uitwisselen met hun omgeving, en verkrijgen ze "super-eigenschappen. De protonen worden supergeleidend, wat betekent dat de elektrische stromen gecreëerd door het bewegen van de protonen vrijwel geen energie verliezen. Een magneetveld kan in een supergeleidend gebied alleen bestaan in vortex-achtige structuren die fluxbundels worden genoemd (type II supergeleiding), of kan zich ophouden in onregelmatige gebieden van gewone protonen (type I supergeleiding). De ongeladen neutronen vormen in de grondtoestand een supervloeistof: een vloeistof met een viscositeit van vrijwel nul, die alleen rotatie kan bevatten door microscopisch kleine vortices te vormen. De supervloeistof in neutronensterren bevatt vermoedelijk vele miljoenen van miljarden vortices. Om de kern heen ligt een schil van materie met dichtheid lager dan de nucleaire dichtheid. Deze regio bestaat naar verwachting gedeeltelijk uit vaste atoomkernen die een kristalstructuur aannemen, en wordt daarom de korst genoemd. Met uitzondering van de buitseste laag, ongeveer een kilometer dik, komen in de korst ook ongebonden neutronen in superfluide toestand voor. Net als de vaste korst van de aarde (die overigens veel minder sterk is) kan de vaste korst van een neutronenster barsten en vervormen in zogenaamde sterbevingen. Tot slot ligt buiten de korst van de neutronenster de magnetosfeer, een uitgestrekte gemagnetiseerde regio die vermoedelijk gevuld is met plasma (elektronen, positronen en ionen).

Neutronensterren zijn "dode" sterren. Dat wil zeggen dat er geen kernfusie plaatsvindt die energie levert voor een grote lichtkracht, zoals bij gewone sterren. In plaats daarvan nemen we straling waar die geproduceerd wordt in specifieke gebieden in de magnetosfeer, en soms thermische straling van het hete stropervlak. Deze thermische straling is vaak niet uniform, wat aangeeft dat de korst warmere en koudere regio's bevat. Dit wordt toegeschreven aan anisotrope warmtegeleiding als gevolg van het sterke inwendige magneetveld. Een van de belangrijkste karakteristieken van neutronensterren is dat ze zeer snel draaien: de snelst draaiende neutronensterren draaien honderden keren per seconde om hun as. Als de emissiegebieden bij elke rotatie onze gezichtslijn kruisen ontvangen wij daardoor straling in de vorm van pulsen, waarbij de frequentie van de pulsen gelijk is aan de rotatiefrequentie van de ster. De meeste neutronensterren worden ontdekt aan de hand van zulke pulsen in radiostra-
ling. Dit leidde tot de naam pulsars (pulsating radio stars), die nu wordt gebruikt voor alle neutronensterren met gepulseerde emissie. Door een serie pulsen waar te nemen kunnen de rotatiefrequentie en de evolutie van deze frequentie worden bepaald. Deze techniek wordt pulsar timing genoemd. Er is een snelgroeiende hoeveelheid pulsar timing data beschikbaar, die belangrijke informatie over de karakteristieken van neutronensterren bevat, aangezien de rotatie van deze sterren wordt beïnvloed door het dynamische gedrag van, en de interactie tussen, de kern, korst en magnetosfeer.

Doordat neutronensterren in de loop der tijd energie verliezen, een klein deel waarvan wij waarnemen als straling, neemt de rotatiesnelheid langzaam af. Deze afname is over het algemeen langzaam en regelmatig, wat het mogelijk maakt om deze afname nauwkeurig te meten, en de rotatiefrequentie nauwkeurig te voorspellen. Sommige neutronensterren, voornamelijk relatief jonge pulsars, gaan echter af en toe opeens sneller draaien. Deze gebeurtenissen staan bekend als glitches, en gebeuren over het algemeen zo snel dat ze niet in de tijd gevolgd kunnen worden, maar in plaats daarvan voorkomen in de data als een plotselinge stap omhoog in de rotatiefrequentie.

Glitches worden naar verwachting veroorzaakt door de overdracht van impulsmoment van de supervloeistof van neutronen aan de rest van de ster. Waar alle geladen delen van de ster sterk met elkaar verbonden zijn door elektromagnetische krachten, kunnen de superfluide neutronen gedeeltelijk ontkoppelen en in sterkere mate draaien dan de rest van de ster. Dit gebeurt doordat de vortices in de supervloeistof vastgepind raken in sommige delen van de kern en de korst, die daardoor meer vortices bevatten dan nodig is om de rotatiefrequentie van de buitenste korst te evenaren. De supervloeistof binnenin de neutronenster kan dus een grotere mate van rotatie bevatten dan verwacht zou worden aan de hand van de waargenomen frequentie. Gedacht wordt dat zo nu en dan een grote hoeveelheid vortices vrij komt uit de vastgepinde regio’s in een lawine-achtige gebeurtenis, waarbij het impulsmoment van deze vortices wordt overgedragen aan de korst. Hierdoor gaat de korst sneller draaien en vindt er een glitch plaats, waarvan de grootte afhangt van het formaat van de superfluide regio en de hoeveelheid vortices in de lawine.

Het is niet altijd makkelijk om een glitch te identificeren in de timing data van een pulsar. Andere soorten onregelmatigheden kunnen verward worden met een glitch, of een echte glitch verhullen. Bovendien is het niet voor alle pulsars mogelijk om de rotatiefrequentie zo vaak te meten als nodig is om glitches te kunnen detecteren. In Hoofdstuk 2 presenteren we detectielimieten voor glitches waarbij deze effecten meegenomen zijn in de analyse (Sectie 2.2). We gebruiken deze limieten, samen met een nieuwe geautomatiseerde methode voor het detecteren en meten van glitch-achtige variaties (Sectie 2.4), om glitches te bestuderen in de pulsar J0534+2200, de beroemde Krabpulsar. Deze bron is de neutronenster in het midden van de Krabnevel, die gevormd is in een supernova die in het jaar 1054 werd waargenomen op aarde.
Krabpulsar ondergaat regelmatig glitches van zeer verschillende groottes, en wordt al 30 jaar bijna dagelijks geobserveerd, zodat het mogelijk is om ook de kleinere glitches goed te bestuderen. De kleinste mogelijke glitches zouden in theorie moeten voortkomen uit een lawine van maar een paar vortices, en zouden daardoor niet te detecteren zijn in timing data. Tot onze verassing vinden wij echter dat vortexlawines uit minimaal enkele miljarden vortices moeten bestaan om de typische glitches in deze pulsar te veroorzaken (Sectie 2.7). Dit resultaat zal helpen om het activatiemechanisme van het gezamenlijk vrijkomen van vortices te begrijpen, een van de belangrijkste open vragen rondom het glitch-fenomeen.

De rotatie van pulsars na een glitch hangt af van hoe de verschillende delen van de ster reageren op de versnelde rotatie. De geladen deeltjes in de kern en de magnetosfeer zullen de versnellende korst snel volgen, maar de supervloeistof van neutronen is minder sterk gekoppeld aan de rest van de ster en reageert langzamer, op een tijdschaal die kan oplopen tot enkele tientallen jaren. Doordat de glitch het snelheidsverschil tussen de supervloeistof en de rest van de ster kleiner maakt, kan de supervloeistof ontkoppelen van de andere delen van de ster. De afremming van de neutronenster na de glitch verloopt dan sneller, omdat het totale impulsmoment dat afgeremd wordt kleiner is. Dit leidt er meestal toe dat de rotatiefrequentie ten minste gedeeltelijk hersteld naar de waarde die voorspeld zou zijn op basis van waarnemingen van voor de glitch. Waarnemingen van deze post-glitch relaxatie kunnen gebruikt worden om informatie te verkrijgen over de eigenschappen van de supervloeistof van neutronen en de stercomponenten waar deze mee in interactie treedt. Theoretische modellen van de post-glitch relaxatie zijn daarbij van grote waarde.

Niet alle glitches lijken op dezelfde manier te herstellen, zelfs binnen een en dezelfde pulsar. Soms verandert de mate van afremming van de ster helemaal niet (step-like glitches), of neemt deze met minder dan één procent toe. In andere gevallen neemt de afremming toe door de glitch. Dit wordt vaak gezien in magnetars of na de zeer grote giant glitches. Meestal verdwijnt deze toename in de mate van afremming geheel of gedeeltelijk, op een tijdschaal die kan variëren van een paar uur tot tientallen jaren. Over het algemeen gebeurt dit exponentieel. Een veelbelovende manier om glitches te bestuderen is met behulp van theoretische modellen waarbij de neutronenster wordt beschreven als een hydrodynamisch twee-vloeistoïdensysteem, gericht op het beschrijven van de gekoppelde dynamica van de supervloeistof en de rest van de ster. Er is aangetoond dat zo’n model, gecombineerd met realistische microfysische berekeningen, de exponentiële relaxatie kan verklaren die gezien wordt na giant glitches (Haskell et al. 2012). In Hoofdstuk 2 gebruiken wij dit model om de dynamica van de supervloeistof verder te bestuderen. We laten zien dat onze aanpak een natuurlijke verklaring levert voor het sterke herstel dat vaak waargenomen wordt bij magnetars, alsmede voor veel andere post-glitch karakteristieken. Bovendien be-
kijken we subtielere gebeurtenissen dan een catastrofaal vrijkomen van vortices, en laten we zien dat hiermee step-like glitches verklaard kunnen worden, en dat deze modellen kunnen leiden tot ongebruikelijke vormen van post-glitch herstel, en een grote diversiteit in rotatiegedrag in het algemeen.

De reactie van de supervloestoef op een andere perturbatie, zoals een plotseling stijging van de temperatuur of een snelle verandering van het magnetoveld, is vermoedelijk verbonden met andere dynamische gebeurtenissen in het leven van de neutronenster, zoals bijvoorbeeld sterbevingen, of de uitbarstingen van hoge-energie straling (bursts) die typisch zijn voor magnetars. Deze magnetosferische bursts verkrijgen hun energie uit het sterke magnetoveld van een magnetar, en vallen vaak samen met een glitch. De evolutie van de rotatie van een magnetar wordt vervolgens mogelijk ook door andere inwendige of magnetosferische factoren beïnvloed, en is vaak onregelmatig. In tegenstelling tot wat wordt waargenomen bij magnetars, werd van gewone radiopulsars lang gedacht dat er niets gebeurde in de magnetosfeer tijdens een glitch, omdat er geen veranderingen in de radio- emissie gelijktijdig met de glitch werden waargenomen. Dergelijke veranderingen werden in 2007 echter wel gevonden na een grote glitch in de radiopulsar J1119-6127, vergezeld van een atypische afname van de afremming van de pulsar op latere tijdstippen. Deze bijzondere interessante bron is de focus van Hoofdstuk 4, waarin we een grondige analyse presenteren van de timing data van deze pulsar verzameld in de laatste zestien jaar. Hier introduceren we tevens een nieuwe methode om de langetermijnafremming van de neutronenster te bestuderen (Sectie 4.2.2). Onze studie toont een zeer ongebruikelijke post-glitch relaxatie, en maakt het mogelijk om de fysische mechanismen achter de glitches van pulsar J1119-6127 in detail te bestuderen (Sectie 4.3). Omdat deze neutronenster net als magnetars zeer sterk magnetisch is leveren deze observaties extra bewijs dat het sterke magnetoveld de glitches mogelijk verbindt aan magnetosferische fenomenen.

Waar dit soort case-studies van ongebruikelijk post-glitch gedrag kunnen helpen om minder belichte aspecten van het fenomeen glitches te verkennen, kunnen case-studies van typische glitches zorgen voor betere limieten op de theorieën over het glitchmechanisme, en de reactie van de supervloestoef in het algemeen. De precieze details van hoe vortices bewegen, zowel in gebieden waar ze vrij zijn als waar ze vast zitten, hangen af van onbekende microfysica. Door theoretische voorspellingen te vergelijken met observaties van veelvoorkomende post-glitch relaxaties kunnen we hier meer inzicht in krijgen. De Velapulsar is hier een geschikte bron voor, omdat deze pulsar regelmatig goed waarneembare giant glitches vertoond, waargenomen in radio-observaties met een hoge tijdsresolutie. In Hoofdstuk 5 analyseren we drie maanden aan dergelijke observaties volgend op de grootste bekende glitch van deze pulsar. We modelleren de post-glitch relaxatie aan de hand van verschillende theore-
tische voorspellingen voor het gedrag van de supervloeistof, voor verwachte waarden van de sterkte van de interactie tussen de supervloeistof en de geladen delen van de ster. Onze resultaten geven een sterke aanwijzing dat het grootste deel van de supervloeistof in de korst, en tenminste een deel van de supervloeistof in de kern, ontkoppeld tijdens de glitch en bijdroeg aan de waargenomen relaxatie. Onze resultaten laten echter zien dat de kern minder effectief is in het vastpinnen van vortices dan sommige theoretische modellen voor type II supergeleidende protonen voorspellen. Dit is een waardevol gegeven voor verdere microfysische studies van het bewegen en vastgepind worden van vortices.
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