The Many Phases of Gamma-Ray Burst Afterglows

ACADEMISCH PROEFSCHRIJT

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'πάντων χρημάτων μέτρον ἐστιν ἄνθρωπος'
– Πρωταγόρας
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Introduction

Gamma-ray bursts (GRBs) are the most powerful sources of electromagnetic (EM) radiation in the universe. They are the result of physical processes that convert huge amounts of energy (an appreciable fraction of the rest-mass energy of the Sun) into $\gamma$-ray photons within very short periods of time. We observe a few of them per week with durations that span a range of several orders of magnitude centered around a few seconds. Typically occurring at cosmological distances, GRBs are very rare events and, as far as we know, not recurrent.

The extremity characterising all physical quantities we can measure (photon energy, duration, luminosity, total energy output) implies that the observed radiation reveals some very unusual places in the universe. Never resolved and even poorly localized in many cases, sources of GRBs can only be understood via reverse engineering from the point-source observations we have. We now believe that the $\gamma$ rays originate from an outflow of (in general) magneto-hydrodynamic nature. This outflow is itself a product of a catastrophic event that has led to the formation of a black hole (BH) or a magnetar. Rapid accretion onto the newly formed compact object causes the ejection of mass and energy at relativistic velocities. Part of the copious amounts of kinetic energy available is converted to the $\gamma$ rays we observe while some of it energizes the surrounding medium that in turn produces the afterglow. GRB afterglows are observed at longer wavelengths, typically for a much longer time than the prompt ($\gamma$-ray) emission. Their discovery and subsequent study has provided a wealth of information contributing significantly towards understanding the GRB phenomenon.

Owing largely to their extreme nature and manifestation, GRBs are interesting to study in their own right. However, they can also be viewed as probes and tools. As is many times the case in astrophysics, by studying GRBs we are given a chance to take a look at an experiment of physical conditions far surpassing our own capabil-
itories. This experiment includes: the birth of a black hole – the most gravitationally profound object in the universe, the launching of some of the fastest outflows seen across all astrophysical sources, and the formation of shocks – the most probable site of particle acceleration, where the highest-energy cosmic rays are produced. Apart from ‘labs’, GRBs also act as ‘flashlights’ illuminating otherwise dark areas of the universe throughout its evolution. In that way they enable us to catch glimpses of the physical conditions describing the interstellar and intergalactic medium at various epochs after the big bang.

In the remainder of this Chapter I will highlight important aspects of GRBs and analyse key points relevant to the Chapters that follow. In the following Section I present a short historical background of GRB observations. I proceed then in Section 1.2 to analyse basic features of the leading theoretical model accounting for the whole range of available observations. In Section 1.3 I discuss advances that have shaped our current understanding of GRB progenitors. In Section 1.4 I turn to the currently most important research topics of the field. In Section 1.5 I present a simple derivation of synchrotron spectra from relativistic blast waves in order to highlight the important physical parameters that we will be concerned with in the remaining Chapters. Finally, in Section 1.6 I link specific research topics to the work I have carried out during my PhD, introducing in that way the aims and results of the Chapters that follow.

1.1 Observational timeline

GRBs were discovered in the 1960s by the U.S. Air Force Vela satellites, which were flown to monitor compliance with the Nuclear Test Ban Treaty. It took several years for the announcement of the scientific discovery (Klebesadel et al., 1973), during which it was established that neither the Earth nor the Sun could be the sources of those mysterious bursts, albeit with limited directional constraints. For 30 years γ-rays remained the only source of information on GRBs. However, as observations of BATSE onboard the Compton Gamma-ray Observatory (launched in 1991) piled up it became clear that the sources’ distribution was uniform and inhomogeneous (Fishman & Meegan, 1995), strongly suggesting a cosmological origin. This picture was soon confirmed (Metzger et al., 1997) by the measurement of redshifts placing GRBs gigaparsecs away from Earth.

Redshift measurements of GRBs were facilitated by a major observational breakthrough; the discovery of X-ray, optical and radio counterparts to the γ rays (Costa et al., 1997; Groot et al., 1997; Frail et al., 1997). This was possible mainly thanks to the Beppo-SAX satellite (launched in 1996) which, within hours after a burst, provided location of the source that resulted in follow-up observations from ground-
1.1 Observational timeline

Figure 1.1: Early sample of GRB light curves (Mészáros, 2006). Diverse complexity, durations and profiles are evident. Variability can reach the level of milliseconds implying a very compact source.

based instruments. This longer-wavelength radiation (dubbed afterglow) led to more accurate localization of GRBs on the sky, something which allowed for the identification and study of their host galaxies (Bloom et al., 1998). Furthermore, by opening observation windows in other wavelengths the analysis of afterglows has greatly broadened our understanding of GRB events. This has mainly come in the form of support for the fireball model (see next Section) which has been enjoying great popularity in the community for the last 15 years, or so. The localization and follow-up observations of afterglows in the late 1990s also led to the discovery of associated su-
1 Introduction

pernova explosions, both in nearby and distant sources (Galama et al., 1998; Bloom et al., 1999; Hjorth et al., 2003). These associations have strongly influenced the progenitor models of GRBs by linking them to the death of massive stars.

The launch of Swift in 2004 provided some of the missing pieces of the puzzle. As its name suggests it can slew quickly once it detects a burst and observe at X rays, UV and optical wavelengths. By doing so it has unveiled the phase between the tail of the $\gamma$-ray emission and the beginning of the afterglow, at timescales of minutes to hours. During that phase, mainly X-ray but also optical light curves display a rather complex behaviour, (Nousek et al., 2006) whose interpretation should enrich our knowledge beyond the standard fireball model. Swift’s sensitive gamma-ray detector has recorder bursts at $z > 8$ (Tanvir et al., 2009) but has also found a significant underluminous population of bursts, revealing the broadness of the GRB energy distribution.

One of Swift’s primary objectives has been to study further the different GRB populations. By the beginning of the 1990s (Kouveliotou et al., 1993) there was already evidence of a bimodal distribution in the population of GRBs in terms of duration and spectral hardness. Long/soft GRBs had been most common, with their subsequent afterglow detections placing them inside star-forming galaxies. Short/hard bursts, on the other hand, were not adequately localized as no afterglow had been observed. This has changed since 2005 when the first afterglow from a short GRB was observed (Gehrels et al., 2005). Afterglow observations of short GRBs have proven that they are also at cosmological distances and have shed light in the nature of their progenitors.

The year 2008 saw the launch of the Fermi Gamma-ray Space Telescope, equipped with sensitive detectors over a broad range of photon energies, but most importantly covering four orders of magnitude in $\gamma$ rays (McEnery et al., 2012). Fermi has contributed significantly to many areas of research and gamma-ray astronomy in particular (see, for example, Abdo et al. 2009 for the discovery of several pulsars through their $\gamma$-ray emission). In the field of GRBs it has unveiled the high-energy behaviour of the prompt emission. This has been found to lag the low-energy $\gamma$ rays and last longer (Zhang et al., 2011). The overall spectra display various components including some of thermal origin (Guiriec et al., 2011). This inference has accompanied theoretical developments towards a more thorough understanding of the exact mechanisms that shape the observed bursts of $\gamma$ rays.

These have been the pivotal observations in the field of GRBs. In the following Sections I present how their interpretation has resulted in refining our views on these objects.
1.2 Fireballs in space

By the end of the previous century ever-growing amounts of data along with progress on the theoretical side strongly favoured the fireball shock scenario (Cavallo & Rees, 1978; Paczyński, 1986; Goodman, 1986; Rees & Mészáros, 1992, 1994; Sari & Piran, 1997b) as a general framework that provided adequate description of the data. What this model essentially does is to suggest a series of processes for energy conversion that can accommodate the observations. The general picture is the following: a violent event deposits a large amount of energy (a fraction of a solar rest mass) in a small volume leading to the formation of an expanding fireball. A small fraction of the initial budget will be converted to kinetic energy of the outflow. Once the outflow has become transparent, heating of the particles (due to internal shocks, magnetic dissipation or even particle collisions) will result in the efficient release of some of the available kinetic energy in the form of $\gamma$ rays. Meanwhile, part of the remaining kinetic energy will slowly generate a blast wave that shocks and heats matter from the circumburst medium (CBM). The shocked CBM will then radiate at progressively longer wavelengths (afterglow) until the flux levels get buried in the background noise. Below, I analyse critical aspects of the fireball scenario.
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1.2.1 Relativistic sources

One of the major consequences of revealing the cosmological distances associated with GRBs was the implied total energy release, if isotropically emitted, which in some cases is equivalent to a solar rest mass in γ rays alone. This energy crisis can be somewhat resolved under the assumption that the emission is not isotropic; we will come back to this very important point later on. But even then, the combination of fast variability displayed in the light curves (see Fig. 1.1) and total amount of released energy implies a compact region with energy many times higher than that corresponding to its rest mass. The outcome of this state will be an expanding fireball that reaches relativistic speeds, by converting (at least in original versions of the model) heat to kinetic energy through pressure.

In fact, relativistic motion can be inferred for the source just by the simple fact that we observe high-energy (> 511 keV) photons which would otherwise fail to escape the compact region of emission due to pair creation. This is what is known as the “compactness problem”, the resolution of which demands the source of γ rays to be moving at high Lorentz factors (Γ > 100) towards the observer (Schmidt, 1978; Krolik & Pier, 1991; Lithwick & Sari, 2001). All GRB outflows that have been analysed according to these limitations have been found to be ultrarelativistic with Lorentz factors sometimes greater than 1000.

1.2.2 Prompt emission

During the 1990s it was realized that external shocks (arising due to the interaction of the outflow with the CBM) could not produce the observed variability of GRB light curves (Rees & Mészáros, 1994; Sari & Piran, 1997b). Instead, shocks between different parts of the outflow are expected to form naturally if the central engine operates unsteadily. This realization gave rise to the internal-external fireball shock model according to which internal shocks (within the outflow) are responsible for the acceleration of particles that produce the prompt emission, while external shocks (from the ‘collision’ of the outflow with the CBM) are the energy source of the afterglow radiation.

The radiation mechanism of the prompt emission was initially assumed to be synchrotron, mainly due to the fact that observed power-law spectra (Band et al., 1993) suggested a non-thermal mechanism, but also because of the widespread occurrence of synchrotron radiation in similar astrophysical sources (AGN, X-ray binaries, supernovae etc.). However, thermal signatures have been found in spectra, in the past decade (e.g. Ryde 2005), possibly revealing the long hidden photosphere of these objects. Deviations from a pure power-law spectrum have only reinforced attempts to investigate further the types of processes that could result in efficient heating of
particles at radii relevant to the $\gamma$-ray emission (Giannios, 2006; Beloborodov, 2010) or even reconsider processed photospheric emission (Pe'er & Ryde, 2011). Almost half a century after their discovery, debate over the dominant physical processes of GRB prompt emission continues as intense as it ever was.

1.2.3 Afterglow

Cosmological models for GRBs predict naturally a signal from the interaction of relativistic outflows with the CBM (e.g. Rees & Mészáros 1992). In fact, detailed calculations of spectra and light curves from this interaction proceeded observations of afterglow radiation itself (Mészáros & Rees, 1997). Its subsequent discovery not only confirmed the cosmological distances of GRBs, but it also provided support to the fireball model. This support has been implicit, but consistent. It is based on successful modelling of the observations as radiation from decelerating relativistic blast waves. An important ingredient of the modelling has been the work by Blandford & McKee (1976) who derived a self-similar solution describing the dynamics of such a spherical blast wave. On the radiation side, the observed power-law spectra of afterglows reveal a non-thermal origin. Varying degrees of polarization have been measured (van Paradijs et al., 2000), strongly hinting at synchrotron as the dominant radiative process.

Advances in sensitivity and response time of space- and ground telescopes have been instrumental in collecting a wealth of observational data sets from numerous afterglows. These have been studied extensively to uncover details of their behaviour and to extract physical parameters through modelling. Afterglows are especially handy for such an undertaking. The reason is that the details (duration, irregularity and so on) of the central engine do not affect the manifestation of afterglows after some point (typically hours to days after trigger, in the observer frame). This fact, combined with multiwavelength coverage, poses rigorous constraints on physical quantities that can be deduced from the data through modelling. These quantities are: kinetic energy of the blast wave, CBM density, as well as microphysics parameters that regulate energy conversion at the shock.

One of the most important physical quantities, with repercussions on many aspects of GRB theory (energetics, rate, dynamics), is the opening angle of jets. The simple scenario of spherical outflow raises concerns regarding the total energy of GRB explosions as well as the inferred efficiencies of the processes that are responsible for converting part of this energy into observed radiation. Beyond easing the energy-budget issue, jets have also been observationally deduced from simultaneous achromatic breaks in the light curves of some afterglows (e.g. Harrison et al. 1999). Such breaks occur when the bulk Lorentz factor of the outflow $\Gamma_{\text{bulk}} \sim 1/\theta_j$ ($\theta_j$ is the semi-opening angle of the jet) and the entire volume of the jet becomes causally con-
Figure 1.3: One of the very first efforts to constrain physical parameters by comparing predictions of the fireball model to afterglow data (Wijers et al., 1997). Even for the first afterglow discovered (GRB 970228), once the source was localized observations at 6 different frequencies were possible. Despite the simplicity of the models, early efforts to fit observations were relatively successful and provided crucial feedback on theoretical developments. Within a short period of time a few bursts' afterglow fits had considerably reinforced the foundations of the fireball model.

connected. Working out the details of the dynamics (and the exact effects on the spectra) after the jet break, has been the subject of intense study (Rhoads, 1999; Granot et al., 2001; Kumar & Granot, 2003; Zhang & MacFadyen, 2009; van Eerten et al., 2011). These details are important to interpret part of the observations but also to understand the manner in which the outflow is transitioning towards non-relativistic velocities. For that stage another self-similar description of the dynamics is available (Sedov, 1959) and as observations of afterglows can be continuous over, in some cases, the
order of years (e.g. GRB 970508, GRB 030329 etc.) it is crucial to devise a formalism in which all these different phases are smoothly connected. Achieving that will result in a robust theoretical model based on which issues like jet opening angles, and consequently energetics, rate etc., should be resolved.

1.3 GRB progenitors

I have so far refrained from addressing one of the core questions of GRB research: What causes these events? The answer is neither short, nor fully known. That is one reason for postponing this discussion, the other being the relatively complex and ongoing evolution of the subfield, which depends heavily on many and accurate observations and meticulous analysis. In this Section I will present the most popular models and review how progress made in the past decades has resulted in the current picture.

For about 20 years after their discovery, while the distance to GRBs remained unknown, a plethora of progenitor models had been proposed. Only as evidence for cosmological distances started piling up did the sample reduce to a handful of plausible scenarios. By the early 1990s the classification of GRBs into long/soft and short/hard subtypes did not necessarily imply at the time a diversity in their respective progenitors, as other properties (like isotropic distribution) were similar (Kouveliotou et al., 1993). However, as afterglow observations became possible, the study of their progenitors entered a new era. We now have substantial evidence pointing towards two separate cases of events, roughly separated by duration, but with comparable γ-ray luminosity. An important advancement, that afterglow detections allowed for, has been the better localization of the source, which has in many cases (in some others crucially not) resulted in associations with host galaxies. Study of those galaxies has revealed clues that have helped us discern between different hypotheses.

1.3.1 Long bursts

While the possibility of GRBs being galactic sources was still considered, a popular scenario in the literature (Paczyński, 1986; Goodman, 1986; Eichler et al., 1989), applicable in the case of extragalactic origin, was that of merging neutron stars (NS). This idea gained even further support (Narayan et al., 1992) as it became clearer that GRBs lie at cosmological distances. At the same time a “failed supernova” (now known as collapsar) was suggested (Woosley, 1993) as another possible progenitor, in particular for long GRBs (LGRBs). This postulate suggests a link between SNe and GRBs.

This latter scenario is by now considered almost a certainty for LGRBs and that is based on two different kinds of evidence. The first one is of statistical nature
and derives from the strong correlation of LGRBs with star-forming galaxies (van Paradijs et al., 2000). The second, and perhaps more convincing, is the connection of some LGRBs with Ic SNe, deduced from sky positions (Galama et al., 1998), light curves (Bloom et al., 1999), or spectra (Hjorth et al., 2003; Stanek et al., 2003). Evidence for further GRB-SNe associations have been found since (e.g. Cobb et al. 2010; Levesque et al. 2012), establishing a solid link between the two.

In the collapsar model, the iron core of a very massive rotating star is collapsing towards the end of the star’s life, similarly to the standard SN scenario. The main difference lies in the accretion of a disk/torus, of order stellar mass, that has not been expelled in what would otherwise have been a successful SN explosion (hence the original name “failed supernova”). The details of energy release are debatable (neutrino transport is one possibility; MacFadyen & Woosley 1999, while magnetic extraction is another; Barkov & Komissarov 2008). In any case, the burst’s duration is linked to the accretion of that disk onto the central object and high angular momentum of the core is a necessary ingredient of the process. Interestingly, there is an association of LGRBs with low-metallicity host galaxies (e.g. Levesque et al. 2010). Metals (in Astrophysics lingo, “everything that’s not H or He”) are crucial in efficient angular-momentum loss of the stars by driving massive stellar winds. Therefore, their relative absence from GRB hosts strengthens the collapsar scenario by providing observational evidence for the high rotation rates necessary in the model.

The connection between LGRBs and SN also poses questions regarding the duality of massive-star explosions. Strong detection limits confirm the absence of a SN counterpart in nearby GRBs (Fynbo et al., 2006) demonstrating that the two phenomena do not always occur together. It may well be that a wide range of possibilities between one and/or the other exist, with the final outcome depending on the exact situation. Regardless, establishment of the GRB-SN connection has been a major step in comprehending the origin and cause of those, until recently, enigmatic objects.

1.3.2 Short bursts

The story is quite different when it comes to short GRBs (SGRBs). Their γ-ray behaviour displays four distinct differences from long bursts. They are (obviously) of shorter duration, with harder spectra, occurring less frequently and releasing less energy in total. Another distinctive feature was the failure to detect an afterglow of a SGRB during the Beppo-SAX era. Supported by the fact that the collapsar model naturally predicts durations of order tens of seconds, it seemed that SGRBs may actually be quite different events with progenitors separate from those of the long class.

Actually, the NS-NS merger scenario is more suitable to bursts of duration less than a second and thus remained popular as a progenitor model for SGRBs even after
1.4 Open questions

The establishment of the LGRB-SN connection. There is now inconclusive evidence that this may indeed be the case. This is backed up by afterglow observations (or the lack thereof) of host galaxies harbouring, or associated with, the GRB sources. The first afterglow detections of SGRBs revealed elliptical galaxies with substantially less star formation than the LGRB hosts (Bloom et al., 2006; Gorosabel et al., 2006). By now, there are about 20 SGRB afterglows in the literature (notably less and poorer than for LGRBs), 25% of which lies hostless (Berger, 2010). For the rest, a range of host-galaxy properties has been noted, although with on average significantly less star formation than those of the long class. The offsets they display from the centers of their hosts are considerably larger as well (Fong et al., 2010). All these properties fit nicely into the merger framework.

When two compact objects merge, a black hole is expected to be the outcome. Numerical simulations have shown (Aloy et al., 2005) that accretion of a torus (fraction of the total mass) onto the central BH can result in relativistic collimated knotty outflows. Due to their structure and velocity these outflows are capable of producing SGRBs on the timescales implied by the observations. A compact-object binary is the end phase in the life of a binary system consisting of two massive stars. Large distances from the sites of star formation are expected for those systems. This is due to the kicks the binary experiences during the two SN explosions that the constituent stars undergo. Combined with lifetimes of the order hundreds of Myr (Faber & Rasio, 2012), compact binaries have plenty of time and speed to cross big chunks of their host galaxies, or even completely abandon them before they merge. This expectation compares nicely with the large offsets of SGRBs and even accounts for the hostless population. Outside their formation sites, if not outside the host galaxies, SGRB afterglows are then understandably underluminous as a result of the low CBM densities as well as the intrinsically lower explosion energies. Despite the progress, important questions remain regarding the nature of these objects. A detection of a gravitational wave (GW) signal, characteristic of a compact-binary inspiral, coincident with a SGRB would be the ‘smoking gun’ conclusively proving their connection.

1.4 Open questions

In the previous Sections I have presented an overview of observational and theoretical progress in GRB research. In this Section I turn to discuss issues for which we have limited understanding. These issues cover, more or less, all areas of study (and often more than one at a time), ranging from progenitor types to radiation mechanisms and outflow dynamics.
1 Introduction

Figure 1.4: Histogram of Swift GRB durations along with a fit representing three different populations (Horváth et al., 2008). Inclusion of an intermediate class significantly (99.54%) improves the description of the data, if the distributions are Gaussian. T90 (the time interval during which 90% of the total observed γ-ray flux is obtained) for the intermediate class is centered around 10 s.

1.4.1 Progenitor models and non-EM information carriers

Due to the cosmological distances of GRBs it is clear that we cannot hope to directly observe the objects and processes responsible for their emergence. Instead, we can expect that continuous observations and refinement of the theoretical models will eventually lead to a more solid picture. Evidence is now strong for the connection of LGRBs with SNe and more tentative for the case of SGRBs, for which NS-NS/BH mergers seem to be a favourable proposition.

Things, however, seem to be slightly more complicated than that. It has become clear in recent years that the long-soft dichotomy cannot account for all bursts we observe (Zhang, 2006). Namely there have been many events, with overall characteristics that are at odds with the duration of the burst (Kann et al., 2011). Thus, it is evident that duration is not an absolute criterion for the classification of GRBs. The picture has been further complicated by the statistical inference of a third, intermediate class of GRBs (Horváth et al., 2008). This class is found to comprise about a third of all Swift GRBs. Their typical duration is at the low end of the LGRB distribution and they are often related to LGRB progenitors (de Ugarte Postigo et al., 2011). Regardless of the statistics, the diversity, and in some cases ambiguity, of
GRBs suggests that a generalisation of the current models is needed.

SGRB study has long suffered from low-number statistics, in part due to their lower rate and less energetic engines, but mostly because of their underluminous afterglows. Expected accumulation of more observations should already shed light in their behaviour, confirming or disproving our current ideas. The long-awaited detection of a strong GW signal might provide the first unambiguous link between SGRBs and merging compact objects and might even allow us to distinguish between a NS-NS and a NS-BH scenario. Promising EM counterparts to a compact merger have been recently discussed in the literature (Nakar & Piran, 2011; Metzger & Berger, 2012) and although a $\gamma$-ray detection (SGRB) is crucial, additional possibilities (optical/radio afterglows, kilonova signals) are considered and observational strategies to utilise them have been proposed.

Beyond GWs, there are more types of non-EM signals expected from GRBs. Immediate or secondary products of particle acceleration, both cosmic rays and high-energy neutrinos are expected to originate from strong internal shocks, widely believed to occur at GRB outflows (Waxman & Bahcall, 2000; Dermer & Holmes, 2005). So far searches, however, have only yielded upper limits on their fluxes (Abbasi et al., 2012). The, so far, non-detection of GRBs in these channels may be suggestive of modifications to the standard fireball model. For example, a magnetically dominated outflow would tend to involve fewer baryons and thus accelerate
considerably fewer cosmic rays (some of which will eventually decay to neutrinos) than what would be expected in the 'baryonic jet' scenario. Evidence in favour of the launch of Poynting outflows from GRB central engines has been found both on an observational (Coburn & Boggs, 2003), and (with less dispute) a theoretical level (Drenkhahn & Spruit, 2002; Vlahakis & Königl, 2003; Komissarov et al., 2009). An alternative explanation, viable in light of recent prompt-emission observations, would be that internal shocks are not responsible for the $\gamma$-ray emission and do not operate to the extent believed, something that would displace GRBs from the candidates’ list for ultra-high energy cosmic-ray acceleration. External shocks (the existence of which can be considered confirmed) would need to operate in a highly magnetized medium for efficient acceleration of cosmic rays to be achievable (Vietri et al., 2003).

1.4.2 Physics of prompt emission

The origin of prompt-emission spectra is now the subject of intense debate (Ryde & Pe’er, 2009; Willingale et al., 2010; Zhang et al., 2011), which in part relates to the old ‘synchrotron line of death’ problem (Preece et al., 1998). The latter expression was coined to describe observed power-law spectra with slopes different than the predictions of standard synchrotron theory (Rybicki & Lightman, 1986). In practice, since the early 1990s a spectral function (Band et al., 1993) representing a broken power law has been (and is still being) used to fit the observations. This function, however, lacks theoretical foundations; it is merely a heuristic description. Moreover, in some cases it needs to be supplemented with an additional high-energy component (Abdo et al., 2009) to provide a good description of the spectra. In a broader context, if we assume that the prompt emission originates from shock-accelerated particles, the efficiency implied for energy conversion at the internal shocks is often close to unity (Zhang et al., 2007), something improbable within the current framework (e.g. Daigne & Mochkovitch 1998).

The mechanisms of energy conversion and radiation are physically related and can only be tackled simultaneously. In recent years efforts have been focused to introduce new models where thermal emission (either photospheric or due to energy dissipation at larger radii) is an integral part of the observed spectra (Giannios, 2006; Beloborodov, 2010; Pe’er et al., 2012). These studies explore different physical processes that result in a thermal spectrum, modified due to multiple-temperature plasma or inverse-Compton scattering. Fermi-type acceleration of particles is not excluded (some of the proposed models are largely hybrid) but the efficiency crisis implied by the operation of internal shocks is largely avoided. The wide spectral range of the instruments aboard Fermi will undoubtably aid the effort of distinguishing between the proposed mechanisms in the near future.
1.4.3 Afterglow physics

Contrary to the prompt emission, GRB afterglows are relatively well-behaved. There is extensive evidence for the presence of relativistic external shocks which accelerate particles to power-law energy distributions that map onto the observed spectra through the synchrotron mechanism. This well-founded understanding has allowed, in some cases, for accurate predictions and interpretation of the data. Moreover, due to the robustness of the current framework, it is easier to expand it by adding extra pieces accounting for additional physical processes. More often than not, this extra physics has deeply-rooted consequences on the overall properties of the GRB phenomenon. In this way, afterglows have so far proven instrumental in providing breakthrough observations of GRBs and retain the same outlook for the future. In what follows I present observational properties which are manifested in the afterglow but concern important aspects of GRBs as a whole.

Early afterglow behaviour

Typical timescales for the beginning of afterglow observations in the Beppo-SAX era were of the order of several hours, at best. These observations often displayed the canonical afterglow behaviour expected in the simplest fireball scenario. The rapid response of Swift, however, has enriched our knowledge by revealing the behaviour at timescales of the order of minutes. In Fig. 1.6 all potential phases of an early X-ray light curve are presented, according to Swift-XRT data. A range of possibilities exists.

Phase 1 (steep decay) is present in almost all bursts and is typically attributed to high-latitude prompt emission (Zhang et al., 2006) which due to the finiteness of the speed of light reaches the observer at later times. This phase is regarded to represent the transition from the main burst to the afterglow and is considered a validation of the different origin of the two.

Phase 2 (shallow decay or plateau) is present in an appreciable sample of Swift bursts, often with coupled optical light curves (Panaitescu & Vestrand, 2011). The slopes observed during this phase are not reconcilable with the simple fireball-model predictions. Instead, they are commonly attributed to a long-lasting central engine which continuously supplies the blast wave with energy, affecting in this way the dynamics, and hence, the temporal profile of the emission.

Phase 3 represents the canonical afterglow decay, as deduced from theory and confirmed by most Beppo-SAX observations. Temporal indices often seem to indicate a flat distribution of the CBM density (Zhang et al., 2006). On the face of it, this is at odds with the expectations for a massive-star progenitor, the stellar wind of which would be expected to have shaped the environment around it.
1 Introduction

−α
−α
−α

Figure 1.6: Synthetic X-ray light curve of GRB afterglows in the Swift era, at early (minutes to days) observer times (Nousek et al., 2006). Three segments, connected by breaks, are possible. The temporal behaviour of the light curves in individual segments has been inconclusively linked to various physical mechanisms, like high-latitude emission (1), energy injection (2) and canonical afterglow decay (3).

On top of this ‘standard’ picture there can be flares at any phase. In fact about half of all Swift afterglows display some flaring activity (O’Brien et al., 2006). Segment 2, in particular, is often accompanied by flaring events, during which spectra are similar to those of phase 1 (Zhang et al., 2006). A common interpretation of these flares is that they originate from late central-engine activity.

The details of the complex, but almost universal overall behaviour are still to be worked out. However, a few qualitative conclusions from early-afterglow light curves can already be drawn. First, the prompt emission seems to have a different origin than the afterglow. This confirms the internal-external shock scenario, despite the fact that internal shocks have been challenged as an energy conversion mechanism, mainly on efficiency grounds. Second, it is clear that the central engine’s operation can be long-lived, episodic and in some cases delayed, leading to shallow light curves and flares. This demonstrates that instantaneous explosion models are an over-simplification and calls for a more accurate treatment of the dynamics of GRB outflows.

Jets and late-time behaviour

About 10% of Swift’s X-ray afterglow light curves show clear signs of a steepening at approximately 1 day, with some dispersion around the central value. This behaviour is consistent with radiation from a laterally expanding blast wave, i.e. emission after
the jet break. The sample, however, of bursts that display such behaviour is unexpectedly small. Before Swift, Beppo-SAX had identified about 20 cases of achromatic jet breaks (Mészáros, 2006), strongly pointing towards jets being a universal feature of GRB outflows. Expectations were that Swift would confirm this picture and even reinforce it by detecting jet breaks occurring earlier on, implying narrower jets. In light of the previous discussion on early afterglow behaviour, Swift did discover breaks at early times, and a small fraction of them may indeed be jet breaks (Racusin et al., 2009). However, a notable trend in the majority of Swift X-ray afterglow observations is the absence of jet breaks, an issue that has raised a few eyebrows, to say the least.

This discovery is uncomfortable, more for the diversity in outflows’ geometry that it suggests, rather than the energy crisis implied by some bursts if isotropic. The reason is that, as a sample, Swift GRBs are underluminous, and the events that show no jet break especially so (Racusin et al., 2009). Even if spherical, the total emitted energy in $\gamma$-rays can be as low as $10^{48}$−$10^{49}$ erg. However, a closer look at jet-break mechanisms is necessary. There is evidence that jet breaks’ characteristics change when viewed off-axis (van Eerten et al., 2010) and even that jet breaks are actually expected to be chromatic (van Eerten et al., 2011). What this means is that we may need to upgrade the quality of the observational criteria used in order to characterise the presence of jet breaks, in the near future.

Methodology set aside, it seems quite probable by now that there is a range of possibilities for the geometry of GRB outflows. Whether that range includes spherical outflows is unknown, but should not come as a big surprise if it turns out to be the case. SGRB afterglows are too few to provide good statistics, with only a couple claimed to contain a jet break, but if we were to use the findings to date we would infer larger opening angles than in the LGRB sample (Panaitescu, 2006; Grupe et al., 2006; Burrows et al., 2006). As has been stressed before in this Chapter, the implications of beaming can be significant on fundamental physical aspects that characterise the GRB phenomenon, like energy release, progenitor type and rates. And it is perhaps these aspects that we most eagerly want to know.

1.5 Synchrotron radiation from relativistic blast waves

In all the Chapters that follow we will be concerned with studying various phases of the afterglow. Specifically, we will be focusing on synchrotron radiation originating from relativistic (in general) blast waves. Therefore, it is worthwhile examining a simple version of the problem to introduce the important physical parameters and the methods that will be commonly used in the following Chapters.
1.5.1 Blast-wave dynamics

Let us consider a sudden release of energy within a small volume, embedded in an environment of constant density. As a result of pressure gradient, the volume will expand and if the expansion velocity is higher than the local speed of sound, this will generate a shock that propagates radially within the CBM. If the shock is relativistic (bulk velocity \( \sim c \)) and strong (high pressure difference between shocked and unshocked regions) the energy density and number density of the shocked CBM gas will be (Blandford & McKee, 1976)

\[
\begin{align*}
e_2 &= 4 \gamma^2 \rho_1 c^2, \\
n_2 &= 4 \gamma n_1,
\end{align*}
\]

where the subscripts 1 and 2 refer to the unshocked and shocked regions, respectively, \( \gamma \) is the bulk Lorentz factor of the shocked fluid and \( \rho_1 \) (\( n_1 \)) is the mass (number) density of the unshocked fluid. We have also made the implicit assumption that the enthalpy of the unshocked gas is dominated by its rest-mass energy. Here and throughout this Section, thermodynamic quantities are expresses in their local fluid frame. The jump conditions expressed in eq. (1.1) and (1.2) imply that if the blast wave is adiabatic, the relation between Lorentz factor and radius of the shock is

\[
\gamma \propto r^{-3/2}.
\]  

Using the relativistic Doppler effect for a source moving on the line of sight, we can relate the radius of the shock to the arrival time of photons at a distant observer \( dt_{\text{obs}} \approx dr/2\gamma^2 c \). Through this formula we can translate eq. (1.3) to a scaling expressed in terms of the observer time:

\[
\gamma \propto t_{\text{obs}}^{-3/8}.
\]

All thermodynamic quantities of the blast wave derive from the jump conditions, which are in turn defined by the density of the CBM and shock Lorentz factor. Thus, eq. (1.4) provides a handle on the temporal profiles of mass and energy density, temperature etc. as they would be deduced by an observer. By connecting the profiles of those quantities to a radiation mechanism, one can then calculate the observed spectra as a function of time.

1.5.2 Synchrotron spectra

Synchrotron radiation is the dominant emission mechanism in GRB afterglows. This is supported by the general shape and evolution of the observed spectra, polarization measurements and successful modelling of the observations based on the synchrotron
model. Due to their lower mass, electrons are far more efficient emitters than protons and it is their spectra that are typically considered in modelling. We assume that their energy distribution is a power law with index \(-p\) (with \(p > 2\)). Three more ‘micro-physics’ parameters are needed to complete the description of the spectra. These are the fraction of electrons that are accelerated at the shock \(\xi\), the fraction of internal energy of the shocked gas carried by the power-law electrons \(\epsilon_e\) and the fraction of internal energy of the shocked gas carried by magnetic fields \(\epsilon_B\). When electron cooling and self-absorption are taken into account, the spectrum consists of 4 segments connected through 3 critical frequencies, the derivation of which we demonstrate below.

In the comoving frame the emitted power of a relativistic electron (with Lorentz factor \(\gamma_e\)) inside a magnetic field of strength \(B'\) is (Rybicki & Lightman, 1986)

\[
P' = \frac{4}{3} c \sigma_T \gamma_e^2 U'_B, \tag{1.5}
\]

where \(c\) and \(\sigma_T\) are the speed of light and Thomson cross section, respectively, while \(U'_B = B'^2 / 8\pi = \epsilon_B e_2\).

The bulk of the electron population have energies close to the minimum energy of the power-law distribution \(\gamma_m\). Assuming a pure proton-electron plasma and relating mass and energy density of the shocked gas to the electron distribution we find

\[
\gamma_m = \frac{p - 2}{p - 1} \frac{m_p}{m_e} \epsilon_e \xi^{-1}, \tag{1.6}
\]

where \(m_p\) and \(m_e\) are the proton and electron mass, respectively.

Most of the emitted energy of any electron is radiated at the synchrotron characteristic frequency. For electrons of energy \(\gamma_m m_e c^2\) that frequency has the comoving value

\[
\nu'_m = \frac{3}{4\pi} \gamma_m^2 \frac{q_e B}{m_e c} \sin \alpha, \tag{1.7}
\]

where \(\alpha\) is the pitch angle between magnetic field and electron velocity. When translated to the observer frame, \(\nu_m\) is one of the three critical frequencies of the synchrotron spectrum.

From eq. (1.5) we see that the more energetic electrons have higher energy losses. We can therefore define a critical Lorentz factor for which the energy losses within the expansion timescale \(t\) of the system are significant

\[
\gamma_c = \frac{6\pi m_e c \gamma}{\sigma_T B^2 t}. \tag{1.8}
\]

The characteristic frequency of the electrons with Lorentz factor \(\gamma_c\) introduces another critical frequency in the spectrum \(\nu_c\) above which the spectrum steepens due to the rapid cooling that the contributing electrons undergo.
The final critical frequency of the spectrum is the self-absorption frequency \( \nu_a \), below which absorption of photons due to the inverse process of synchrotron emission causes quenching of the emitted spectrum. An estimate of \( \nu_a \) can be obtained by using formulas for the (frequency-dependent) absorption coefficient (Rybicki & Lightman, 1986) and setting the optical depth of the blast wave to 1.

In Fig. 1.7 we present an example of a synchrotron spectrum. The spectrum shown is just one of the few different types that can emerge according to the ordering of the critical frequencies. The positions of those frequencies are functions of time and therefore, spectral transitions from one type to another are possible during the afterglow evolution. What is important to note is that the slopes of all power-law segments in all possible types of spectra are known from theory. Some of the slopes only depend on the index of the electron power-law distribution, while the rest have values independent of all the physical parameters.
1.6 Thesis outline

Having discussed major advancements and problems in the study of GRBs, it is now time to present my own contribution to the field, first by summarizing the following Chapters of this Thesis. Most of the basic ingredients, necessary for the comprehension of the remainder of this Thesis, have already been introduced, but I will expand more on methods and concepts when I find it necessary.

1.6.1 Dynamics and spectra of transrelativistic outflows

In Chapter 2 we present results from one-dimensional (1D) relativistic hydrodynamic (RHD) simulations of GRB outflows that start off relativistically, but decelerate to non-relativistic velocities. We study synchrotron radiation from these outflows while we expand on previous methods of accurate calculation of spectra and light curves from simulation snapshots. Motivation for this study has been the fact that several afterglow data sets extend to observer times of hundreds of days, a timescale typical of the expected transition to Newtonian dynamics.

Similarly to SNe, GRB outflows go through stages of acceleration, coasting and deceleration. The details differ at early stages, mainly due to the uncertainties of the acceleration mechanism (in both types of events) but also due to the relativistic nature of GRB outflows. Once the latter start decelerating, the bulk Lorentz factor decreases as a function of time. The solution of Blandford & McKee (1976) provides a reasonably good description of the dynamics during deceleration, down to Lorentz factors of a few (Kobayashi et al., 1999). On the other side of the velocity space, the Sedov-Taylor solution (Sedov, 1959; Taylor, 1950) is applicable to outflows with velocities much smaller than the speed of light. The problem lies on linking them and understanding the observational implications (for example, duration) of this transition.

A range of effects, expected to accompany the transrelativistic phase, have been explored. Such effects are, for example, the changing adiabatic index of the shocked gas, which is shown to have an effect early on, as the index deviates from the ultrarelativistic value of 4/3. Furthermore, we demonstrate how radiative transfer can be performed to calculate late-time resolved images of GRB outflows and how the images’ properties depend on the part of the spectrum that is being probed. For the transrelativistic phase, we find that it is manifested in the observed spectra as a slow transition from the relativistic scalings to the non-relativistic ones.
1.6.2 Flux prescriptions at all observer times

In this Chapter we employ the setup developed in the previous one to obtain a set of formulas, based on which one can calculate detailed spectra and light curves of afterglows for a given set of physical parameters. We accomplish this by using numerical results in order to ‘calibrate’ analytic scalings of the radiation during the ultrarelativistic and non-relativistic dynamical phases and connecting the two through heuristic formulas.

We show that this method provides a quite accurate description of the numerical results, at all times. We find that the transrelativistic phase is manifested differently across the spectrum, something which has important consequences for data fitting based on afterglow models. A noteworthy feature of the formalism we introduce is that it allows for investigating the density structure of the CBM, an indirect probe of the progenitor’s nature.

Apart from presenting the formulas we derive, we also make available a software implementation that combines these formulas with a fitting code. The expectation is that the numerical simplicity of the method will motivate researchers to use it in order to fit physical parameters of the standard fireball model to data. This is the first study that uses simulation runs to arrive at flux prescriptions applicable over such a wide range of observer times. In the future similar prescriptions (although not necessarily analytic in nature; van Eerten & MacFadyen 2012b) are expected from studies of 2D jets that should be able to address interesting topics, like jet breaks, directly.

1.6.3 Flux prescriptions applied on GRB afterglow data

In this Chapter we use the flux prescriptions that we have presented previously in order to fit model parameters to data of well-monitored GRB afterglows. Beyond obtaining best-fit values we are keen on understanding the performance of a spherical model in reproducing the features of the observed light curves. We find that from the few bursts examined, one (GRB 970508) can be completely fitted by a spherical model, while the structure of the CBM points towards a stellar-wind progenitor. Other bursts produce overall decent fits although with a few weaknesses and/or extreme physical parameters derived.

The values we derive for the blast wave of GRB 970508 are reasonable. The total blast-wave energy is found to be close to $10^{51}$ erg, which is comparable to the isotropic equivalent of the prompt emission ($\sim 5 \cdot 10^{51}$ erg; Rhoads 1999). We find energy equipartition between power-law electrons and magnetic field, as well as evidence for a non-accelerated electron population.
1.6.4 Thick shells as origins of early shallow decay

In the last Chapter of this Thesis we turn our attention to the early afterglow behaviour of bursts in the *Swift* era. Specifically, we analyze the observational signatures of the so-called “thick-shell” scenario as we assess whether it is a viable proposal for the explanation of early shallow decays and plateaus in optical and X-ray light curves. A ‘thick shell’ represents the situation where the (assumed continuous) ejection of matter from the burster lasts longer than usual, or commonly assumed. The effect of such an ejection profile on the dynamics can be quite important, as it may result in the reverse shock, that propagates inside the ejecta, becoming relativistic before all energy has been given to the blast wave. This may, in turn, induce an intermediate phase between coasting and canonical deceleration, during which the dynamics of the blast wave are governed by energy fluxes across the two shocks. This scenario is a popular, but not fully explored, option for energy injection due to continuous activity of the central engine.

We present simple analytic calculations of the dynamics, the particle population and the thermodynamics of the blast wave as a function of time, during the intermediate dynamical phase and the transition to the canonical decay. From these we can extract the observational properties in such a scenario. We construct a semi-analytic application of the model and use it to highlight the diversity of light curves during energy injection. We investigate the predictions of the model by deriving scalings for the optical flux at the end of energy injection and find that the reverse shock explains better the observational findings. We apply the model on afterglow data sets of GRB 060729 and GRB 090423 and infer basic physical parameters that describe energy injection into the corresponding blast waves.
Abstract   We present a study of the intermediate regime between ultra-relativistic and nonrelativistic flow for gamma-ray burst afterglows. The hydrodynamics of spherically symmetric blast waves is numerically calculated using the AMRVAC adaptive mesh refinement code. Spectra and light curves are calculated using a separate radiation code that, for the first time, links a parametrisation of the microphysics of shock acceleration, synchrotron self-absorption and electron cooling to a high-performance hydrodynamics simulation. For the dynamics we find that the transition to the nonrelativistic regime generally occurs later than expected, that the Sedov-Taylor solution overpredicts the late time blast wave radius and that the analytical formula for the blast wave velocity from Huang et al. (1999) overpredicts the late time velocity by a factor $4/3$. Also we find that the lab frame density directly behind the shock front divided by the fluid Lorentz factor squared remains very close to four times the unshocked density, while the effective adiabatic index of the shock changes from relativistic to nonrelativistic. For the radiation we find that the flux may differ up to an order of magnitude depending on the equation of state that is used for the fluid and that the counterjet leads to a clear rebrightening at late times for hard-edged jets. Simulating GRB 030329 using predictions for its physical parameters from the literature leads to spectra and light curves that may differ significantly from the actual data, emphasizing the need for very accurate modelling. Predicted light curves
at low radio frequencies for a hard-edged jet model of GRB 030329 with opening angle 22 degrees show typically two distinct peaks, due to the combined effect of jet break, non relativistic break and counterjet. Spatially resolved afterglow images show a ring-like structure.

### 2.1 Introduction

Gamma-ray burst (GRB) afterglows can be explained from the interaction between an initially relativistic shock wave of hot fluid and the medium surrounding the burster. On passage of the shock electrons get accelerated to relativistic velocities (even with respect to the already relativistic local fluid flow) and small scale magnetic fields are generated. Under influence of the magnetic field, the electrons will produce synchrotron radiation, which will be seen by the observer. This model has been very successful when applied to broadband afterglow data, but thus far model predictions have been made using simplifying assumptions for the blast wave structure (approximating the blast wave width by a homogeneous slab, e.g. Wijers et al. 1997; Mészáros & Rees 1997; Sari et al. 1998; Rhoads 1999) or from analytical solutions in either the ultrarelativistic or the nonrelativistic regime (e.g. Granot et al. 1999a; Gruzinov & Waxman 1999; Wijers & Galama 1999; Frail et al. 2000).

Since the beginning of the decade, fluid simulations have been performed to study afterglow blast waves and their resulting spectra (see Granot et al. 2001; Downes et al. 2002). More recent simulations have been used to address the specific theoretical issue of the visible effect of the blast wave encountering a density perturbation (Nakar & Granot, 2007; van Eerten et al., 2009). Very recently Zhang & MacFadyen (2009) studied the transition to the transrelativistic regime and the spreading of a collimated outflow, using an adaptive mesh technique for the fluid simulation. They made some simplifying assumptions for the radiation mechanism, when compared to the early analytical efforts (e.g. Granot et al. 1999a), such as approximating the cooling time by the lab frame time and ignoring synchrotron self-absorption.

The aim of this paper is to present a theoretical and qualitative study of the transition regime between relativistic and nonrelativistic blast waves and the effect on the light curves and spectra at various wavelengths, using adaptive mesh relativistic fluid simulations for blast waves from an explosion in a homogeneous medium, while including all details of the synchrotron radiation mechanism that have been used for earlier analytical estimates. Also we present resolved afterglow images. We study spherical blast waves and sharp edged jets obtained by taking conic sections from a spherically symmetric fluid flow.

Obviously these simulations do not yet fully address the complete GRB afterglow picture of a realistic, 2D-dynamical jet, which we address in future work. However,
some GRB afterglows have power law decays that last for months without a jet break, and thus may be (nearly) spherical. These are of course already addressed in the present work. Also, by studying the conic sections from spherical flows, we already address some aspects of jet behaviour, which allows us to probe some outstanding issues, such as whether the receding jet may lead to visible features in the late light curve, and whether a dynamical jet break must be truly achromatic. Finally, any fluid flow behaviour typical to higher-dimensional simulations, like lateral spreading of the jet, is best understood from a direct comparison to one-dimensional simulations and its effects on the light curve will in practice be modeled as a deviation from the heuristic description based on analytical approximations and one-dimensional simulations (i.e. as an additional smooth jet break). A companion paper is in preparation that will discuss the practical consequences for broadband afterglow data fitting from the underlying model from this paper.

This paper is organized as follows. In section 2.2 we discuss our radiation code and how it expands upon an approach outlined earlier in van Eerten & Wijers (2009), hereafter EW09. A proper treatment of synchrotron radiation and shock wave generation of accelerated particles and small scale magnetic fields requires us to trace some additional quantities along with the fluid quantities.

In section 2.3 we provide the details of our simulations that assume typical GRB parameters. We show how the blast wave starts out in the ultrarelativistic regime and smoothly approaches the nonrelativistic regime. We discuss the consequences of different equations of state for the fluid and how our simulations differ from analytical approximations for the nonrelativistic regime. We show how the fluid lab frame density divided by the fluid Lorentz factor squared right behind the shock remains always close to four times that in front of the shock, even though we have differing adiabatic indices in both the relativistic and nonrelativistic regimes. Three additional quantities needed to be traced and we present results for the behaviour of these three: the accelerated electron number density, the magnetic field energy density and the accelerated particle distribution upper cut-off Lorentz factor. We explain how calculation of the latter especially is numerically challenging and how it shapes the spectrum beyond the cooling break.

In section 2.4 we take our results from section 2.3 and calculate spectra and light curves. We calculate spectra at 1, 10, 100, 1,000 and 10,000 days in observer time. We separately discuss the different factors contributing to the shape of the light curves: the equation of state, the evolution of the magnetic field and the evolution of the accelerated particle distribution.

We then turn to the specific case of GRB 0303029 in section 2.5. We take the explosion parameters that have been established for this burst by previous authors to set up a simulation. We qualitatively compare the resulting light curves to radio data.
at different wavelengths, assuming both a spherical explosion and a hard edged jet with opening angle of 22 degrees. We provide spatially resolved radio images and make a qualitative prediction for the expected signal at radio wavelengths that will be observable with the next generation of telescopes, like LOFAR.

We discuss our results in 2.6. In the appendices we provide additional technical details on the numerical implementation of our approach and a discussion on the theoretical limitations and assumptions of our approach.

2.2 The radiation code

In this paper we follow the approach first outlined in EW09, where we calculate spectra and light curves from the output of a relativistic hydrodynamics (RHD) code using a separate radiation code. For the RHD simulations we use AMRVAC, a high performance code that includes adaptive-mesh refinement (AMR) (see Keppens et al. 2003; Meliani et al. 2007). AMRVAC calculates the evolution of the following conserved variables:

\[ D = \gamma \rho', \quad \vec{S} = \gamma^2 h' \vec{v}, \quad \tau = \gamma^2 h' - p' - \gamma \rho' c^2, \]

with \( \gamma \) the Lorentz factor, \( \rho' \) the proper density, \( h' \) the relativistic (i.e. including rest mass) enthalpy density, \( \vec{v} \) the three velocity, \( p' \) the pressure and \( c \) the speed of light. In the entire paper, all comoving quantities will be primed.

In the second stage we use a radiation code to obtain the received flux for a given observer frequency, time and distance, from the local values of conserved variables at any contributing point in the fluid (we also use two auxiliary quantities, \( \gamma \) and \( p' \), that AMRVAC stores as well in order to facilitate its calculation of the time evolution of the conserved variables). The radiation mechanism that is considered is synchrotron radiation and a number of parameters have been introduced in EW09 that capture the underlying radiation and shock microphysics. There are four of these ‘ignorance’ parameters. The fraction of the thermal energy that resides in the tangled-up magnetic field that is generated by the passage of a shock \( \epsilon_B \) usually has a value around 0.01. The fraction of electrons \( \xi_N \) that is accelerated into a relativistic power law distribution in energy also by the passage of a shock is usually of order unity in the relativistic regime. The thermal energy fraction captured by these electrons \( \epsilon_e \approx 0.1 \) and minus the slope of the electron distribution \( p \approx 2.5. \)

The flux calculated by the radiation code is given by

\[ F_\nu = \frac{1 + z}{r_{\text{obs}}^2} \int \frac{d^2 P_V}{d\nu d\Omega} (1 - \beta \mu) c \, dA \, dt_e. \]  

Here \( z \) denotes redshift, \( r_{\text{obs}} \) denotes the observer luminosity distance, \( d^2 P_V / d\nu d\Omega \) the received power per unit volume, frequency and solid angle, \( dA \) the equidistant
The radiation code

The surface element given by the intersection of the fluid grid with that surface from which radiation is poised to arrive exactly at \( t_{\text{obs}} \) and \( t_e \) the emission time. The integral

\[
\int (1 - \beta \mu) c \, dA \, dt_e
\]

is effectively an integral over the entire radiating volume. \( \mu \) is the angle between the local fluid velocity and the observer position, \( \beta \) the fluid velocity in units of \( c \) and the factor \((1 - \beta \mu)\) is a retardation effect due to the moving of the radiating source. The detailed dependency of the received power on the ignorance parameters and local fluid conditions is explained in EW09. However, in that paper only ultra-relativistic flows were addressed and in order to include subrelativistic and nonrelativistic flows as well, a number of features were added to our radiation code. Also we have added synchrotron self-absorption and the possibility to resolve the signal from the fluid into an image on the sky. We now have a generic radiation code that is capable of calculating the spatially resolved synchrotron radiation profile from an arbitrary fluid flow. The additional physics that we have included is explained below, with some of the practical numerical issues discussed separately in appendix 2.8.

2.2.1 Realistic equation of state

In EW09 we applied a fixed adiabatic index \( \Gamma_{\text{ad}} \) equation of state (EOS)

\[
p' = (\Gamma_{\text{ad}} - 1) e'_{\text{th}}, \tag{2.3}
\]

where \( e'_{\text{th}} \) is the thermal energy density. In practice \( \Gamma_{\text{ad}} \) was always set to 4/3. However, when following a fluid from the relativistic regime (with flow velocities \( \sim c \) and thermal energy density dominating the rest mass energy density) down to the classical regime, this fixed adiabatic index becomes too restrictive. We therefore apply a Synge-like EOS (Synge, 1957) that results in an effective adiabatic index varying smoothly from 4/3 to its classical limit 5/3:

\[
p' = \rho' c^2 \left( \frac{e'}{\rho' c^2} - \frac{\rho' c^2}{e'} \right), \tag{2.4}
\]

where \( e' \) denotes the comoving energy density including rest mass, \( e' = \rho' c^2 + e'_{\text{th}} \). This EOS has already been applied in amrvac (see Meliani et al. 2008, 2004). Also, because the radiation code reads both the conserved variables as well as \( p' \) from disc directly, it does not invoke any EOS itself, and no change in the radiation code was needed. The resulting effective adiabatic index is given by

\[
\Gamma_{\text{ad,eff}} = \frac{5}{3} - \frac{1}{3} \left( 1 - \frac{\rho'^2 c^4}{e'^2} \right). \tag{2.5}
\]
The effect of an advanced EOS on the behaviour of the fluid is profound and we discuss this in detail in section 2.4.3.

### 2.2.2 Electron cooling

The shape of the observed spectrum from a single fluid cell, if electron cooling does not play a role, follows directly from the dimensionless function $Q(v'/v'_m)$, first introduced in EW09. It has the limiting behaviour $Q \propto (v'/v'_m)^{1/3}$ for small $(v'/v'_m)$ and $Q \propto (v'/v'_m)^{(1-p)/2}$ for large $(v'/v'_m)$. The received power depends on this shape and on the local fluid quantities via

$$\frac{d^2 P}{dv \, d\Omega} = \frac{(p-1) \sqrt{3} q_e^3}{8 \pi m_e c^2} \frac{\xi_N n B'}{\gamma^3 (1-\beta \mu)^3} Q \left( \frac{v'}{v'_m} \right).$$  \hspace{1cm} (2.6)$$

Here $n$ denotes the lab frame number density (of all electrons, both accelerated and thermal). $B'$ denotes the local comoving magnetic field strength, calculated from the thermal energy density after the passage of a shock. $m_e$ and $q_e$ denote electron mass and charge, respectively. The frequency $v'_m$ is the synchrotron peak frequency, and it is related to the lower cut-off Lorentz factor $\gamma'_m$ of the power law accelerated electrons via

$$v'_m = \frac{3 q_e}{4 \pi m_e c} \gamma'^2 B'.$$  \hspace{1cm} (2.7)$$

If cooling does not play a role, the evolution of $\gamma'_m$ is completely adiabatic, which has as a consequence that the total fraction of the local thermal energy density residing in the power-law accelerated particle distribution remains fixed. $\gamma'_m$ will be related to $e'_\text{th}$ throughout the downstream fluid according to

$$\gamma'_m = \left( \frac{p-2}{p-1} \right) \frac{e'_\text{th}}{\xi_N n' m_e c^2}.$$  \hspace{1cm} (2.8)$$

When cooling does play a role, however, this picture is changed. It now becomes necessary to introduce an upper cut-off Lorentz factor $\gamma'_M$ as well. In a single fluid element, no accelerated electrons with energies above $\gamma'_M$ will be found, because these have cooled to energies at or below $\gamma'_M$. The temporal evolution of any electron Lorentz factor $\gamma'_e$, and therefore of $\gamma'_m$ and $\gamma'_M$ as well, is given by

$$\frac{d \gamma'_e}{dt'} = \frac{\gamma'_e}{3n'} \frac{dn'}{dt'} - \frac{(\gamma'_e)^2 \sigma_T B'^2}{6 \pi m_e c},$$  \hspace{1cm} (2.9)$$

where $\sigma_T$ is the Thomson cross section, $m_e$ the electron mass and $t'$ the comoving time. The final term in this equation reflects synchrotron radiation losses, and if it is omitted only the adiabatic cooling term is left and it can be shown that this
will result in the aforementioned fixed relation between $\gamma'_e$ and $e'_\text{th}$. In light of the previous subsection on the EOS, it may be worth noting that equation (2.9) is derived for a relativistic electron distribution with adiabatic index $4/3$ and that this remains valid even if the bulk of the fluid becomes nonrelativistic. After all, the power-law accelerated electrons are relativistic by definition.

The above has the following consequences for the simulations and radiation code. Because for low values of $\gamma'_e$, the radiation loss term can be neglected next to the adiabatic expansion term, we will not apply equation (2.9) to $\gamma'_m$ and we will continue to calculate $\gamma'_m$ locally using equation (2.8) in the radiation code. For $\gamma'_M$, this is not an option and we numerically solve equation (2.9) in amrvac, resetting $\gamma'_M$ to a high value wherever we detect the passage of a shock. This reset implements the shock-acceleration of particles. In appendix 2.8 we discuss the numerical issues of this approach in some more detail. Also, in appendix 2.9.2 we show that the gyral radius even for the high energy electrons contributing to the observed spectrum (within the frequency range under consideration, $10^8 - 10^{18}$ Hz) is orders of magnitude smaller than the relevant fluid scales.

Finally, we summarize the consequences of electron cooling for the spectrum discussed earlier in EW09. The received power from a fluid element is now given by

$$
\frac{d^2P}{d\nu d\Omega} \propto \frac{\xi_{\text{NN}}B'}{\gamma^3(1-\beta\mu)^3}Q\left(\frac{\nu'}{\nu'_M}, \frac{\nu'}{\nu'_M}\right),
$$

with the relations between the upper and lower cut-off Lorentz factors and their corresponding critical frequencies given by equation (2.7). $Q$ is a generalisation of $Q$ and the flux at frequencies $\nu'$ above $\nu'_M$ drops exponentially. That the resulting spectrum from the entire fluid does not show an exponential drop is due to the fact that there will always be some fluid elements contributing for which $\nu'_M$ is still sufficiently high. The effect of this ‘hot region’ close to the shock front (with a size that depends on the observer frequency) on the composite synchrotron spectrum from a shock will be a steepening of the slope by $-1/2$ instead. The cooling break is found at that frequency for which the width of the hot region becomes comparable to the width of the blast wave.

### 2.2.3 Magnetic field energy evolution

The magnetic field directly behind a shock has been parametrised using

$$
\frac{B'^2}{8\pi} = e'_B = e_B e'_\text{th},
$$
Furthermore we assumed the number of magnetic flux lines threading a surface co-moving with a fluid element to remain invariant, resulting in

\[ e_B' \propto \rho'^{4/3}. \]  

(2.12)

For relativistic fluids this implies that the fraction \( \epsilon_B \) remains fixed downstream, because \( e_{\text{th}}' \propto \rho'^{\Gamma_{\text{ad}}} \). For a changing adiabatic index it is no longer possible to calculate \( e_B' \) a posteriori from \( e_{\text{th}}' \), since the relation between the two is now no longer fixed. It becomes necessary to numerically solve in \textsc{amrvac} the equation

\[ \frac{d}{d\tau'} \frac{e_B'}{\rho'^{4/3}} = 0. \]

(2.13)

Like \( \gamma_M' \), we reset \( e_B' \) whenever a shock is encountered. The practical implementation of the evolving magnetic field is again discussed in appendix 2.8. We note here that the assumption of frozen field lines is not essential, and that we can in principle include different magnetic field behaviour either by adding a source term to equation (2.13) (parametrising for example, magnetic field decay through reconnection) or by implementing a different equation entirely.

### 2.2.4 Changing fraction of accelerated particles

Although \( \xi_N \), the fraction of electrons accelerated by the passage of a shock is often assumed to be of the order unity for highly relativistic blast waves, it has to be lower at late times because otherwise there would not be enough energy available per accelerated electron to create a relativistic distribution (in other words, to ensure that \( \gamma_m' > 1 \)). We have implemented this change in our code by replacing user parameter \( \xi_N \) by \( \xi_{\text{N,NR}} \), that is, the fraction of electrons that is accelerated in the nonrelativistic limit. The fraction at the relativistic limit we set to one. Because \( \gamma' \beta \) is the most direct measure of how relativistic the fluid flow locally is, we have parametrised the simplest possible smooth transition between both limiting cases by

\[ \xi_N = \frac{\beta \gamma + \xi_{\text{N,NR}}}{1.0 + \beta \gamma}. \]

(2.14)

Whenever the passage of a shock is detected, \textsc{amrvac} resets the number density of accelerated electrons \( n_{\text{acc}}' \) according to \( n_{\text{acc}}' = \xi_N n' \), with \( \xi_N \) determined using the equation above. As with the magnetic field energy density, we now need to follow \( n_{\text{acc}}' \) explicitly. Because \( n_{\text{acc}}' \) is a number density, its evolution is described by a continuity equation, following

\[ \frac{\partial}{\partial t} n_{\text{acc}}' \gamma + \frac{\partial}{\partial x^i} n_{\text{acc}}' \gamma v^i = 0, \]

(2.15)

and is therefore easily implemented in \textsc{amrvac}.
2.2 The radiation code

2.2.5 Synchrotron self-absorption

In previous work we have solved equation (2.2) by first integrating over $A$ for a given emission time $t_e$ (and thus for a single snapshot), followed by an integration over $t_e$. If we switch the order of the integrations then the integral over $t_e$ represents the solution to a linear radiative transfer equation without absorption, with the intensity given by

$$I_\nu = \int \frac{d^2 P_V}{d \nu d \Omega} (1 - \beta \mu) c \, dt_e.$$  \hfill (2.16)

The integral over $A$ then represents a summation over all rays. The full linear radiative transfer equation including synchrotron self-absorption has the form

$$\frac{dI_\nu}{dz} = -\alpha_\nu I_\nu + j_\nu,$$  \hfill (2.17)

with $j_\nu \equiv \frac{d^2 P_V}{d \nu d \Omega}$ and $dz \equiv c \, dt_{\text{obs}} = (1 - \beta \mu) c \, dt_e$. The synchrotron self-absorption coefficient is given by

$$\alpha_\nu' = -\frac{1}{8\pi m_e \gamma^2} \int_{\gamma_m^\prime}^{\gamma_M^\prime} \frac{dP_{\text{<e>}}'}{d \gamma'} \gamma_e^2 \frac{\partial}{\partial \gamma_e'} \left[ \frac{N'_e(\gamma_e')}{\gamma_e'^2} \right] d\gamma_e'.$$  \hfill (2.18)

Here $dP_{\text{<e>}}'/d \gamma'$ denotes the emitted power per ensemble electron and $N'_e(\gamma_e')$ the electron number density for relativistic electrons accelerated to $\gamma_e'$. Integrating $N'_e(\gamma_e')$ over possible electron Lorentz factors yields $n'_\text{acc}$ by definition. These quantities are defined and explained in detail in EW09 (see also appendix 2.8.3).

In this treatment of the self-absorption coefficient we only take into account transitions between already occupied energy levels of electrons, leading to the integration limits of equation (2.18) being exactly $\gamma_m'$ and $\gamma_M'$. In this way we ignore stimulated emission arising from a population inversion below $\gamma_m'$. This results in values of the absorption coefficient that are larger by a factor of $3(p+2)/4$ when compared to Granot & Sari (2002).

In our radiation code, we now calculate the linear radiative transfer equation for each individual ray by not integrating over the two-dimensional surface $A$ (to get a single flux value from the collection of rays) until after the final snapshot has been processed. In addition to allowing us to include the effect of self-absorption, we now also get a spatially resolved signal from the fluid, showing the expected ring structure (extending predictions from Granot & Loeb 2001 to the nonrelativistic regime). We use an adaptive-mesh type approach to $A$ in order to ensure an adequate spatial resolution, see appendix 2.8.4.
2.3 Fluid dynamics

In this section we describe the setup of our relativistic fluid simulations and compare the results against the theoretically expected behaviour.

2.3.1 Expected early and late time behaviour

Both the early and late time behaviour of the fluid can be described by a self-similar solution that is determined completely from the explosion energy $E$ and the circum-burst number density $n_0$.

At early stages, the Blandford-McKee (BM) solution (Blandford & McKee, 1976) for relativistic blast waves predicts the following relation between the shock front fluid Lorentz factor $\Gamma$ and the explosion time ($t$, which is the same as the emission time $t_e$):

$$\Gamma^2 = \frac{17E}{16\pi\rho_0^3c^5}. \quad (2.19)$$

The density $\rho_0$ is related to the number density through the proton mass: $\rho_0 = m_p n_0$. The shock radius $R(t)$ is then given by

$$R(t) = ct\left(1 - \frac{1}{16\Gamma^2}\right). \quad (2.20)$$

To lowest order $R(t)$ is just $ct$, while the shock front fluid velocity $\beta \sim 1$. Further analytical equations for the fluid profile (in terms of pressure $p$, Lorentz factor $\gamma$, number density $n$, etc.), behind the shock front can be found in Blandford & McKee 1976.

At late stages the evolution of the blast wave is described by the Sedov-Taylor (ST) solution (Sedov, 1959; Taylor, 1950). For a fixed adiabatic index $5/3$, the shock radius is now given by

$$R(t) \approx 1.15 \left(\frac{Et^2}{\rho_0}\right)^{1/5}, \quad (2.21)$$

which follows directly from dimensional analysis (except for the numerical constant). In this classical approximation, the speed of light $c$ does not appear. The shock front Lorentz factor is approximately one, while $\beta$ can be found from $\beta \equiv dR(t)/c\,dt$. Again analytical formulae for the fluid profile exist in the literature (Sedov, 1959).

At some point in time the evolution of the blast wave will no longer be adequately described by the BM solution but will become more and more dictated by the ST solution. An estimate for the turning point (Piran, 2005) can be made by equating the explosion energy to the total rest mass energy that is swept up:

$$E = \rho_0 c^4 \frac{4}{3} \pi R_{NR}^3. \quad (2.22)$$
Solving for $R_{\text{NR}}$ returns the approximate radius at which the original explosion energy in the blast wave is no longer dominant over its rest mass energy.

Analytical estimates for the bulk fluid flow velocity including the intermediate regime also exist. One such example is found in Huang et al. (1999) and we discuss it in more detail as well as compare their prediction for $\gamma(t)$ right behind the shock front directly against our simulation results in section 2.3.3.

### 2.3.2 Setup of simulations

We have performed a number of simulations using the typical values for a GRB exploding into a homogeneous medium. We set up our simulations starting from the BM solution. The isotropic explosion energy $E = 1 \cdot 10^{52}$ erg, the medium number density $n_0 = 1\text{cm}^{-3}$. We have set the initial shock Lorentz factor to 10 (and the fluid Lorentz factor therefore $\sim 7$, differing by a factor $\sqrt{2}$). Although both amrvac and the radiation code are able to deal with far higher Lorentz factors, the focus for this research is on the transition to the nonrelativistic regime and for that purpose this relatively low Lorentz factor is sufficient. We have continued the simulations until the fluid proper velocity in the lab frame $\beta \sim 0.01$.

We have used both the advanced equation of state and a fixed adiabatic index at $4/3$ and $5/3$. In the advanced EOS simulation we have also calculated the other quantities mentioned in the previous section: $\epsilon_B, \xi_N$ and $\gamma'_M$. The value at the shock front for $\epsilon_B$ was set to the standard 0.01 and the non-relativistic limit for $\xi_N$ was set at $\xi_{N,NR} = 0.1$. Sufficiently high values for $\gamma'_M$ at the shock front are chosen, generally on the order of $10^7$.

In amrvac it is the number of refinement levels that determines the accuracy of the simulation. We have used 17 levels of refinement and 120 cells at the lowest refinement level. The grid was initially taken to run from $10^{16}$ cm to $10^{19}$ cm. The effective spatial resolution due to adaptive mesh refinement was therefore $\sim 1.27 \cdot 10^{12}$ cm. This should be compared against the width of the blast wave at the start of the simulation, when it is the smallest. This is approximately equal to $R(t)/\Gamma^2 \sim 3 \cdot 10^{15}$ cm, for a starting shock Lorentz factor of 10.

Convergence of our results has been checked by performing simulations at different refinement levels and by simulations running for a shorter time on a smaller grid (thereby increasing the resolution). For the light curves and spectra we have used simulations with a shorter running time of $12.2 \cdot 10^3$ days. At this stage the fluid velocity directly behind the shock is still six percent of light speed, but we have full coverage up to 10,000 days in observer time. The corresponding grid size is $1 \cdot 10^{17}$ cm to $6.7 \cdot 10^{18}$ cm, leading to an effective resolution of $8.3 \cdot 10^{11}$ cm. On a stan-
The solid line in figure 2.1 shows $\beta \gamma$ at the shock front for the advanced EOS simulation. The expected scaling behaviour at the early stage is dictated by $\Gamma \propto t^{-3/2}$ and at the late stage by $\beta \propto t^{-3/5}$. We have plotted this asymptotic behaviour as well, setting the early stage scaling coefficient from the initial value at $\beta \gamma \sim 7$ and the late stage scaling coefficient at $\beta \gamma \sim 0.016$ (this point lies far to the right outside the plot). The shock velocity is shown to smoothly evolve from the BM solution to the ST solution. The meeting point of the asymptotes at $t \approx 1290$ days lies at $\beta \gamma \approx 0.244$. At this point $\beta \gamma$ for the fluid $\approx 0.33$, so the fluid is still moving at a significant fraction of the speed of light.

1 For example, an Intel dual core 1600 MHz processor with 4 GB of RAM.
2.3 Fluid dynamics

According to equation (2.22), the predicted radius for the transition to occur is 
\( R_{\text{NR}} \approx 0.38 \) parsec for the initial explosion energy and circumburst density that we have used, corresponding to a lab frame time \( t_{\text{NR}} \approx 450 \) days. We therefore conclude that the transition point from the relativistic to the nonrelativistic regime is far later than predicted by \( t_{\text{NR}} \).

Also plotted in fig. 2.1 is the predicted value for \( \beta \gamma \) from Huang et al. (1999), which we have implemented as follows. The starting point is

\[
\frac{dy}{dm} = -\frac{\gamma^2 - 1}{M_{\text{ej}} + 2\gamma m}, \tag{2.23}
\]

the differential equation proposed by the authors to depict the expansion of GRB remnants, simplified to the adiabatic case. Here \( m \) denotes the rest mass of the swept-up medium and \( M_{\text{ej}} \) the mass ejected from the GRB central engine. Our approach starting from the BM solution is a limiting case where \( M_{\text{ej}} \downarrow 0 \). The \( M_{\text{ej}} \) term was included by Huang et al. (1999) to incorporate a coasting phase. When solving equation (2.23) we will use a very high (~10^7) initial bulk fluid Lorentz factor \( \gamma_0 \) and by assuming \( M_{\text{ej}} \sim E/2\gamma_0 c^2 \) we converge on the limiting scenario used in our simulations. Equation (2.23) can be analytically solved to yield

\[
(\gamma - 1)M_{\text{ej}}c^2 + (\gamma^2 - 1)mc^2 = E, \tag{2.24}
\]

which (numerically) leads to \( \gamma(t) \) once we apply

\[
m = \frac{4}{3}\pi R^3 n_0 m_p, \tag{2.25}
\]

and

\[
R(t) = \int_0^t \beta(\tau)c \, d\tau. \tag{2.26}
\]

Here \( t \) is measured in the simulation lab frame (i.e. it does not refer to observer time).

The resulting curve for \( \beta \gamma \) initially lies below the simulation result, but ends up above at 4/3 times the simulation value. The initial and final slopes for the analytical \( \beta \gamma \) curve are correct by construction. We conclude that the approach from Huang et al. (1999) initially underestimates the BM phase and significantly overestimates the late stage flow velocity. The transition point between the relativistic and nonrelativistic regime also lies at an earlier time for the analytical curve, closer to the analytically predicted \( t_{\text{NR}} \).

**Blast wave radius**

In figure 2.2 we plot the blast wave radius as a function of lab frame time for three different simulations: fixed adiabatic index at 4/3 and 5/3 and using the advanced
Figure 2.2: The resulting blast wave radii as a function of lab frame time for different simulations. The steady slope line shows the radius as predicted by the ST solution. The different simulations end up in the asymptotic regime with different radii: the $\Gamma_{\text{ad}} = 5/3$ ends up above the ST solution, the advanced EOS below the ST solution between the others and $\Gamma_{\text{ad}} = 4/3$ the lowest. The bottom curve shows the effective adiabatic index for the advanced EOS, minus $4/3$. It starts at approximately zero at the left of the plot and proceeds to its asymptotic limit $1/3$ in the nonrelativistic case.

EOS. Also we plot the radius as predicted from equation (2.21) and, for the advanced EOS simulation, the difference between the effective adiabatic index and its relativistic limit $\Gamma_{\text{ad}} = 4/3$. The latter illustrates how relativistic the fluid still is in terms of temperature (as opposed to flow velocity). At the intersecting point for the $\gamma\beta$ asymptotes at 1290 days the effective adiabatic index is already quite close ($\Gamma_{\text{ad,eff}} \approx 1.63$) to its nonrelativistic limiting value. After 3800 days, when the time evolution of all the radii is dictated by $R(t) \propto t^{2/5}$ from the ST solution we see a systematic difference between the different radii. At this time the ST radius is 1.358 parsec, the $\Gamma_{\text{ad}} \equiv 4/3$ radius is 1.197 parsec, the $\Gamma_{\text{ad}} \equiv 5/3$ radius is 1.388 parsec and the advanced EOS radius 1.313 parsec. Taking the advanced EOS radius as a standard, this implies that ST overpredicts the radius by 3.4 percent, $\Gamma_{\text{ad}} \equiv 5/3$ overpredicts the radius by 5.7 percent and $\Gamma_{\text{ad}} \equiv 4/3$ underpredicts the radius by 8.9 percent. Because all radii follow the same temporal evolution at this stage, these errors are systematic and will
2.3 Fluid dynamics

Figure 2.3: Comoving number density profile. The profiles were taken at times corresponding to emission arrival times for the closest part of the shock front (i.e. with velocity directly towards the observer) at 10, 100, 1,000 and 10,000 days. Listed in increasing number and including the initial profile, these times correspond to lab frame times of 137, 387, 761, 2,227 and 12,583 days. Later times correspond to curves peaking further to the right in the plot.

remains the same throughout the further evolution of the blast wave. This implies that, if the quantity \( \frac{E}{\rho_0} \) is derived from the radius using the Sedov-Taylor equation (2.21), it will be overpredicted by 18 percent. Equation (2.21) can be replaced by

\[
R(t) \approx 1.11 \left( \frac{E t^2}{\rho_0} \right)^{1/5}, \tag{2.27}
\]

to get a relation between radius and explosion parameters in the nonrelativistic phase, where the numerical constant has been corrected for the overprediction.

Density and energy profiles

In figure 2.3 we have plotted the comoving fluid number density profile (of the protons or the electrons, not both) at five different moments in time. Because later on we discuss spectra and light curves up to an observer time of 10,000 days, we have chosen emission times corresponding to arrival times of the shock front (using \( t_{\text{obs}} = t - R(t)/c \)) up to 10,000 days as well. The earliest fluid profile shows the
Gamma-ray burst afterglows from trans-relativistic blast waves simulations

Figure 2.4: Lab frame number density divided by $\gamma^2$. This effectively scales the shock profile to 4 at the shock front. The same lab frame times as in fig. 2.3 have been used.

initial conditions calculated from the BM solution when the shock Lorentz factor is 10. After some time, the number density at the shock front can be seen to tend to the value predicted from the shock-jump conditions for a strong classical shock, which is $n_0(\Gamma_{ad} + 1)/(\Gamma_{ad} - 1) = 4n_0$ for the classical value of the adiabatic index $5/3$.

What is shown in figure 2.4 is an interesting feature of the blast wave, which is that the lab frame density directly behind the shock divided by the squared fluid Lorentz factor directly behind the shock, $D/\rho_0 \gamma^2 = 4p_0$ throughout the entire simulation. This can be seen analytically to hold from the Rankine-Hugoniot relations in both the ultrarelativistic and nonrelativistic case, even though the adiabatic indices have different fixed values, from

\[
\frac{D}{\rho_0 \gamma^2} = \frac{\Gamma_{ad} + 1}{\gamma} / \frac{\Gamma_{ad} - 1}{\Gamma_{ad} - 1},
\]

which can be viewed as the relativistic generalization of the classical compression ratio and holds for arbitrary $\gamma$. When we use an advanced EOS, where we let $\Gamma_{ad}$ smoothly evolve from $4/3$ to $5/3$, we see from the figure that this generalized compression ratio remains very close to four even at intermediate times. We make use of this feature for the shock detection algorithm (see appendix 2.8.2).

In figure 2.5 we have plotted the thermal energy density at the same times as the
2.3 Fluid dynamics

Figure 2.5: Thermal energy density profile for the same lab frame times as in fig. 2.3.

number density. Unlike the density at the shock front, the thermal energy density is not expected to tend to a fixed value. The ST solution instead predicts a steep decline \( \propto R(t)^{-3} \), which is why the final shock front thermal energy density is many orders of magnitude smaller than the initial shock front thermal energy density.

Magnetic field and particle acceleration

We now turn to those quantities calculated in \texttt{amrvac} solely to aid in the construction of spectra and light curves, and that have no feedback on the dynamics. In figure 2.6 we have plotted \( \epsilon_B \). Because we assumed the number of field lines through a fluid surface element to remain frozen (see equation 2.12), the magnetic field energy density declined less rapidly than the thermal energy and as a consequence the local fraction \( \epsilon_B \) increased. A discussion on the merit of our assumption about the magnetic field behaviour is outside the scope of this work (and from particle-in-cell simulations it can certainly be argued that it is not perfect, see e.g. Chang et al. 2008). However, our plot does show that at least it does not lead to unphysical values or strong inconsistencies. The maximum value for \( \epsilon_B \) found in fig. 2.6 is 0.037 (up from 0.01 at the shock front), which is not unreasonably large and, besides, occurs far downstream in a region that will contribute negligibly to the observed flux. We emphasize that \( \epsilon_B \) is a relative measure and that both thermal and magnetic energy densities drop
steeply, both with respect to earlier times and with respect to their value at the shock front at any time. The numerical method presented in this paper for parametrizing the magnetic field energy density is quite general and can be readily modified to study different parametrizations.

In figure 2.7 we have plotted the fraction $\xi_N$ of electrons that are accelerated to a power-law distribution. This fraction was taken to smoothly decrease from unity in the relativistic regime down to 0.1 for our simulation in the nonrelativistic regime. The rightmost profile, with the shock front arriving at 10,000 days, has $\xi_N$ down to 0.16.

In figure 2.8 we have plotted the normalized values for $\gamma'_M$, the upper cut-off Lorentz factor of the power-law particle distribution. Although formally $\gamma'_M$ should be reset to infinity at the shock front, we picked a value corresponding to a cut-off above $10^{18}$ Hz through the entire simulation, for a fluid element heading directly towards the observer (see also section 2.2.2 and appendix 2.8). In our simulation settings, this results in $\gamma'_M$ peak values of the order $10^7$, and because these values are arbitrary as long as they are sufficiently large, we have normalized the $\gamma'_M$ profiles. The profiles show two things. First, they show the steep decline directly after the injection of new hot electrons. This steep decline and the $\sim 7$ orders of magnitude difference between shocked and unshocked $\gamma'_M$ are numerically challenging, which
2.3 Fluid dynamics

Figure 2.7: $\xi_N$, the fraction of electrons accelerated to a power law distribution for the same lab frame times as in fig. 2.3.

Figure 2.8: Upper cut-off Lorentz factor of the power law distribution $\gamma_m'$, normalized to 1 at the shock front, for the same lab frame times as in fig. 2.3.
is why we implemented the logarithm of $\gamma'_{\text{M}}$ in our code instead (again see appendix 2.8). Second, the width of the profile is a measure for the size of the hot region discussed in section 2.2.2. It can be seen from the figure that the width of the profile increases over time. The width will nevertheless remain smaller than the width of the density profile by far. In our simulations, we resolve the $\gamma'_{\text{M}}$ profile and use it to determine the local refinement level.

## 2.4 Spectra and light curves

Using the simulation data described in the previous section we have calculated spectra and light curves at various observation times and frequencies. We have saved a total number of 10,000 snapshots of the fluid profile, with $10.8 \cdot 10^4$ seconds between consecutive snapshots, corresponding to a resolution $c \, dt \sim 3 \cdot 10^{15}$ cm. Although this resolution is of the same order as the initial shock width, it is still sufficient at the early stage because the shock initially nearly keeps up with its own radiation. The effective resolution is given by $c \, dt/\Gamma^2 \sim 3 \cdot 10^{13}$ cm, which is only a factor 10 larger than the spatial resolution of $10^{12}$ cm and corresponds to a temporal resolution of $dt \sim 1000$ seconds. It is therefore ensured that the blast wave in the initial stage is covered by over a hundred snapshots.

### 2.4.1 Expected spectral and temporal behaviour

The scaling behaviour for the critical frequencies and the flux is well known from analytical estimations assuming a homogeneous radiating slab directly behind the shock front with fluid properties determined via either the BM or ST solution (see e.g. Wijers & Galama 1999; Frail et al. 2000; Granot & Sari 2002). We summarize the interstellar medium (ISM) scalings below, with $\nu_m$ denoting the peak frequency, $\nu_a$ the synchrotron self-absorption critical frequency and $\nu_c$ the cooling break frequency. At the observer times and frequencies in this paper we find either $\nu_a < \nu_m < \nu_c$ or $\nu_m < \nu_a < \nu_c$.

In the relativistic limit, the corresponding scalings are

$$
\nu_a \propto \begin{cases} 
\nu_m^0, & \nu_a < \nu_m \\
\nu_m^{-(3p+2)/(p+4)}, & \nu_a > \nu_m 
\end{cases} 
$$

$$
\nu_m \propto t^{-3/2}, \quad (2.30)
$$

$$
\nu_c \propto t^{-1/2}, \quad (2.31)
$$

for the critical frequencies. Note that $t$ now refers to observer times. The flux above
2.4 Spectra and light curves

both peak and self-absorption break scales as

$$F \propto \begin{cases} v^{(1-p)/2} t^{(1-p)/4} & v < v_c \\ v^{-p/2} t^{(2-3p)/4} & v > v_c \end{cases} \quad (2.32)$$

If $v_a < v_m$ we get the following flux scaling below the peak break:

$$F \propto \begin{cases} v^2 t^{1/2} & v < v_a \\ v^{1/3} t^{1/2} & v > v_a \end{cases} \quad (2.33)$$

If $v_a > v_m$ we have below the self-absorption break

$$F \propto \begin{cases} v^2 t^{1/2} & v < v_m \\ v^{5/2} t^{5/4} & v > v_m \end{cases} \quad (2.34)$$

In the nonrelativistic limit the scalings are

$$v_a \propto \begin{cases} t^{6/5} & v_a < v_m \\ (3p-2)/(p+4) & v_a > v_m \end{cases} \quad (2.35)$$

$$v_m \propto t^{-3}, \quad (2.36)$$

$$v_c \propto t^{-1/5}, \quad (2.37)$$

for the critical frequencies. The flux above both peak and self-absorption break scales as

$$F \propto \begin{cases} v^{(1-p)/2} t^{(21-15p)/10} & v < v_c \\ v^{-p/2} t^{(4-3p)/2} & v > v_c \end{cases} \quad (2.38)$$

If $v_a < v_m$ we get the following flux scaling below the peak break:

$$F \propto \begin{cases} v^2 t^{-2/5} & v < v_a \\ v^{1/3} t^{8/5} & v > v_a \end{cases} \quad (2.39)$$

If $v_a > v_m$ we have below the self-absorption break

$$F \propto \begin{cases} v^2 t^{13/5} & v < v_m \\ v^{5/2} t^{11/10} & v > v_m \end{cases} \quad (2.40)$$

The summary above shows that only the temporal behaviour of the break frequencies and fluxes is altered by the transition to the nonrelativistic regime. We therefore do not expect spectra calculated from our simulations covering the transition to differ in slope from the slopes calculated above. The light curve slope, however, may differ.
2.4.2 Spectra

In figure 2.9 we have plotted spectra for a number of different observation times, ranging from 1 day to 10,000 days. For comparison we have also plotted the different power law slopes for 1 day as predicted by Granot & Sari (2002), where we have added a dependency on $\xi_N$. We plot predictions for both $\xi_N = 1$ and $\xi_N = 0.1$. It can be seen that the simulated spectrum still lies closer to the $\xi_N = 1$ prediction, just as we would expect for an early time spectrum. Because of shifts in both flux level and position of the spectral breaks for different values of $\xi_N$, the flux does not always lie in between the analytical predictions. For example, because the peak frequency $\nu_m$ for the simulation lies close to that of the $\xi_N = 1$ prediction and flux lies also closer to $\xi_N = 1$ but below, the resulting flux at higher frequencies ends up below both predictions.

Figure 2.9 proves that our method works and that the asymptotic behaviour for the spectral slopes matches the predicted slopes. For frequencies above the self-absorption break and below the cooling break this merely confirms that the synchrotron spectral function $Q(\nu'/\nu_m')$ has been implemented correctly. The flux at
frequencies above the cooling break however, shows the consequence of a finite and evolving upper cut-off $\gamma'_M$. A slope is reproduced that matches the prediction. It has been explained above and in EW09 how this slope now arises as a product of the interplay between the hot region and the blast wave width.

At the low frequencies, where synchrotron self-absorption plays a role, the simulations also reproduce a spectral slope that corresponds to what was expected from analytical calculations. The flux level is now dictated by the radiative transfer equation through a medium that is no longer completely transparent at these frequencies. As discussed in section 2.2.5, the resulting flux will differ by a factor of a few from Granot & Sari (2002), due to a difference in approach when calculating the absorption coefficient from the particle distribution.

We emphasize that fig 2.9 covers 10 orders of magnitude in frequency, 8 orders of magnitude in flux and four orders of magnitude in observer time. As we expected from analytical calculations, the spectral slopes in the different power law regimes do not change over time. The transitions between the different regimes are smooth. An explicit calculation of the sharpnesses of the transitions will be presented in a follow-up paper.

### 2.4.3 Light curves

We will use optical light curves to illustrate the consequences of the different assumptions and model parameters. In fig. 2.10 we present simulated light curves for simulations that differ only in the EOS used. Electron cooling and self-absorption have been disabled, $e_B$ is fixed at 0.01 and $\xi_N = 0.1$ everywhere throughout the simulation. This allows for a clear view on both the effect of the EOS and of the trans-relativistic break. The latter can be found at $\sim 1000$ days for all three simulations. This is somewhat earlier than the transition time determined from the fluid flow in section 2.3.3, which we determined to be around 1290 days (the difference is due to relativistic beaming). *The transition time is also later than what is usually assumed for the nonrelativistic transition by nearly a factor three.*

The difference in flux from the different EOS assumptions can be traced to the different thermal energy profiles (and hence, for fixed $e_B$, to magnetic field energies that differ with the same ratios), with $\Gamma_{ad} \equiv 4/3$ having the highest $e'_\text{th}$. This is illustrated in fig. 2.11. The difference in peak thermal energy densities between the fixed adiabatic index simulations is a factor of two, as expected from the ratio $(5/3 - 1)/(4/3 - 1)$. Because the flux depends on the thermal energy via the magnetic field strength and $\gamma_m$ (see equations 2.6 and 2.7), the flux for $\Gamma_{ad} \equiv 4/3$ is higher than that for $\Gamma_{ad} \equiv 5/3$. The light curve for the advanced equation of state lies between the two limiting cases, starting close to the $4/3$ curve but moving to the $5/3$ as the flow becomes nonrelativistic. *This additional decrease in flux has the consequence that the*
Figure 2.10: Comparison of optical (at $t \cdot 10^{14} \text{Hz}$) light curves for different equations of state. The top curve has $\Gamma_{\text{ad}} = 4/3$, the center curve the advanced EOS and the bottom curve $\Gamma_{\text{ad}} = 5/3$. For clarity as few complications as possible are included: cooling and self-absorption are switched off, $\epsilon_B$ is fixed at 0.01 and $\xi_N = 0.1$ everywhere. Also plotted are the expected relativistic slope $3(1 - p)/4$ and nonrelativistic slope $(21 - 15p)/10$.

**Advanced equation of state light curve will be slightly steeper in the transrelativistic phase than the fixed adiabatic index light curves.**

In figure 2.12 we show the effects of the detailed evolution calculation of $\xi_N$. Aside from the full simulations, we also perform two simulations that keep $\xi_N$ fixed throughout at either 1 or 0.1 but are otherwise identical to the full simulation. At early times in the radio, before the peak frequency has passed, the $\xi_N \equiv 1$ curve lies above the full simulation curve, where at early times in the optical it lies below.

Figure 2.13 shows the fractional difference between complete and fixed $\epsilon_B \equiv 0.01$ simulation light curves (the light curves themselves lie very close to each other on a plot using a logarithmic scale), calculated via $(F_{\text{fixed}} - F_{\text{complete}})/F_{\text{complete}}$, where $F$ is the flux. The figure shows that the late time light curves for the fixed $\epsilon_B$ end up below the light curves that trace the evolution of the magnetic field. This can be understood from the fact that evolving $\epsilon_B'$ according to $\epsilon_B' \propto (\rho')^{4/3}$ implies a relative rise of the magnetic field energy density relative to (but still far below) the thermal energy when the flow becomes nonrelativistic. Fixing $\epsilon_B$ forces the magnetic field energy density to follow $\rho^\Gamma$. The flux spans many orders of magnitude over time.
2.4 Spectra and light curves

Figure 2.11: Direct comparison between thermal energy density $\epsilon_{th}'$ profiles for the different equations of state. The top profile has $\Gamma_{ad} = 4/3$, the center curve the advanced EOS and the bottom profile has $\Gamma_{ad} = 5/3$. All snapshots are taken at 515 days simulation time. The difference in radius between the blast waves is 2 percent.

A quantitative comparison between the slopes from the radio and optical light curves for the full simulations is shown in figure 2.14. The horizontal lines in the plot indicate expected asymptotic values for the power law scalings. In the relativistic limit, the expected slope is $1/2$ before passage of $\nu_m$ and $-1.125$ after (using $p = 2.5$). After the cooling break passes, a further steepening to $-1.375$ is expected. In the nonrelativistic regime the expected slopes before and after passage of the cooling break are $-1.65$ and $-1.75$ respectively. The plot shows that the relativistic slopes are matched very well. The radio light curve quickly tends to $1/2$ and after passage of the peak frequency it moves in $\sim 95$ days to $-1.125$, where it remains until the onset of the nonrelativistic break time. The optical light curve starts out in the intermediate regime from the passage of $\nu_m$, with the passage of the cooling break coming too early for the light curve to settle into the pre-cooling break slope of $-1.125$. The post-cooling break slope $-1.375$ is obtained instead and is again maintained until the nonrelativistic break. The light curve slopes in the nonrelativistic regime are less steep than expected. A number of factors play a role here, as discussed above. The advanced EOS leads to a steepening of the decay during the transition phase
Figure 2.12: Top: Comparison between complete simulation light curve (solid line) at radio frequency $4.8 \cdot 10^9$ Hz and simulation curves where $\xi_N$ is kept fixed at 1.0 (dashed line) and 0.1 (dotted line) throughout. The complete curves start out close to $\xi_N \equiv 1$ but slowly evolve towards $\xi_N \equiv 0.1$. Bottom: same as left, only now for optical frequency $5 \cdot 10^{14}$ Hz. As with the radio light curves, the full curve starts near $\xi_N \equiv 1$ but turns to $\xi_N \equiv 0.1$. 

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(which lasts well over 10,000 days), whereas the increase in $e_B'$ relative to $e_{\text{th}}'$ and the decrease in $\xi_N$ (leading to an increase in energy per particle) lead to less steep decay. The change in slope in the nonrelativistic regime is the result of the interplay between these different factors, with the end result being a slope less steep than expected. The final nonrelativistic slopes differ significantly from those expected from analytical models, and this has a large impact on fitting models to observational data.

### 2.5 GRB 030329

In the preceding section we have systematically explored the different aspects of transrelativistic blast wave afterglows with respect to dynamics and radiation for standard values of the input parameters. We now qualitatively compare radio data for GRB 030329 to simulation results using physical parameters for this GRB established by earlier authors as input. GRB 030329 is one of the closest and brightest GRBs for which an afterglow was found. Because of this brightness, the afterglow could be monitored for an extended period of time at various wavelengths and after six years its radio signal is still being observed (Kamble et al., 2009). GRB 030329 is a good example to use to illustrate the various aspects of the radiation code.
Figure 2.14: Power law behaviour of the optical and radio light curves. The lines plot $\alpha$, assuming for every two consecutive data points the relationship $F_{i+1} = F_i (t_{i+1}/t_i)^\alpha$. The solid line refers to the radio light curve and the dashed line to the optical light curve. The horizontal lines denote $0.5, 3(1-p)/4 = -1.125, (2-3p)/4 = -1.375, ((21-15p)/10) = -1.65, (4-3p)/2 = 1.75$ from top to bottom.

The redshift of GRB 030329 has been determined to be $z = 0.1685$ (Greiner et al., 2003), which leads to a luminosity distance of $2.4747 \cdot 10^{27}$ cm (for a flat universe with $\Omega_M = 0.27, \Omega_L = 0.73$ and $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Various authors have determined the physical properties of the GRB from analytical model fits to the data (e.g. Willingale et al. 2004; Berger et al. 2003; Sheth et al. 2003; Van Der Horst et al. 2005; Huang et al. 2006; Van Der Horst et al. 2008) with various assumptions for the jet structure. Here we take the physical parameters established by Van Der Horst et al. (2008). From their conclusion for the jet break time and cooling frequency at this time, and assuming equipartition between accelerated particle energy and magnetic field energy (i.e. a fixed $\epsilon_e \equiv \epsilon_B$ in their model), we arrive at $E = 2.6 \cdot 10^{51}$ erg (for a spherical explosion), $n_0 = 0.78 \text{ cm}^{-3}, p = 2.1, \epsilon_e = \epsilon_B = 0.27$. We assume a homogeneous medium and we set the hydrogen mass fraction in this medium to unity. Van Der Horst et al. (2008) fix $\xi_N$ at unity, but we use a nonrelativistic limit $\xi_{N,NR} = 0.1$. Because GRB 030329 shows clear evidence of a collimated outflow, it is no longer sufficient to assume a spherical explosion. When calculating emission...
from a jet, we assume a hard-edged jet with opening angle 22 degrees and no lateral spreading.

We have plotted light curves at 15 GHz, 4.8 GHz and 1.4 GHz in figure 2.15, including data points from the Westerbork Synthesis Radio Telescope (WSRT) (4.8 GHz and 1.4 GHz, Van Der Horst et al. 2005) for comparison and the Very Large Array (VLA) (15 GHz, Berger et al. 2003). Two things are clearly visible. First, our simulated light curves still differ strongly from the data, although largely the same input parameters have been used for the blast wave simulations as those that were derived from fitting to the dataset using an analytical model for the blast wave. The different assumptions in Van Der Horst et al. (2005) account for this in part, but nevertheless this demonstrates once more the need for detailed fit prescriptions from
2 Gamma-ray burst afterglows from trans-relativistic blast wave simulations

Figure 2.16: Left: Simulated light curve at 200 MHz for GRB 030329, top curve for spherical explosion and bottom curve for hard-edged jet with opening angle 22 degrees. We have drawn the following slopes from left to right: $1/2$, $5/4$ and $11/10$. LOFAR sensitivity for 25 core and 25 remote stations after four hours of integration time is 0.273 mJy and indicated by the horizontal line. Center: Simulated light curve at 120 MHz for GRB 030329. LOFAR sensitivity is 0.145 mJy. Right: Simulated light curve at 75 MHz for GRB 030329. LOFAR sensitivity is 4.2 mJy, too high to be shown in the plot.

simulations (a similar conclusion was drawn in EW09 for the ultra-relativistic case).
Second, the counterjet contribution will stand out clearly for a hard-edged jet model. For now, the comparison between simulation and data is still qualitative. Newer data are available and once the simulation input parameters are fine-tuned with respect to the data as well (as opposed to estimated using an analytical fit to the data), it should be possible to address the rise of the counterjet in a more quantitative fashion.

Because of the equipartition constraint on $\epsilon_B$ and $\epsilon_e$, both were given a relatively high value of 0.27 at the shock front. In the nonrelativistic regime, the magnetic energy density will grow relative to the thermal energy density further downstream (although both will decrease strongly in absolute value). At $t_e \sim 34.7$ yrs, the last time covered by our simulation (set up to cover 10,000 days in observer time) we find that $\epsilon_B$ has risen to approximately 0.36 at the back of the blast wave (0.43 where the blast wave density again equals the upstream density). Even further downstream, when the density has fallen three orders of magnitude below the upstream density, $\epsilon_B$ peaks at 1.28. This is not unphysical, but merely an indication that magnetic fields have become dynamically important in a region of the fluid which has no consequence for the light curve.

2.5.1 Low frequency array light curves and resolved images

Figure 2.16 shows predicted light curves at the very low frequencies that can be explored in the near future by radio telescopes such as the Low Frequency Array (LOFAR), assuming four hours of integration time, 25 core stations and 25 remote stations (Nijboer & Pandey-Pommiers, 2009). GRBs are among the prime targets for LOFAR’s Transient Key Project (Fender et al., 2006). Most of the time all light curves lie below the self-absorption break. This, in combination with the $\nu_m$ break,
Figure 2.17: Radio images at 200 MHz. Left: after \( \sim \) 16 days. The intensity increases monotonically outward. The outer radius is \( 1.91 \cdot 10^{17} \) cm. Center: after \( \sim \) 270 days. The intensity decreases monotonically outward. The outer radius is \( 9.7 \cdot 10^{17} \) cm. Right: after \( \sim \) 4300 days. A central bright ring with radius \( \sim 10^{18} \) cm appears. At larger radii the intensity decreases monotonically. The outer radius is \( 3.4 \cdot 10^{18} \) cm.

a hard-edged jet model and the turnover to the nonrelativistic regime, leads to an interesting double peak structure of the light curve. First the signal rises, according to the relativistic rise in the self-absorption regime that predicts a slope of \( 1/2 \). After \( \sim 4 \) days a clear jet break is seen and the resulting drop in slope leads to a decreasing signal again. Around circa 150 days the critical frequency \( \nu_m \) passes through the observed frequency band. The slope of the spherical explosion changes accordingly towards the predicted relativistic \( 5/4 \). Around approximately 600 days the blast wave has become nonrelativistic and the counterjet starts to contribute (but is still overwhelmed by the forward jet). The predicted nonrelativistic slope for the spherical explosion is now \( 11/10 \).

We have included LOFAR detection thresholds for four hours of integration time. These sensitivity limits are higher than those presented in Van Der Horst et al. (2008), because LOFAR has been scaled down in the meantime. The spherical explosion energy is an overestimation of the actual explosion energy and the flux levels corresponding to the jet simulations lie closer to what will actually be received. However, from fig. 2.15 it is clear that our qualitative comparison systematically underestimates the actual flux levels. Also, the integration time used in LOFAR can easily be increased, even up to days. Fig. 2.16 therefore does not mean that GRB 030329 will not be observable by LOFAR, but only that a larger integration time than four hours is likely required.

For 200 MHz we have calculated spatially resolved images as well, for spherical explosions. Three images are presented in figure 2.17, for three different observer times. They show three qualitatively different types of behaviour. At 15 days a limb-brightened image is observed, whereas at 240 days the image on the sky becomes
limb-darkened. At 3900 days another structure is visible and a brighter ring exists within the image, at a radius of $\sim 10^{18}$ cm. This is a result of the self absorption break $\nu_A$ being different for different emitting regions of the blast wave. These images are fully consistent with predictions from Granot (2007) for the ultra-relativistic case.

### 2.6 Summary and conclusions

In this paper we present the results of detailed dynamical simulations of GRB afterglow blast waves decelerating from relativistic to nonrelativistic speeds, as well as spectra and light curves calculated from these simulations using a method first described in van Eerten & Wijers (2009) (EW09) that we have extended to include more details of synchrotron radiation. We summarize our results and conclusions below.

We have performed, for the first time, hydrodynamical simulations of decelerating relativistic blast waves using adaptive mesh refinement techniques including a parametrisation for a shock accelerated electron distribution radiating via synchrotron radiation. From these simulated blast waves we have calculated light curves and spectra at various observer times and frequencies. An advanced equation of state was used for the dynamical simulations, with an effective adiabatic index smoothly varying between the relativistic and nonrelativistic limit. Three additional parameters were traced during hydrodynamical evolution: maximum accelerated particle Lorentz factor, magnetic field energy density and accelerated particles number density. We assumed that fewer particles were accelerated by shocks that are less relativistic. To obtain the observed flux including synchrotron self-absorption, a set of linear transfer equations were solved for beams traversing through the blast wave. This method expands upon EW09 by including self-absorption and dynamically calculated electron cooling.

We have used standard assumptions for the GRB explosion energy ($\sim 10^{52}$ erg) and circumburst particle number density ($\sim 1$ cm$^{-3}$) for a homogeneous medium and particle acceleration and magnetic field parameters. By directly comparing against various analytical models and expected limiting behaviour, we draw a number of conclusions about the dynamics of our simulations:

- We find that the transition of $\beta\gamma$ directly behind the shock front from the relativistic to the nonrelativistic regime occurs later than expected, around $\sim 1290$ days rather than $\sim 450$ days, for the standard model parameters.

- An analytical calculation of $\beta\gamma$ according to Huang et al. (1999) is found to overestimate the late time values by a factor 4/3.

- Directly applying the Sedov-Taylor solution to late time afterglow evolution is found to overestimate the radius by a few percent and keeping the adiabatic...
index fixed throughout the evolution of the blast wave will lead to systematic differences of as much as ten percent.

- The density jump across the shock may be arbitrarily high for relativistic shocks, but will be a factor of four in the nonrelativistic regime. This is known from the shock jump conditions. Our simulations show that the quantity $D/\gamma^2$, a combination of lab frame density and Lorentz factor directly behind the shock, will remain close to four times the unshocked density throughout the entire simulation, even though the effective adiabatic index evolves from relativistic to nonrelativistic.

- If we assume the number of magnetic field lines through the surface of a fluid element a constant, the magnetic field energy will become relatively larger compared to the thermal energy. It will remain a small fraction however (assuming only a small amount of energy is used for magnetic field creation across the shock). Our approach allows for different assumptions on the magnetic field energy evolution.

- The upper cut-off Lorentz factor $\gamma'_M$ for the shock-accelerated relativistic power law electron distribution decreases on a distance scale much smaller than the width of the blast wave due to synchrotron losses and determines the shape of the spectrum near and above the cooling break.

Using the output from the dynamical simulations, we calculate the flux. The following general conclusions are drawn for the radiation:

- Calculated light curves show a transition between the relativistic and nonrelativistic regime at around 1000 days in observer time, again later than expected.

- The observed fluxes for different assumptions on the equation of state may differ by a factor of a few. This is a direct consequence of the amount of thermal energy (and therefore magnetic field energy) directly behind the shock front.

- Implementing a changing effective adiabatic index has the consequence that the resulting light curve will slowly evolve from the relativistic limiting value to the nonrelativistic value. This transition takes tens of years in observer time and will lead to a steeper decay in the afterglow light curve than predicted by analytical models assuming a fixed index.

- This steepening is a smaller effect than the combined effect of evolving the magnetic energy density and the accelerated particle number density. When all effects are included, the final light curve slopes differ markedly from the
analytically expected values. This implies a significant complication for late
time afterglow modeling.

We have applied our approach to GRB 030329 as well, using physics parameters
derived by Van Der Horst et al. (2008) using an analytical model. It is shown that
the resulting radio light curves differ up to an order of magnitude between simulation
and analytical model, although this can be partly attributed to some different assump-
tions. Assuming a hard edged jet with an opening angle of 22 degrees, our simulated
light curves show a rebrightening due to the counterjet around 1000 days. Simulated
curves at radio frequencies that will be observable using LOFAR show that four hours
of integration time is likely not sufficient to distinguish the signal from the noise and
a larger integration time is required. Finally, spatially resolved images show a bright
ring that, depending on the precise power law regime that is observed, may be located
not only in the center or on the edge but also at intermediate radii within the afterglow
image. This is consistent with earlier work by Granot (2007) on afterglow images in
the relativistic phase.

A recent paper (Zhang & MacFadyen, 2009) has appeared discussing afterglow
blast waves decelerating to nonrelativistic velocities using twodimensional simu-
lations. The authors find that lateral expansion of a relativistic GRB jet is a very slow
process and that the jet break is mostly due to the edges of the jet becoming visible.
This implies the hard edged jet model that we have applied to GRB 030329 is suffi-
cient to model the jet break at \( \sim 4 \) days. Zhang & MacFadyen (2009) do not include
synchrotron self-absorption and calculate the cooling break by assuming the cooling
time throughout the entire blast wave equal to the grid time.

The approach to calculating light curves and spectra from generic fluid simu-
lations that we present in this paper assumes that synchrotron radiation is the dominant
radiative process, that particle acceleration takes place in a region far smaller than
the blast wave width and that the feedback on the dynamics from the radiation is
negligible. We briefly address these issues in appendix 2.9.

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self-absorption.
2.8 Numerical implementation

2.8.1 Appendix A1: Partial differential equations

AMRVAC was written to solve a system of coupled partial differential equations. When adding additional equations to the solver, it is therefore best to use partial differential equations. In the case of the magnetic field energy $e_B'$, we start by rewriting equation (2.13) as

$$\frac{\partial}{\partial t} e_B' + v_i \frac{\partial}{\partial x^i} e_B' = 0.$$  (2.41)

If we multiply this equation by $\rho'\gamma$ and add to this the continuity equation

$$\frac{\partial}{\partial t} \rho' + \frac{\partial}{\partial x^i} \rho' v^i = 0,$$  (2.42)

which we first multiply by $e_B' / \rho'^{4/3}$, we obtain

$$\frac{\partial}{\partial t} \gamma e_B' + \frac{\partial}{\partial x^i} \gamma e_B' v^i = 0.$$  (2.43)

This is the type of conservation equation that AMRVAC is specialized in, and it is therefore the quantity $\gamma e_B'$ that we calculate in AMRVAC.

For the evolution of the upper cut-off $\gamma'_M$ we follow a similar procedure. We start by simplifying equation (2.9) to

$$\frac{d}{dt} \frac{\rho'^{1/3}}{\gamma'_M} = \alpha \frac{\rho'^{1/3} B^2}{\gamma},$$  (2.44)

where $\alpha = \sigma_T / 6\pi m_e c$, and $t$ refers to lab frame time (i.e. emission time). The Lorentz factor in the source term arises when we write the comoving time derivative in the lab frame. We can now follow a procedure similar to what we did for the magnetic energy density, but first we rewrite the equation above once more for numerical reasons. The quantity $\gamma'_M$ varies over many orders of magnitude in a very short time span and any quantity that depends on $\gamma'_M$ linearly is therefore difficult to deal with numerically. We solve this by rewriting equation (2.44) into

$$\frac{d}{dt} \ln \frac{\rho'^{1/3}}{\gamma'_M} = \frac{\alpha \gamma'_M B^2}{\gamma}.$$  (2.45)

Although there still is a linear dependence on $\gamma'_M$ in the source term, in practice this equation provides a better starting point for AMRVAC. From combining with the
continuity equation we get

\[ \frac{\partial}{\partial t} \gamma \rho' \ln \frac{\rho'^{1/3}}{\gamma'_M} + \frac{\partial}{\partial x} v^i \gamma \rho' \ln \frac{\rho'^{1/3}}{\gamma'_M} = \alpha \gamma'_M \rho' B'^2, \] (2.46)

with \( \gamma \rho' \ln \rho'^{1/3} / \gamma'_M \) the quantity of interest. A similar approach to tracing the effect of cooling in the context of relativistic blast waves has also been taken by Downes et al. (2002). Although in our formalism \( \gamma'_M \) at the shock front should be reset to infinity, and therefore \( \gamma \rho' \ln \rho'^{1/3} / \gamma'_M \) to minus infinity, we just take a very low value for \( 1 / \gamma'_M \) in order to minimize numerical diffusion. In our simulations, this arbitrarily low value corresponds to a hard cut-off of the spectrum above \( \nu \sim 10^{18} \) Hz, at frequencies sufficiently far above our observation range to be of no consequence. The ‘real’ \( \gamma'_M \) catches up with the numerical \( \gamma'_M \) almost instantaneously.

### 2.8.2 Appendix A2: Shock detection method

In total, AMRVAC now calculates the evolution of three additional quantities: \( n_{\text{acc}} \) (using equation 2.15), \( \gamma e'_B / \rho'^{1/3} \) (using equation 2.43) and \( \gamma \rho' \ln (\rho'^{1/3} / \gamma'_M) \) (using equation 2.46). All three quantities get reset wherever a shock is detected. Both the reset values of \( n'_{\text{acc}} \) and \( e'_B \) depend on the fluid variables directly behind the shock front and it is therefore important that we determine the position of the shock front as accurately as possible. Mathematically speaking, a shock is a discontinuity in the flow variables with a sudden increase in entropy across the discontinuity. In practice, however, finding a shock in a numerical approximation is more involved, both due to numerical shock diffusion and because, strictly speaking, there is a shock discontinuity across every grid cell boundary.

This has the consequence that if we try to find shocks by checking for discontinuities or for entropy jumps, we will find both shocks all over the numerical diffused shock region and at a random variety of positions where the numerical noise happens to rise above a predetermined shock threshold. This then implies that we keep on resetting the additional quantities over some region, something which is especially unwanted in the case of \( \gamma'_M \), given our approach where we take a fluid cell to contain a collection of electrons that have been shocked exactly at the same time and we critically rely on the size of the hot region (see section 2.2.2 and EW09, appendix D, for details).

Because the shocked particle number density and the magnetic field density depend directly on the fluid variables, using, for example, a jump in \( \beta \gamma \) as a trigger, as has been done by Downes et al. (2002), is not an option in these cases either. Although it serves as an excellent indicator of the front of a shock, it will not point us to a location where we can find information on the strength of the shock, but to an arbitrarily defined position just in front of that.
In this paper we solve the issues of shock detection with two shock detection algorithms, both of them making use of the fact that $D/\gamma^2$ directly behind the shock is four times the density just in front of the shock. For $n'_\text{acc}$ and $e'_B$ we define the shock front to be at the peak of the Lorentz factor profile, in the region where $D/\gamma^2 > 3.5\rho_0$. The numerical constant is arbitrary and could be taken closer to 4. With this method we ensure that the shock is detected at those positions where the fluid quantities are sufficiently close to their peak values, although multiple shock peaks may be detected in close proximity of each other due to numerical noise.

For $\gamma'_M$ it is essential that we only detect a single shock front. Here we care less about the precise fluid variable values. For the purpose of resetting $\gamma'_M$ we define the shock front to be at that position where $D/\rho_0\gamma^2$ crosses the value 3.5. For a single shock front, this only happens once. Although, in principle, $\gamma\beta$ could have been used instead of $D/\rho_0\gamma^2$, the latter offers the significant advantage that it does not change in scale over the course of the simulation and always remains close to 4, whereas $\gamma\beta$ becomes arbitrarily small.
Figure 2.18 illustrates the use of the Lorentz factor profile peak as a shock detector. It shows that the numerical diffusion is really very small and that $\gamma'_M$ changes over a significantly smaller spatial scale than $\rho'$.

### 2.8.3 Appendix A3: Synchrotron self-absorption

Equation (2.18) can also be expressed as

$$\alpha'_{\nu'} = K' \nu'^{-2} \int_{\gamma'_m}^{\gamma'_M} d\gamma'_e \mathcal{P}\left(\frac{\nu'}{\nu'_{cr,e}}\right) \times \left[ (p + 2) \gamma'_e^{-(p+1)} \left(1 - \frac{\gamma'_e}{\gamma'_M}\right)^{p-2} + (p - 2) \frac{\gamma'_e^{-p}}{\gamma'_M} \left(1 - \frac{\gamma'_e}{\gamma'_M}\right)^{p-3} \right],$$

where

$$K = C \frac{\sqrt{3} q_e^3 B'}{8 \pi m_e^2 c^2}.$$

Here we have used the fact that $N_e(\gamma'_e) \propto \gamma'_e (1 - \gamma'_e / \gamma'_M)$ (i.e. a slightly modified powerlaw distribution). The scaling factor ($C$) of $N_e(\gamma'_e)$ is determined in terms of $\gamma'_M$ and $\gamma'_m$ from the requirement that the total number of accelerated electrons constitutes a fixed fraction $\xi_N$ of the available electrons. The symbol $\mathcal{P}$ denotes the pitch angle (the angle between magnetic field and particle velocity) averaged version of the synchrotron function, a dimensionless function representing the shape of the synchrotron spectrum for a single electron in the same way $Q$ represents the spectrum of a distribution of particles. The $\nu'_{cr,e}$ in the argument is connected to $\gamma'_e$ via equation (2.7) (see also EW09).

By changing variables from $\gamma'_e$ to $y = \nu' / \nu'_{cr,e}$ we obtain:

$$\alpha'_{\nu'} = \frac{K}{2} \left(\frac{4 \pi m_e c}{3 q_e B'}\right)^{\frac{p}{2}} \nu'^{-\frac{p+4}{2}} \times \left[I_1(y_M, y_m) + I_2(y_M, y_m)\right].$$

where the quantities $I_1(y_M, y_m)$ and $I_2(y_M, y_m)$ are:

$$I_1 \equiv (p + 2) \int_{y_M}^{y_m} dy \mathcal{P}(y) y^{\frac{p+2}{2}} \left[1 - \left(\frac{y}{y_M}\right)^{1/2}\right]^{p-2}$$

and

$$I_2 \equiv (p - 2) y_M^{1/2} \int_{y_M}^{y_m} dy \mathcal{P}(y) y^{\frac{p+3}{2}} \left[1 - \left(\frac{y}{y_M}\right)^{1/2}\right]^{p-3}.$$
2.8 Numerical implementation

As in the case of \( Q(y_M, y_m) \) (see van Eerten & Wijers 2009) values of \( I_1(y_M, y_m) \) and \( I_2(y_M, y_m) \) are tabulated for moderate \( y_M \) and \( y_m \), whereas their limiting behavior, for extreme values of \( y_M \) and \( y_m \) is analytically estimated. Namely, if \( y_M / y_m \to 1 \) the integrals of both \( I_1 \) and \( I_2 \) reduce to the expression inside the integral, evaluated at \( y_m \), multiplied by the appropriate range in \( y \)-space, i.e. \( (y_M - y_m) \).

For \( y_M \ll 1 \) the integrals’ behaviour becomes hard to analytically estimate, especially for general values of \( y_m \) and \( p \). Instead, we fit approximate expressions to the values extrapolated from the tables.

In the case that \( y_m / y_M \gg 1 \) and \( y_m \) is outside the tabulated values, we can break the integral into two parts by using the last tabulated value \( \bar{y}_m \). For \( I_2 \) the formula is:

\[
I_2 = (p-2) y_M^{1/2} \times \left( \int_{y_M}^{\bar{y}_m} dy \, P(y) y^{-3} \left[ 1 - \left( \frac{y_M}{y} \right)^{1/2} \right]^{p-3} Q(y_m) y_m^{p+1} \right) - Q(\bar{y}_m) \bar{y}_m^{p+1} \tag{2.52}
\]

For \( I_1 \) the result is identical, only the terms inside the square brackets (including \( Q(x) \)) have to be evaluated for \( p \to p + 1 \). Finally, for \( y_M \gg 1 \) the result of both integrals is approximated by zero.

2.8.4 Appendix A4: Adaptive mesh and linear radiative transfer

We do not integrate over \( A \) in equation (2.2) directly, but resolve the different rays instead. After the integral over \( t_c \) is finished (i.e. the bundle of linear radiative transfer equations is solved) we can integrate over \( A \) to obtain the flux, while the unintegrated result provides a resolved picture of the emission from the fluid. For spherically symmetric fluid flow or for an observer positioned along the symmetry axis of a jet, the intensity on surface \( A \) is symmetric around a central point. Because the fluid itself moves at nearly the speed of light, it is not a priori clear how many rays need to be included and how they should be spaced along \( A \) in order to obtain a good resolution. An efficient response to this dilemma is to apply the adaptive-mesh refinement concept to \( A \). The equidistant surface (EDS) \( A \) contains a grid with every grid cell containing the intermediate results for a single ray. Every four neighbouring cells in each direction on the EDS are grouped together in a single block. The EDS area \( dA \) that each cell represents may differ, and if the resolution threatens to become too low to adequately capture the radiation profile, a block will be split in half along each direction, spawning new blocks that represent half the size of the parent block along each direction. The refinement criterion that is used is that the combined flux from
Figure 2.19: Intensity and refinement levels perpendicular to the axis between the observer and the source. The maximum refinement level drops quickly to zero away from the edge of the jet. The intensity has been rescaled to an arbitrary scale suitable for direct comparison between intensity profile and refinement levels. Note that the lowest refinement level is zero. The refinement levels shown refer to those used in the radiation calculation, not those in the fluid simulation.

A given block must not differ by more than 1 percent (or a lower threshold, as set by the user) from the combined flux from a coarsened version of the block where only the odd cells are taken into account (with the odd cells representing an appropriately increased surface element). Neighbouring blocks may differ one refinement level at most. We have plotted an example of this strategy in figure 2.19. In practice we set the maximum refinement level similar to that of the fluid simulation. We also use the fluid simulation grid refinement structure to determine the starting refinement structure of the EDS at each iteration for the transfer equation solver, in order to make sure that we will also capture the blast wave when it still has a small radius.

2.9 Applicability of our model

The radiation code is written to be generally applicable to output from relativistic fluid dynamics simulations. However, a number of assumptions and simplifications have been made that are dependent on the physical context. In this appendix we
briefly discuss the consequences and relevance of our assumptions in the case of GRB afterglow blast waves decelerating down to nonrelativistic speeds. We discuss the relevance of an alternative radiative process, inverse Compton scattering, of our assumption that particle acceleration takes places in a region much smaller than the blast wave width and of adiabatic expansion of the blast wave with the radiation losses having no effect on the dynamics.

### 2.9.1 Importance of inverse Compton scattering

A limitation to the applicability of our approach arises from the fact that inverse Compton (IC) radiation, which is not calculated, becomes important when the ratio \( P'_\text{syn} / P'_{\text{IC}} \) approaches, or drops below unity. This ratio is also equal to the ratio between the corresponding energy fields that power the emission \( e'_\text{B} / e'_\text{ph} \) (Rybicki & Lightman 1986), with \( e'_\text{ph} \) being the energy density of the (synchrotron) radiation field. The effect of IC emission on the emitted spectra has been thoroughly investigated in Sari & Esin (2001). In this paper we focus only on its influence on cooling rates, as the high-energy synchrotron spectrum is expected to dominate IC emission for a wide range of physical parameters and radii.

Instead, however, of calculating the entire photon energy density due to synchrotron radiation we can use the fact that the cross-section for IC scattering drops fast beyond the Thomson limit (Blumenthal & Gould 1970). Thus, we can define an ‘effective’ photon field for an electron of Lorentz factor \( \gamma'_e \) as

\[
e'_{\text{ph,eff}}(\gamma'_e) = \frac{4\pi}{c} \int_0^{\nu'_{\text{Thom}}} I'_{\nu',\text{syn}} \, d\nu',
\]

where \( \nu'_{\text{Thom}} = \frac{m_e c^2}{\gamma'_e h} \) (with \( h \) denoting Planck’s constant) is the photon frequency for which the scattering occurs marginally within the Thomson regime for a head-on collision and \( I'_{\nu',\text{syn}} \) is the synchrotron specific intensity. Approximating the specific intensity by

\[
I'_{\nu',\text{syn}} \sim \xi_N n' B' \left( \frac{\nu'}{\nu'_m} \right)^{(1-p)/2} \frac{R}{\Gamma},
\]

and employing the analytical relations of the BM solution we find that right behind the shock front

\[
\frac{P'_\text{syn}}{P'_{\text{IC}}} \approx 7.5 \cdot 10^{16} f(p) (5 \cdot 10^{-8})^{3-p} \xi_N^{p-2} \epsilon_B^{3-p} \times
\]

\[
\epsilon_e^1 n_0^{-p+1} (\gamma'_e)^{\frac{3-p}{2}} R^{-1} \Gamma^{\frac{5-3p}{2}},
\]

(2.53)
where \( f(p) = (p-1)^{p-2}(p-2)^{1-p}(3-p) \), \( \Gamma \) is the Lorentz factor of the shock front and \( R \) the shock radius. By plugging in standard values of this paper (\( \xi_N = 1, \ p = 2.5, \ n_0 = 1 \text{ cm}^{-3}, \ \epsilon_B = 10^{-2}, \ \epsilon_e = 10^{-1}, \ E = 10^{52} \text{ erg} \)) and making further use of the BM equations we find for \( \gamma'_m \)

\[
\frac{P'_\text{syn}}{P'_\text{IC}} \approx 1.8 \cdot 10^{-10} R^{0.5}.
\] (2.56)

This means that IC will dominate synchrotron energy losses for the lowest energy electrons throughout the relativistic phase of the fluid. A comparison of IC to adiabatic cooling, using the synchrotron loss term from equation (2.9) and

\[
\left( \frac{\text{d} \gamma'_m}{\text{d} t} \right)_\text{syn} / \left( \frac{\text{d} \gamma'_m}{\text{d} t} \right)_\text{IC} = \frac{P'_\text{syn}}{P'_\text{IC}},
\]

gives

\[
\left( \frac{\text{d} \gamma'_m}{\text{d} t} \right)_\text{ad} / \left( \frac{\text{d} \gamma'_m}{\text{d} t} \right)_\text{IC} = 10^{-43} R^{2.5}.
\] (2.57)

The corresponding radius after which adiabatic expansion will sharply take over is about \( 1.6 \cdot 10^{17} \text{ cm} \). Moreover, for an electron of energy \( \gamma'_e = 10^4 \gamma'_m \) (i.e. on the order of \( \gamma'_M \)), synchrotron losses will prevail at approximately \( 3 \cdot 10^{17} \text{ cm} \). Therefore it is only early on, and certainly not close to the subrelativistic transition, that IC cooling will affect the evolution of \( \gamma'_M \) or \( \gamma'_m \) for the assumptions made in this paper.

### 2.9.2 Gyration radius

The gyroradius for an electron with Lorentz factor \( \gamma_e \) is given by

\[
r'_g = \gamma_e m_e c^2 / q_e B'.
\]

Using the BM solution and equation (2.7) we find that for the most energetic electrons at the shock front that contribute to received flux within the frequency range under consideration (\( 10^8 - 10^{18} \text{ Hz} \)) this radius lies below

\[
r_g = 5.6 \cdot 10^{-75} \nu_{\text{cut-off}}^{1/2} R^{33/8} \text{ cm},
\] (2.58)

where \( \nu_{\text{cut-off}} \) is the cut-off frequency in the lab frame, used to set \( \gamma'_M \) at the shock front (i.e. a frequency safely above \( 10^{18} \text{ Hz} \)). This should be compared against the size of the hot region, a measure of the spatial distance over which electrons cool significantly. The cooling time for high energy electrons that cool on a scale much shorter than the scale over which the fluid variables change, is approximately equal to

\[
t_{\text{cool}} \approx 6\pi m_e c / \sigma_T (B')^2 \gamma'_M
\]

when the electron cools down to \( \gamma'_M \). Using the self-similar parameter as an intermediate step, this can be linked to a spatial size of the hot region for \( \gamma'_M \):

\[
\delta_{\text{hot}} = 8 \cdot 10^{-52} \nu_{\text{cut-off}}^{-1/2} R^{33/8} \text{ cm}.
\] (2.59)
Thus, for $\nu_{\text{cut-off}} = 10^{18-21}$ Hz the gyroradius of the most energetic electrons is many orders of magnitude smaller than the size of the corresponding hot region, justifying our assumption that particle acceleration takes place locally near the shock front (which we have implemented by a local injection of hot electrons) and the use of an advection equation to model the evolution of $\gamma_M$.

### 2.9.3 Feedback on the dynamics

A last issue is the possibility of the radiative energy losses becoming comparable to the initial energy load of the fireball ($E$) that would imply a considerable impact on the dynamics of the flow. This could be explicitly quantified by calculating the total radiative output during the simulations and comparing it to the explosion energy. However, we can address this issue in a more qualitative manner by noting that the low energy electrons cool predominantly by causing the expansion of the volume they are occupying (slow cooling), even at the shock front. Moreover, for values of $p > 2$ (which is the case under consideration) these electrons are the main energy carriers. In combination with the fact that the total energy residing in relativistic electrons is limited by $\epsilon_e$ (typically of the order of 10%), we are confident that the total energy radiated through synchrotron, especially in the subrelativistic regime, will be orders of magnitude smaller than $E$, and thus not affect considerably the dynamics.
Practical flux prescriptions for gamma-ray burst afterglows, from early to late times

K. Leventis, H.J. van Eerten, Z. Meliani, R.A.M.J. Wijers

Abstract  We present analytic flux prescriptions for broadband spectra of self-absorbed and optically thin synchrotron radiation from gamma-ray burst afterglows, based on one-dimensional relativistic hydrodynamic simulations. By treating the evolution of critical spectrum parameters as a power-law break between the ultrarelativistic and non-relativistic asymptotic solutions, we generalize the prescriptions to any observer time. Our aim is to provide a set of formulas that constitutes a useful tool for accurate fitting of model-parameters to observational data, regardless of the dynamical phase of the outflow. The applicability range is not confined to gamma-ray burst afterglows, but includes all spherical outflows (also jets before the jet-break) that produce synchrotron radiation as they adiabatically decelerate in a cold, power-law medium. We test the accuracy of the prescriptions and show that numerical evidence suggests that typical relative errors in the derivation of physical quantities are about 10 per cent. A software implementation of the presented flux prescriptions combined with a fitting code is freely available on request and on-line \(^1\). Together they can be used in order to directly fit model parameters to data.


3.1 Introduction

Gamma-ray bursts (GRBs) are believed to be produced by powerful relativistic outflows resulting from the catastrophic death of massive stars (Woosley, 1993), or the merger of two compact objects (Eichler et al. 1989). The burst itself (prompt emission) likely arises from internal shocks occurring due to the variability of the central engine (Rees & Mészáros, 1994; Sari & Piran, 1997b), while the afterglow emission comes from the interaction of the same outflow with the medium surrounding the burster (Rees & Mészáros, 1992; Paczyński & Rhoads, 1993). Although the dominant radiation process behind the prompt emission is not yet clear, it is well established that the afterglow radiation is dominated by synchrotron emission from shock-accelerated electrons (Mészáros & Rees, 1993; van Paradijs et al., 2000).

The prompt emission is typically very brief and concentrated at high energies. On the other hand, afterglows are often visible over many more orders of magnitude both in time- and frequency-space (see Mészáros 2006 for an extensive review of GRB research). Thus, studying the afterglow radiation allows us to put a multitude of constrains both on the microphysics (e.g. the fraction of internal energy going to the magnetic fields and the power-law accelerated electrons) governing the shocked plasma (Spitkovsky, 2008; Sironi & Spitkovsky, 2009), as well as on the basic physical parameters describing the phenomenon macroscopically, like blast-wave energy, density and structure of the surrounding medium.

It is these macroscopic parameters that determine the dynamical evolution of the outflow. However, a full analytic description of the dynamics is only possible when the spatial component of the four-velocity of the outflow $\beta\gamma$ is either much greater (Blandford & McKee, 1976) or much smaller (Sedov, 1959) than 1. Therefore, relativistic hydrodynamic (RHD) simulations (Kobayashi et al., 1999; Meliani et al., 2007; Zhang & MacFadyen, 2009; De Colle et al., 2012) are the most accurate means of studying the intermediate dynamical regime linking the ultrarelativistic and Newtonian solutions (see however Huang et al. 1999). Van Eerten et al. (2010a) have numerically studied the light curves of outflows advancing through all three dynamical regimes and have shown that the transition is slow, i.e. deviations from the expected relativistic behaviour appear well before the Newtonian asymptotes are reached, mainly due to the changing adiabatic index of the shocked gas. For typical burst parameters (isotropic blast-wave energy $E_{\text{iso}} = 10^{52}$ ergs, ambient medium number density $n_0 = 1 \text{ cm}^{-3}$) the Sedov-Taylor scalings set in at a few thousand days, observer time, implying that an appreciable portion of the afterglow (typically around hundreds of days) emanates from outflows with dynamics that cannot be described analytically by either of the two asymptotic solutions.

1 The URL is http://www.astro.uva.nl/research/cosmics/gamma-ray-bursts/software/.
Soon after the discovery of the first afterglows (Costa et al., 1997; Groot et al., 1997) efforts were made to calculate broadband synchrotron spectra and light curves as a function of burst parameters (Wijers et al., 1997; Sari et al., 1998; Panaitescu & Kumar, 2000). The common way to do this is by tying the dynamical evolution of the blast-wave in regimes where this is feasible to radiation models that, according to the jump conditions at the shock front, calculate the resulting spectra. Despite the success of early efforts in capturing general features of the observed spectra, the progressive refinement of the models has led to very different estimates of the physical parameters of individual bursts. For example Wijers & Galama (1999) and Granot & Sari (2002) have both fitted GRB 970508 and their derived values differ up to 3 orders of magnitude. Furthermore, the applicability of most of these models is restricted to a particular dynamical phase and only recently have there been a few attempts at addressing the entire evolution of spectra and light curves through the performance of simulations (Zhang & MacFadyen, 2009; van Eerten et al., 2012; Wygoda et al., 2011; De Colle et al., 2012). Even so, these models do not contain a treatment of self-absorption (apart from van Eerten et al. 2012), necessary to model low-frequency observations with e.g. the Expanded Very Large Array EVLA (Perley et al. 2011), the Low-Frequency Array LOFAR (Morganti et al. 2011) and the upcoming Karoo Array Telescope MeerKAT (Booth et al. 2009) and Square Kilometre Array SKA (Carilli & Rawlings 2004), and do not provide flux prescriptions. Van Eerten et al. (2012) do provide a broadband fit code, but it requires the use of a parallel computer network.

The purpose of this work is to provide accurate analytic flux prescriptions, based on one-dimensional RHD simulations, that are applicable to both the ultrarelativistic and Newtonian phase but also, and perhaps more importantly, to observer times when the outflow is transitioning from the former to the latter. Apart from the typical, initially ultrarelativistic outflows of GRBs, the formulas we present are applicable to Newtonian as well as relativistic (Soderberg et al. 2010) outflows from supernova explosions in the adiabatic phase (Chevalier, 1977, 1982; Draine & McKee, 1993) and mildly relativistic outflows originating from binary neutron star (NS) mergers, expected to produce detectable electromagnetic (EM) counterparts to gravitational wave detections (Nakar & Piran, 2011; Metzger & Berger, 2012). They can also be applied to relativistic outflows resulting from the tidal disruption of stars by a super-massive black hole Bloom et al. 2011; Metzger et al. 2012, under the limiting assumption of quasi-spherical outflow. The presented model naturally accounts for the exact shape of the synchrotron spectrum (including self-absorption, but ignoring cooling) and the structure of the blast-wave. Furthermore, it can be applied to a range of power-law density structures of the circumburst medium, a possibility previously studied by van Eerten & Wijers (2009) and De Colle et al. (2012). This allows for
modelling of more complex environments, expected on a theoretical basis (Ramirez-Ruiz et al. 2005) and deduced observationally (Curran et al. 2009). With such a tool a light curve can be fitted without the need of costly simulations and the restrictions of models specialising in specific dynamical phases, or preset structures of the circum-burst medium. In order to obtain the flux prescriptions we combine three elements: (1) analytic formulas for flux scalings during the Blandford-McKee and the Sedov-Taylor phases, (2) one-dimensional, hydrodynamic simulations, using the adaptive mesh refinement code \textit{amrvac} (Meliani et al., 2007; Keppens et al., 2012), that span the whole range of the dynamics (from ultrarelativistic to Newtonian velocities) and (3) a radiative-transfer code that uses simulation snapshots and a parametrisation of the microphysics to calculate instantaneous spectra.

This paper is organised as follows: in Section 3.2 we briefly describe the setup of the performed simulations and the subsequent calculations of spectra and light curves. In Section 3.3 we present formulas that describe the flux as a function of physical parameters in both the relativistic and the Newtonian phase of the outflow. That includes specifying the flux at any given power-law segment, as well as a description of the sharpness of the spectral breaks that occur at critical frequencies. We then proceed in Section 3.4 to connect the two dynamical regimes (relativistic and Newtonian) by treating the transition from the former to the latter as a prolonged temporal \textit{break} the characteristics of which can be linked to the physical parameters of the burst and its environment. In Section 3.5 we describe how one can make use of the flux prescriptions to obtain spectra at any given time. We also show comparisons between spectra based on simulations and spectra constructed using the provided prescriptions. Finally, we present an application of this model to mildly relativistic outflows from binary neutron star mergers in order to assess the recent predictions of Nakar & Piran (2011) concerning the detectability of the produced radio signals. In Section 3.6 we discuss our results and the implications of this work for GRB afterglow models.

3.2 Numerical treatment

3.2.1 Simulations

We have made use of the \textit{amrvac} adaptive-mesh-refinement numerical code to run a series of simulations for different values of physical parameters. These simulations span a wide range of the four-velocity at the shock front, from ultrarelativistic values ($\sim 70$) down to $\sim 0.05$.

In total 7 different simulation runs were used to arrive at the presented prescriptions. They can be characterized by the blast-wave energy $E_{52}$ (in units of $10^{52}$ erg),
### Table 3.1: Parameters of simulations used to derive the flux prescriptions. Left column enumerates the performed simulations.

<table>
<thead>
<tr>
<th>Sim</th>
<th>$E_{52}$</th>
<th>$k$</th>
<th>$n_0$</th>
<th>$\Gamma_{in}$</th>
<th>$R_{max}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
<td>60</td>
<td>$3 \cdot 10^{19}$</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>0.0</td>
<td>4.0</td>
<td>60</td>
<td>$10^{19}$</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>60</td>
<td>$4 \cdot 10^{19}$</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>0.75</td>
<td>0.5</td>
<td>28</td>
<td>$5 \cdot 10^{19}$</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>60</td>
<td>$4 \cdot 10^{19}$</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>70</td>
<td>$10^{19}$</td>
</tr>
<tr>
<td>7</td>
<td>0.01</td>
<td>2.0</td>
<td>1.0</td>
<td>10</td>
<td>$7 \cdot 10^{19}$</td>
</tr>
</tbody>
</table>

starting Lorentz factor of the shock $\Gamma_{in}$, maximum radius of the simulation-box $R_{max}$, slope of the power-law density distribution of the surrounding medium $k$ and value of the number density at $10^{17}$ cm, $n_0$. In Table 3.1 we present the values of these parameters for each run.

### Resolution

The 1D simulation box typically extends from $10^{16}$ cm up to a few times $10^{19}$ cm, although both limits were modified accordingly for physical models with different external density profiles and blast-wave energies. Utilizing the adaptive-mesh-refinement approach in \textsc{amrvac} we have used a maximum of 20 refinement levels which set the effective resolution of the grid. For 120 cells at the lowest refinement level, this amounts to a resolution of $\sim 4.77 \cdot 10^{11}$ cm per cell. For comparison, the corresponding width of the initial BM shell for Simulation 1 is (van Eerten et al. 2010) $R_{in}/(6 \Gamma_{in}^2) = 5 \cdot 10^{12}$ cm, with $R_{in}$ denoting the radius of the shock.

We have checked for convergence against runs of different refinement levels (see also van Eerten & MacFadyen 2012b; De Colle et al. 2012) and have also checked our results against theoretically predicted values in the early part of the outflow when the resolution demands are the highest. They have been found in good agreement.

### Equation of state

In all the simulations we have used a ‘realistic’ equation of state (EOS) with an effective adiabatic index (Meliani et al. 2004) that lies between the ultrarelativistic...
and non-relativistic limits (4/3 and 5/3, respectively):

\[ \Gamma_{\text{ad,eff}} = \frac{5}{3} - \frac{1}{3} \left( 1 - \frac{\rho' c^2}{u'^2} \right). \] (3.1)

In the above equation \( \rho' c^2 \) is the comoving rest-mass energy density and is weighed against the total energy density (including rest-mass) of the gas \( u' \).

This Synge-type (Synge 1957) EOS has also been used in van Eerten et al. (2010), where the effects have been analysed and comparisons to constant \( \Gamma_{\text{ad,eff}} \) have been made. In short, its effect on the observed flux is that of a very gradual transition from values close to (but not at) those corresponding to an ultrarelativistic EOS to values approaching those of a non-relativistic EOS.

### 3.2.2 Radiative-transfer code

The snapshots generated by the simulation runs were post-processed using a radiative-transfer code (van Eerten & Wijers, 2009; van Eerten et al., 2010). During the post-processing an array of beams is created and propagated at the speed of light through the three-dimensional generalisation of the 1D snapshots towards the observer. The elements of this array take the value of the specific intensity \( I_\nu \). At each step the solution to the equation of radiative transfer is applied to each beam and the value for the intensity is updated through the equation

\[ I_\nu = I_0 e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu}), \] (3.2)

where \( I_0 \) is the value of the intensity at the previous step, \( \tau_\nu \) is the optical depth and \( S_\nu \) the source function. We note that the optical depth of a single simulation-cell can be larger than unity.

The array of intensities is constructed so that the positions of its elements lie on the surface from which light-signals arrive at the observer simultaneously. The density of the beams is determined through an adaptive-mesh approach ensuring sufficient resolution. Once all beams have crossed the entire blast-wave integration of the intensity over the surface yields the flux

\[ F_\nu = \frac{(1+z)}{d^2} \int_A I_\nu \, dA, \] (3.3)

where \( d \) is the luminosity distance to the observer, \( (1+z) \) the cosmological correction and \( A \) the surface defined by the beams.

In the case of on-axis jets and spherical outflows, like the ones considered in this study, eq. (3.3) can be reduced to a 1D integral due to the axisymmetry of the source.
3.3 Flux prescriptions in the asymptotic dynamical regimes

In this Section we demonstrate how to combine blast-wave dynamics in each of the self-similar regimes with synchrotron radiation theory to arrive at scalings that describe the observed flux as a function of frequency and time.

3.3.1 Shock dynamics

We first outline the dynamics of the outflow. As mentioned in Section 3.1, one can obtain power-law scalings for the four-velocity as well as the mass and energy densities right behind the shock front as a function of blast-wave radius and time in the two extreme regimes of dynamical behaviour of the afterglow.

Ultrarelativistic phase

In the ultrarelativistic (also known as Blandford-McKee, hereafter BM) phase these scalings take the form (Blandford & McKee 1976):

\[ \gamma_2 \propto \Gamma_{sh} \propto t^{-\frac{3+k}{2}}, \]  
\[ e_2 \propto \Gamma_{sh}^2 n_1, \]  
\[ n_2 \propto \Gamma_{sh} n_1. \]

where \( \gamma_2 \) is the Lorentz factor of the shocked plasma measured in the lab frame (which coincides with the frame of the surrounding medium), \( e_2 \) describes its internal energy density and \( n_2 \) its number density. Here and throughout this paper the quantities \( e \) and \( n \) will be measured in the comoving frame of the fluid they are describing. \( \Gamma_{sh} \) is the Lorentz factor of the propagating shock wave, while \( n_1(r) = n_0 (r/r_0)^{-k} \) is the density of the unshocked medium surrounding the burster as a function of radius. In all the results presented the characteristic distance \( r_0 \) is put at \( 10^{17} \) cm. The time ‘\( t \)’ appearing in eq. (3.4) is the lab-frame time and is to be distinguished from the observed arrival time of light signals which is affected by light travel-time effects. While in the BM phase we can assume \( r \sim ct \) for the radius of the shock.

Newtonian phase

At the phase where the outflow has become Newtonian (also known as Sedov-Taylor phase, hereafter ST) a similar approach can be taken to describe its kinetic and thermodynamic evolution. Dimensional analysis implies
3 Practical flux prescriptions for gamma-ray burst afterglows, from early to late times

\[ r(t) \propto \left( \frac{E t^2}{n_1(r)} \right)^{1/5} \]  

(3.7)

for the scaling of the radius of the shock as a function of time, blast-wave energy and structure of the surrounding medium. This leads to

\[ \beta(t) \propto t^{-\frac{3-k}{5-k}}, \]  

(3.8)

\[ e_2 \propto t^{-\frac{6}{5-k}}, \]  

(3.9)

\[ n_2 \propto n_1, \]  

(3.10)

where \( \beta = \frac{dr}{c dt} \) is the bulk velocity of the shock, in units of \( c \).

3.3.2 Optically thin and self-absorbed synchrotron radiation

The next step is to combine these scalings with formulas that calculate optically thin as well as self-absorbed synchrotron radiation. We assume that electrons are the primary radiating particles and express their post-shock energy distribution as a power-law \( N(E) \propto E^{-p} \). The lower limit of the distribution \( E_m = \gamma_m m_e c^2 \) corresponds to a comoving synchrotron frequency \( \nu'_m = \frac{3}{4\pi} \gamma_m^2 Q_e B'_m c \sin \alpha \), where \( Q_e \) and \( m_e \) are the electron charge and mass, respectively, \( B'_m \) is the comoving value of the magnetic field and \( \alpha \) is the pitch angle between magnetic field and velocity of the electron. In the case of optically thin radiation the flux at a given frequency \( \nu' \), in the comoving frame will be

\[ F'_{\nu'} \propto (p - 1) N B' Q(y_m), \]  

(3.11)

where \( N \) is the total number of power-law accelerated electrons, \( y_m = \frac{\nu'}{\nu'_m} \) and

\[ Q(x) \equiv x^\frac{1-p}{2} \int_0^x y^{\frac{p-3}{2}} \mathcal{P}(y) \, dy. \]  

(3.12)

The function \( Q \) contains all the spectral information. The function \( \mathcal{P} \) appearing in eq. (3.12) is the synchrotron function \( F(x) \) (Rybicki & Lightman 1986) integrated over all pitch angles, for an isotropic pitch-angle distribution.

In the case of optically thick synchrotron radiation the comoving flux is given by

\[ F_{\nu} \propto \frac{j_{\nu}}{\alpha_{\nu}} r^2, \]  

(3.13)
3.3 Flux prescriptions in the asymptotic dynamical regimes

where $j'_\nu$ and $\alpha'_\nu$ are the comoving emissivity and absorption coefficient, respectively, and $r^2$ is a measure of the radiating surface. The expressions for $j'_\nu$ and $\alpha'_\nu$ have the form

$$j'_\nu \propto (p-1)\xi n_2 B' Q(y_m),\quad (3.14)$$

$$\alpha'_\nu \propto \frac{(p-1)^2(p+2)}{p-2} \xi^2 n_2^2 \epsilon_e^{-1} e_2^{-1} B' \nu^{-2} Q(p+1, y_m),\quad (3.15)$$

where $\xi$ is the fractional number of electrons accelerated to a power-law distribution, $\epsilon_e$ is the fraction of internal energy carried by the accelerated electrons and $Q(p+1, y_m)$ is evaluated using eq. (3.12) by replacing $p$ with $(p+1)$. The effect of absorption is the introduction of another critical frequency in the spectrum, $\nu_a$. The ordering of $\nu_m$ and $\nu_a$ determines the shape of the spectrum. In Fig. 3.1 and 3.2 the two different spectra are represented schematically in order to illustrate the break frequencies as well as the slopes of the power laws that they connect.

To arrive at the relations above we have demanded that the distribution of the electrons obeys

$$\int_{E_m}^{\infty} N(E) dE = \xi n_2$$

and

$$\int_{E_m}^{\infty} N(E) E dE = \epsilon_e e_2,$$\quad (3.17)

where we have used the implicit condition that $p > 2$.

In this study we ignore the effect of electron-cooling on the spectra since the fast timescales associated with it translate to distances much shorter than the typical size of a simulation-cell. This work focuses on observer times when the influence of cooling on the observed spectra is negligible.

### 3.3.3 General form of flux scalings

Equations (3.4)-(3.10) allow us to calculate the conditions right behind the shock front as a function of time. From these equations we can compute instantaneous spectra by utilizing standard formulas for synchrotron radiation. However, eq. (3.4)-(3.10) do not specify the structure of the shocked plasma well behind the shock. Such a specification would have allowed us to convolve the different parts of the outflow that contribute to the observed radiation at any given observer time. Such an approach has been taken, for example, by Granot & Sari (2002).

Our approach is based on the fact that in the self-similar regimes the scaling behaviour of the emitted radiation can be calculated by considering a homogeneous
slab that obeys the scalings of the shocked fluid right behind the shock. However, in order to correctly calibrate the scalings (i.e. provide the correct flux levels) one has to capture the shock structure behind the front and the most reliable way to do this is by simulations.

The calibration is done by introducing a polynomial in terms of \( p \) and \( k \) – the spectral index of the power-law accelerated electrons and the index describing the structure of the surrounding medium, respectively. The former quantity (\( p \)) determines the electron distribution, everywhere behind the shock front, for a given set of thermodynamic parameters, while the latter (\( k \)) affects the structure of the decelerating blast-wave. Two standard values for \( k \) are often assumed in the literature, namely 0 (constant density medium) and 2 (constant stellar wind environment). However, fits to \( k \) (Yost et al., 2003; Curran et al., 2009) often indicate different conditions, motivating us to use it as a free parameter. Values for the factors of the calibrating polynomial are then derived by demanding that they satisfy the system of equations resulting from runs of models with different \( p \) and \( k \). The range of values we have explored are \([2.1, 3]\) for \( p \) and \([0, 2]\) for \( k \). Therefore this is also the range under which the presented prescriptions are applicable.
3.3 Flux prescriptions in the asymptotic dynamical regimes

![Graph showing flux prescriptions in the asymptotic dynamical regimes](image_url)

Figure 3.2: Spectrum 2. Normalised form of the spectrum when $\nu_m < \nu_a$.

Having put all of the ingredients together, the equation describing the flux at any given power-law segment of the synchrotron spectrum, either in the BM or the ST phase, has the general form

$$F_{\nu} = C_{\text{pol}} h(p) \xi^{q_1} \xi^{q_2} \epsilon_B^{q_3} \epsilon_r^{q_4} n_0^{q_5} E_{52}^{q_6} \nu^{q_7} \nu^{q_8} (1+z)^{q_9} d_{28}^{-2}, \quad (3.18)$$

where

$$\log C_{\text{pol}} = g_0 + g_p p + g_{pp} p^2 + g_k k + g_{kk} k^2 \quad (3.19)$$

and $h(p)$ is a function of $p$, different for each power-law segment. It takes the following values:
3 Practical flux prescriptions for gamma-ray burst afterglows, from early to late times

\[ h(p) = \begin{cases} 
\frac{(p-2)}{(p-1)(p+2)} \frac{3p+2}{3p-1}, & F_\nu \propto \nu^2 \\
\frac{1}{(p+2)} \frac{G(p)}{G(p+1)}, & F_\nu \propto \nu^{5/2} \\
\frac{p-1}{3p-1} \left( \frac{p-2}{p-1} \right)^{-2/3}, & F_\nu \propto \nu^{1/3} \\
(p-1) G(p) \left( \frac{p-2}{p-1} \right)^{p-1}, & F_\nu \propto \nu^{(1-p)/2} 
\end{cases} \]  

(3.20)

where

\[ G(p) = \frac{\Gamma\left(\frac{5}{4} + \frac{p}{4}\right) \Gamma\left(\frac{p}{4} + \frac{19}{12}\right) \Gamma\left(\frac{p}{4} - \frac{1}{12}\right)}{\Gamma\left(\frac{7}{4} + \frac{p}{4}\right) (p+1)}. \]

(3.21)

\( G(p) \) as well as other factors appearing in eq. (3.20) originate from the limiting behaviour of \( Q(x) \). For details see van Eerten & Wijers (2009).

Equation (3.18) shows all the possible physical dependencies of the flux that this model is taking into account. We have introduced the fraction of internal energy carried by the magnetic field \( \xi_B \). This quantity, along with \( \xi_e, \xi \) and \( p \) constitute a group describing the microphysics of the shocked electrons and enter via synchrotron theory. Therefore, the exponents \( q_\xi, q_e, q_B \) and the prefactor \( h(p) \) remain the same regardless of the dynamics of the outflow. On the other hand there are two quantities describing the burster and its environment: \( n_0 \) and the blast-wave energy \( E_{52} \) (measured in units of \( 10^{52} \) erg), two quantities describing the frequency (\( \nu_{\text{obs}} \)) and the time (\( t_{\text{obs}} \)) of the observation and two more describing the cosmological distances usually associated with GRBs: the redshift \( z \) and the luminosity distance \( d_{28} \) (in units of \( 10^{28} \) cm).

We note that the inclusion of \( \xi \) in our description of the microphysics has the implication that one cannot uniquely determine the values of all model parameters at once. This is a consequence of the degeneracy of the model which for a set of primed parameters \( E_{52}' = \frac{E_{52}}{f}, n_0' = \frac{n_0}{f}, \xi_e' = f \xi_e, \xi_B' = f \xi_B, \xi' = \xi \), produces the same spectrum as the set of unprimed parameters. This degeneracy was first pointed out by Eichler & Waxman (2005) and can also be seen in eq. (3.24), (3.25) and Tables 3.9-3.12 presented in Section 3.4. As a result, a value for one of the parameters must be assumed during fitting in order to determine the others.

All the \( q \)-exponents appearing in eq. (3.18) are determined analytically. They are in general unique for a particular power-law segment in a given dynamical phase of the outflow. This also holds for all the \( g \)-factors appearing in eq. (3.19). Their values, however, are determined by matching them to numerical results (for a variety
of \((p, k)\) values) and solving the resulting systems of equations. They are in fact the calibration of the flux scalings.

### 3.3.4 Flux scalings

**Flux scalings during the BM phase**

A very similar approach has been taken by van Eerten & Wijers (2009). These authors have explored optically thin synchrotron radiation from relativistic outflows, taking into account all the possible spectra that result from either fast or slow cooling (Sari et al. 1998). Here we expand on that by including self-absorption. Table 3.2 contains the values of the \(q\)-exponents (analytically derived dependencies), while Table 3.3 contains the values of the \(g\)-factors (numerically determined calibration).

The values of the \(q\)-exponents are in agreement with the formulas presented in Granot & Sari (2002), apart from the fact that we have chosen to include an extra parameter \(\xi\). The normalisation of the flux scalings results in slightly lower fluxes (of order 30\%) compared to Granot & Sari (2002) a difference which can be attributed to the varying adiabatic index of the simulations (van Eerten et al. 2010). Below \(\nu_m\) we have ignored stimulated emission associated with a population inversion of the electron distribution at the low-energy limit, something which Granot & Sari (2002) have included in their model. That accounts for a factor of approximately \([3(p + 2)/4]\) difference in flux between those predictions and the present ones.

**Flux scalings during the ST phase**

For the Newtonian phase of the outflow, we repeat the same procedure as in the BM phase. The analytically derived dependencies are presented in Table 3.4 while the factors of the calibrating polynomials are presented in Table 3.5. We note that the values of \(q_r\) and \(q_t\) are in agreement with those presented in Frail et al. (2000), apart from their equation (A18) where the scaling for \(\nu_{\text{obs}} \ll \nu_m\) is in error. This error also appears in van Eerten et al. (2010).

### 3.3.5 The sharpness of spectral breaks

In practice the spectral breaks are not infinitely sharp as shown in Fig. 3.1 and 3.2 but show a gradual transition from one power-law index to another. To complete our description of instantaneous spectra, we need to provide a formula for the sharpness of spectral breaks.

An approach commonly used (Granot & Sari, 2002; van Eerten & Wijers, 2009) is to describe the flux close to a break by the following equation (Beuermann et al. 1999)
### Table 3.2: $q$-exponents in the BM phase. Analytically derived $q$-exponents for self-absorbed and optically thin synchrotron radiation in the BM phase. The quantities on the left correspond to different physical parameters on which the flux depends (see eq. 3.18). Each column describes their values for a given power-law segment.

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_{5/2}$</th>
<th>$F_{1/3}$</th>
<th>$F_{(1-p)/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_\xi$</td>
<td>−1</td>
<td>0</td>
<td>$\frac{5}{3}$</td>
<td>$2 - p$</td>
</tr>
<tr>
<td>$q_e$</td>
<td>1</td>
<td>0</td>
<td>$-\frac{2}{3}$</td>
<td>$p - 1$</td>
</tr>
<tr>
<td>$q_B$</td>
<td>0</td>
<td>$-\frac{1}{4}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{p+1}{4}$</td>
</tr>
<tr>
<td>$q_n$</td>
<td>$-\frac{2}{4-k}$</td>
<td>$-\frac{2}{4-k}$</td>
<td>$\frac{2}{4-k}$</td>
<td>$\frac{2}{4-k}$</td>
</tr>
<tr>
<td>$q_E$</td>
<td>$\frac{2}{4-k}$</td>
<td>$\frac{4+k}{4(4-k)}$</td>
<td>$\frac{10-4k}{3(4-k)}$</td>
<td>$\frac{12+4p-kp-5k}{4(4-k)}$</td>
</tr>
<tr>
<td>$q_v$</td>
<td>2</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1-p}{2}$</td>
</tr>
<tr>
<td>$q_t$</td>
<td>$\frac{2}{4-k}$</td>
<td>$\frac{20-3k}{4(4-k)}$</td>
<td>$\frac{2-k}{4-k}$</td>
<td>$\frac{12+3kp-5k-12p}{4(4-k)}$</td>
</tr>
<tr>
<td>$q_z$</td>
<td>$\frac{10-3k}{4-k}$</td>
<td>$\frac{36-11k}{4(4-k)}$</td>
<td>$\frac{10-k}{3(4-k)}$</td>
<td>$\frac{12+4p-kp}{4(4-k)}$</td>
</tr>
</tbody>
</table>

### Table 3.3: $g$-factors in the BM phase. Numerically determined $g$-factors for self-absorbed and optically thin synchrotron radiation in the BM phase. The quantities on the left correspond to different factors of the normalizing polynomial (see eq. 3.19). Each column describes their values for a given power-law segment.

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_{5/2}$</th>
<th>$F_{1/3}$</th>
<th>$F_{(1-p)/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_0$</td>
<td>−18.350</td>
<td>−26.170</td>
<td>−3.232</td>
<td>−6.689</td>
</tr>
<tr>
<td>$g_p$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.810</td>
</tr>
<tr>
<td>$g_{pp}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.075</td>
</tr>
<tr>
<td>$g_k$</td>
<td>0.237</td>
<td>0.185</td>
<td>−0.262</td>
<td>−0.286</td>
</tr>
<tr>
<td>$g_{kk}$</td>
<td>0.133</td>
<td>0.120</td>
<td>−0.014</td>
<td>0.020</td>
</tr>
</tbody>
</table>
### 3.3 Flux prescriptions in the asymptotic dynamical regimes

**Table 3.4:** $q$-exponents in the ST phase. Analytically derived $q$-exponents for self-absorbed and optically thin synchrotron radiation in the ST phase. The quantities on the left correspond to the different physical parameters on which the flux depends (see eq. 3.18). Each column describes their values for a given power-law segment.

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_{5/2}$</th>
<th>$F_{1/3}$</th>
<th>$F_{(1-p)/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_\xi$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$\frac{5}{3}$</td>
<td>$2 - p$</td>
</tr>
<tr>
<td>$q_e$</td>
<td>$1$</td>
<td>$0$</td>
<td>$-\frac{2}{3}$</td>
<td>$p - 1$</td>
</tr>
<tr>
<td>$q_B$</td>
<td>$0$</td>
<td>$-\frac{1}{4}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{p+1}{4}$</td>
</tr>
<tr>
<td>$q_n$</td>
<td>$\frac{4}{5-k}$</td>
<td>$-\frac{11}{4(5-k)}$</td>
<td>$\frac{3}{13(5-k)}$</td>
<td>$\frac{19-5p}{4(5-k)}$</td>
</tr>
<tr>
<td>$q_E$</td>
<td>$\frac{4}{5-k}$</td>
<td>$\frac{6+k}{4(5-k)}$</td>
<td>$\frac{7-4k}{3(5-k)}$</td>
<td>$\frac{10p+6k(p+5)}{4(5-k)}$</td>
</tr>
<tr>
<td>$q_v$</td>
<td>$2$</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1-p}{2}$</td>
</tr>
<tr>
<td>$q_t$</td>
<td>$\frac{2k-2}{5-k}$</td>
<td>$\frac{11}{2(5-k)}$</td>
<td>$\frac{24-10k}{3(5-k)}$</td>
<td>$\frac{3(7-5p)+4k(p-2)}{2(5-k)}$</td>
</tr>
<tr>
<td>$q_z$</td>
<td>$\frac{17-5k}{5-k}$</td>
<td>$\frac{24-7k}{2(5-k)}$</td>
<td>$\frac{6k-4}{3(5-k)}$</td>
<td>$\frac{5(2p+k)-3(2+kp)}{2(5-k)}$</td>
</tr>
</tbody>
</table>

**Table 3.5:** $g$-factors in the ST phase. Numerically determined $g$-factors for self-absorbed and optically thin synchrotron radiation in the ST phase. The quantities on the left correspond to different factors of the normalizing polynomial (see eq. 3.19). Each column describes their values for a given power-law segment.

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_{5/2}$</th>
<th>$F_{1/3}$</th>
<th>$F_{(1-p)/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_0$</td>
<td>$-16.510$</td>
<td>$-25.645$</td>
<td>$-5.513$</td>
<td>$-8.789$</td>
</tr>
<tr>
<td>$g_p$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$8.528$</td>
</tr>
<tr>
<td>$g_{pp}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0.230$</td>
</tr>
<tr>
<td>$g_k$</td>
<td>$0.126$</td>
<td>$-0.017$</td>
<td>$-0.044$</td>
<td>$0.157$</td>
</tr>
<tr>
<td>$g_{kk}$</td>
<td>$-0.009$</td>
<td>$-0.014$</td>
<td>$0.008$</td>
<td>$0.070$</td>
</tr>
</tbody>
</table>
Table 3.6: $s$-factors in the BM phase. Numerically determined $s$-factors (see eq. 3.23) for all possible breaks in the BM phase. Each column describes a specific break, with the two associated spectral indices denoted on top.

<table>
<thead>
<tr>
<th>Break</th>
<th>$s_0$</th>
<th>$s_p$</th>
<th>$s_k$</th>
<th>$s_{kk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \rightarrow \frac{5}{3}$</td>
<td>-2.91</td>
<td>-0.11</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$5 \rightarrow \frac{1-p}{2}$</td>
<td>1.24</td>
<td>-0.145</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$2 \rightarrow \frac{1}{3}$</td>
<td>1.64</td>
<td>0</td>
<td>-0.18</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{3} \rightarrow \frac{1-p}{2}$</td>
<td>1.83</td>
<td>-0.41</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$$F_\nu(\nu_{\text{obs}}) = A \left[ \left( \frac{\nu_{\text{obs}}}{\nu_0} \right)^{-a_1 s} + \left( \frac{\nu_{\text{obs}}}{\nu_0} \right)^{-a_2 s} \right]^{-1/s},$$  \hspace{1cm} (3.22)

where $(\nu_0, A)$ are the coordinates of the meeting point of the two power laws associated with the break, $a_1$ and $a_2$ are the asymptotic power-law indices before and after the break, respectively, and $s$ is the so called ‘sharpness parameter’. We have performed $\chi^2$-minimization fitting in logarithmic space to obtain values of $s$ for specific runs and used those to arrive at a description of the sharpness in terms of a polynomial of $p$ and $k$. This polynomial has the general form

$$s = s_0 + s_p p + s_k k + s_{kk} k^2.$$  \hspace{1cm} (3.23)

Its factors have been determined by solving the system of equations resulting from the application of eq. (3.22) to models with different $p$ and $k$ parameters. In Tables 3.6 and 3.7 we present the values of $s_0$, $s_p$, $s_k$ and $s_{kk}$ in the BM and the ST phase, respectively.

In Fig. 3.3 a best fit to the shape of the spectrum around $\nu_m$ is shown, for one of the run models. We also plot the flux for two different values of $s$ to illustrate the notable effect it can have on flux levels.

### 3.4 The transrelativistic regime

The results of the previous Section constitute a full description of the possible synchrotron spectra (ignoring cooling) during the ultrarelativistic and non-relativistic dynamical phases of the afterglow evolution. However, we have not yet addressed a large portion of the afterglow’s overall behaviour, namely the transrelativistic regime.
3.4 The transrelativistic regime

Figure 3.3: The effect of sharpness on the flux close to a spectral break. This fragment of the simulation-based spectrum focuses on the flux around $\nu_m$. The best fit is shown along with two more curves that have the same parameters but different sharpness. Simulation-based spectrum has the following model parameters: $E_{\text{52}} = 1$, $n_0 = 1$, $p = 2.5$, $k = 0$, $\xi = 10^{-2}$, $\epsilon_c = 10^{-1}$, $\epsilon_B = 10^{-2}$, $d_{28} = 1$, $z = 0.56$, $t_{\text{obs}} = 100$ days.

Table 3.7: $s$-factors in the ST phase. Numerically determined $s$-factors (see eq. 3.23) for all possible breaks in the ST phase. Each column describes a specific break, with the two associated spectral indices denoted on top.

<table>
<thead>
<tr>
<th></th>
<th>$2 \rightarrow \frac{5}{2}$</th>
<th>$5 \rightarrow \frac{1 - p}{2}$</th>
<th>$2 \rightarrow \frac{1}{3}$</th>
<th>$\frac{1}{3} \rightarrow \frac{1 - p}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>-5.50</td>
<td>3.50</td>
<td>2.63</td>
<td>1.88</td>
</tr>
<tr>
<td>$s_p$</td>
<td>0.73</td>
<td>-0.71</td>
<td>-0.24</td>
<td>-0.46</td>
</tr>
<tr>
<td>$s_k$</td>
<td>0.10</td>
<td>-0.07</td>
<td>-0.31</td>
<td>0.11</td>
</tr>
<tr>
<td>$s_{kk}$</td>
<td>0</td>
<td>-0.11</td>
<td>-0.07</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
During this stage the dynamics deviates considerably from the BM solution, without having settled yet into the ST solution. As mentioned in Section 3.1, this phase of the afterglow typically spans a few orders of magnitude in observer time, while there is no full description of its dynamics, even in the simple, spherical case.

An approach we have investigated and found useful is that of treating the trans-relativistic phase as a ‘break’ during which the temporal evolution of the spectrum’s critical parameters displays a smooth transition from the relativistic power-law behaviour to the non-relativistic one. These parameters could be, for example, the values of the flux at every possible power-law segment. However, the same level of accuracy can be achieved by using the positions of the critical frequencies and the flux at one of them, instead. Based on values for these parameters one can construct the spectrum (Sari et al., 1998; Wijers & Galama, 1999) because the slopes of the power-law segments are known for a given ordering of $\nu_a$ and $\nu_m$.

### 3.4.1 Peak flux

A convenient frequency to measure the flux is at $\nu_m$ of spectrum 1. This is because we can assume that the bulk of the electrons radiate most of their power at that frequency. Using eq. (3.11) in combination with the scalings for the dynamics in each of the two extreme phases of the outflow, we find for the flux at $\nu_m$

$$F_{m-BM} = C_{pol} \xi e_B^{1/2} n_0^{4/2(4-k)} E_{52}^{8-3k/2(4-k)} t_{obs}^{-(k/2)} (1+z)^{8-k/2(4-k)} d_{28}^{-2}, $$  

in the BM phase and

$$F_{m-ST} = C_{pol} \xi e_B^{7/2} n_0^{7/2(5-k)} E_{52}^{8-3k/3(3-k)} t_{obs}^{1-k} (1+z)^{2+k/3} d_{28}^{-2}, $$  

in the ST phase. The $g$-factors of $C_{pol}$ for both dynamical phases are presented in Table 3.8.

### 3.4.2 Critical frequencies

The behaviour of the critical frequencies can easily be deduced (in either the BM or ST phase) by equating the flux formulas on both sides of a spectral break. For each of $\nu_m$ and $\nu_a$ there will be two such expressions corresponding to the two possible spectra for the two different orderings of the frequencies. In general, the value of a critical frequency will be given by the following formula

$$\nu_{cr} = f_n s_n^{q_n} e_e^{q_e} e_B^{q_B} n_0^{q_0} E_{52}^{q_E} t_{obs}^{q_T} (1+z)^{q_z}. $$  

The numerical factors $f_n$ result from equating the fluxes of the power-laws at each side of the spectral break. Tables 3.9 and 3.10 summarize the formulas for the critical
### 3.4 The transrelativistic regime

Table 3.8: *g*-factors for \( F_{m-BM} \) and \( F_{m-ST} \). Numerically determined *g*-factors for \( F_{m} \) (measured at \( \nu_{m1} \)), both in the BM and ST phase.

<table>
<thead>
<tr>
<th></th>
<th>( F_{m-BM} )</th>
<th>( F_{m-ST} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_0 )</td>
<td>0.531</td>
<td>−0.674</td>
</tr>
<tr>
<td>( g_p )</td>
<td>0.487</td>
<td>0.305</td>
</tr>
<tr>
<td>( g_{pp} )</td>
<td>−0.060</td>
<td>−0.019</td>
</tr>
<tr>
<td>( g_k )</td>
<td>−0.291</td>
<td>−0.055</td>
</tr>
<tr>
<td>( g_{kk} )</td>
<td>0.004</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 3.9: \( f_n \) and \( q \)-exponents for critical frequencies in the BM phase, while \( \nu_a < \nu_m \) (spectrum 1). The \( q \)-exponents carry the analytically derived dependencies, while the \( f_n \)-factors carry the flux-calibrating \( C_{pol} \) and \( h(p) \) (see eq. 3.18, 3.19 and 3.20).

<table>
<thead>
<tr>
<th></th>
<th>( \nu_{a1} )</th>
<th>( \nu_{m1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_n )</td>
<td>( \left( \frac{C_{13} h_{13}}{C_{22} h_{22}} \right)^{\frac{3}{5}} )</td>
<td>( \left( \frac{C_{(1-p)/2} h_{(1-p)/2}}{C_{13} h_{13}} \right)^{\frac{6}{3p-1}} )</td>
</tr>
<tr>
<td>( q_\xi )</td>
<td>( \frac{8}{5} )</td>
<td>−2</td>
</tr>
<tr>
<td>( q_e )</td>
<td>−1</td>
<td>2</td>
</tr>
<tr>
<td>( q_B )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( q_n )</td>
<td>( \frac{12}{5(4-k)} )</td>
<td>0</td>
</tr>
<tr>
<td>( q_E )</td>
<td>( \frac{4-4k}{5(4-k)} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( q_t )</td>
<td>( \frac{-3k}{5(4-k)} )</td>
<td>( \frac{-3}{2} )</td>
</tr>
<tr>
<td>( q_z )</td>
<td>( \frac{8k-20}{5(4-k)} )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

For clearer presentation we have labelled as \( \nu_{a1} \) and \( \nu_{m1} \) the critical frequencies \( \nu_a \) and \( \nu_m \), respectively, when \( \nu_a < \nu_m \) (i.e. when spectrum 1 applies), while they are labelled as \( \nu_{a2} \) and \( \nu_{m2} \) in the opposite case when spectrum 2 applies.
Table 3.10: $f_n$ and $q$-exponents for critical frequencies in the BM phase, while $\nu_m < \nu_a$ (spectrum 2).

<table>
<thead>
<tr>
<th></th>
<th>$\nu_m2$</th>
<th>$\nu_a2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$</td>
<td>\left( \frac{C_2 h_2}{C_{5/2} h_{5/2}} \right)^2 \left( \frac{C_{(1-p)/2} h_{(1-p)/2}}{C_{5/2} h_{5/2}} \right)^{\frac{2}{4+p}}</td>
<td></td>
</tr>
<tr>
<td>$q_\xi$</td>
<td>-2</td>
<td>$\frac{4-2p}{4+p}$</td>
</tr>
<tr>
<td>$q_e$</td>
<td>2</td>
<td>$\frac{2(p-1)}{4+p}$</td>
</tr>
<tr>
<td>$q_B$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{p+2}{2(4+p)}$</td>
</tr>
<tr>
<td>$q_n$</td>
<td>0</td>
<td>$\frac{8}{(4+p)(4-k)}$</td>
</tr>
<tr>
<td>$q_E$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{8+4p-kp-6k}{2(4+p)(4-k)}$</td>
</tr>
<tr>
<td>$q_t$</td>
<td>$-\frac{3}{2}$</td>
<td>$\frac{3kp-2k-12p-8}{2(4+p)(4-k)}$</td>
</tr>
<tr>
<td>$q_z$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{10k+4p-24-kp}{2(4+p)(4-k)}$</td>
</tr>
</tbody>
</table>

Table 3.11: $f_n$ and $q$-exponents for critical frequencies in the ST phase, while $\nu_a < \nu_m$ (spectrum 1). The expressions contained in $f_n$ need to be evaluated using the formulas applicable to the ST regime.

<table>
<thead>
<tr>
<th></th>
<th>$\nu_a1$</th>
<th>$\nu_m1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$</td>
<td>\left( \frac{C_{1/3} h_{1/3}}{C_{5/2} h_{2}} \right)^{\frac{3}{5}}</td>
<td>\left( \frac{C_{(1-p)/2} h_{(1-p)/2}}{C_{1/3} h_{1/3}} \right)^{\frac{6}{5(5-k)}}</td>
</tr>
<tr>
<td>$q_\xi$</td>
<td>$\frac{8}{5}$</td>
<td>-2</td>
</tr>
<tr>
<td>$q_e$</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>$q_B$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$q_n$</td>
<td>$\frac{5}{5-k}$</td>
<td>-$\frac{5}{2(5-k)}$</td>
</tr>
<tr>
<td>$q_E$</td>
<td>-$\frac{5+4k}{5(5-k)}$</td>
<td>$\frac{10-k}{2(5-k)}$</td>
</tr>
<tr>
<td>$q_t$</td>
<td>$\frac{30-16k}{5(5-k)}$</td>
<td>$\frac{4k-15}{5-k}$</td>
</tr>
<tr>
<td>$q_z$</td>
<td>$\frac{21k-55}{5(5-k)}$</td>
<td>$\frac{10-3k}{5-k}$</td>
</tr>
</tbody>
</table>
3.4 The transrelativistic regime

Table 3.12: \( f_n \) and \( q \)-exponents for critical frequencies in the ST phase, while \( \nu_m < \nu_a \) (spectrum 2).

<table>
<thead>
<tr>
<th></th>
<th>( \nu_{m2} )</th>
<th>( \nu_{a2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_n )</td>
<td>( \left( \frac{C_1 h_2}{C_{5/2} h_{5/2}} \right)^2 )</td>
<td>( \left( \frac{C_{(1-p)/2} h_{(1-p)/2}}{C_{5/2} h_{5/2}} \right)^{2/4+p} )</td>
</tr>
<tr>
<td>( q_\xi )</td>
<td>( -2 )</td>
<td>( \frac{4-2p}{4+p} )</td>
</tr>
<tr>
<td>( q_e )</td>
<td>( 2 )</td>
<td>( \frac{2(p-1)}{4+p} )</td>
</tr>
<tr>
<td>( q_B )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{p+2}{2(4+p)} )</td>
</tr>
<tr>
<td>( q_n )</td>
<td>( \frac{5}{2(5-k)} )</td>
<td>( \frac{30-5p}{2(4+p)(5-k)} )</td>
</tr>
<tr>
<td>( q_E )</td>
<td>( \frac{10-k}{2(5-k)} )</td>
<td>( \frac{10p-kp-6k}{2(4+p)(5-k)} )</td>
</tr>
<tr>
<td>( q_t )</td>
<td>( \frac{4k+15}{5-k} )</td>
<td>( \frac{10-8k-15p+4kp}{(4+p)(5-k)} )</td>
</tr>
<tr>
<td>( q_z )</td>
<td>( \frac{10-3k}{5-k} )</td>
<td>( \frac{12k+10p-30+3kp}{(4+p)(5-k)} )</td>
</tr>
</tbody>
</table>

3.4.3 Evolution of critical parameters

A practical way of describing the temporal evolution of the parameters needed to construct a spectrum at any point is that of a smoothly broken power law. We can use eq. (3.22), this time characterizing a temporal break in the following manner

\[
\Phi(t_{\text{obs}}) = A \left[ \left( \frac{t_{\text{obs}}}{t_0} \right)^{-a_1 s_1} + \left( \frac{t_{\text{obs}}}{t_0} \right)^{-a_2 s_1} \right]^{-1/s_1}, \tag{3.27}
\]

where \((t_0, A)\) is the meeting point of the asymptotes and \( \Phi \) is the value of any of the critical parameters. In this version of eq. (3.22) \( a_1 \) and \( a_2 \) are the BM and ST slopes, respectively. We can rewrite the above equation in the following way

\[
\Phi(t_{\text{obs}}) = \left( \Phi_{\text{BM}}^{-s_1} + \Phi_{\text{ST}}^{-s_1} \right)^{-1/s_1}, \tag{3.28}
\]

where both \( \Phi_{\text{BM}} \) and \( \Phi_{\text{ST}} \) have to be evaluated at \( t_{\text{obs}} \).

In the previous Sections we have established not only the scalings of the critical parameters we wish to follow, but their actual values as a function of observer time in both extreme dynamical regimes. This allows us to insert them directly into eq. (3.28), where the only unknown left is the sharpness \( s_1 \). As in the case of spectral breaks we have performed \( \chi^2 \)-minimization fitting in logarithmic space and have arrived at a description of ‘\( s_1 \)’ in terms of a polynomial of the following form
Simulation-based values of $F_m$

**Fitting function**

**BM-slope**

**ST-slope**

---

**Figure 3.4**: A broken power-law fit to the evolution of $F_m$. Plotted are the BM and ST asymptotes. Simulation-based data have the following model parameters: $E_{52} = 1$, $n_0 = 1$, $p = 2.5$, $k = 0$, $\xi = 10^{-2}$, $\epsilon_e = 10^{-1}$, $\epsilon_B = 10^{-2}$, $d_{28} = 1$, $z = 0.56$.

\[ s_t = s_0 + s_p p + s_{pp} p^2 + s_k k + s_{kk} k^2. \]  

Results for the values of these $s_t$-factors are presented in Table 3.13.

An example fit of $F_m$ is shown in Fig. 3.4. The behaviour of $F_m$ displays clear deviations from the BM scalings already before 100 days observer time, for $E_{iso} = 10^{52}$ erg, $n_0 = 1$ and $k = 0$. It settles to values sufficiently close (within 10%) to the ST solution at around 5000 days. The *duration* of the transrelativistic regime is represented in $s_t$, for a given set of physical parameters. Table 3.13 demonstrates that the sharpness of every parameter is generally unique. Based on that we conclude that duration and features of the transrelativistic phase will be manifested differently across the spectrum.

### 3.4.4 Evolution of the sharpness of spectral breaks

Our findings so far enable us to determine the values of the critical frequencies and $F_m$ at any given time for a burst of given physical properties. This allows for an
3.4 The transrelativistic regime

Table 3.13: $s_t$-factors for the evolution of critical parameters. Numerically determined $s_t$-factors (see eq. 3.29) describing the evolution of critical parameters (break-frequencies and maximum flux) from the BM to the ST phase. Each column describes a specific parameter, the name of which is denoted on top.

<table>
<thead>
<tr>
<th></th>
<th>$F_m$</th>
<th>$\nu_{a1}$</th>
<th>$\nu_{a2}^a$</th>
<th>$\nu_{m1}$</th>
<th>$\nu_{m2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>−1.49</td>
<td>−0.61</td>
<td>22.50</td>
<td>−0.89</td>
<td>0.43</td>
</tr>
<tr>
<td>$s_p$</td>
<td>0.09</td>
<td>0</td>
<td>−5.00</td>
<td>1.12</td>
<td>0</td>
</tr>
<tr>
<td>$s_{pp}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>−0.21</td>
<td>0</td>
</tr>
<tr>
<td>$s_k$</td>
<td>−0.76</td>
<td>−0.12</td>
<td>−2.00</td>
<td>0.14</td>
<td>0</td>
</tr>
<tr>
<td>$s_{kk}$</td>
<td>0.12</td>
<td>−0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$^a$The $s$-factors for $\nu_{a2}$ have not been determined by solving the system of equations resulting from measuring its value in different models but comprise a rather heuristic approach that minimizes the deviations from the numerically determined values.

accurate calculation of the flux at any given power-law segment of the spectrum. What is left to specify is the flux close to a spectral break for a general $t_{\text{obs}}$. To achieve that we need to provide a quantitative description for the evolution of the sharpness of spectral breaks from the BM to the ST phase.

As it turns out the sharpness of every spectral break follows a ‘characteristic path’ as it evolves from the relativistic values to the Newtonian ones. This path is qualitatively independent of the physical properties of the burst and is unique for every spectral break. In Fig. 3.5 we present these paths for all possible breaks. From the data gathered we have identified three timescales that are represented in this figure: $t_i$, $t_{\text{NR}}$ and $t_f$. In fact, we can simplify things further by setting $t_i = t_{\text{NR}} / 100$ and $t_f = 10 t_{\text{NR}}$ which is generally valid up to a few percent, regardless of the physical parameters of a burst.

Determination of $t_{\text{NR}}$ carries (as in the case of fluxes and critical frequencies) an analytic and a numerical component. The analytic part is motivated by considerations of the dynamics and is similar to other estimates of an observer time marking the transition to the Newtonian phase (Livio & Waxman, 2000; Piran, 2004). Specifically it is identified as the observer time at which the shock Lorentz factor drops to the value of 2, following the BM solution.
3 Practical flux prescriptions for gamma-ray burst afterglows, from early to late times

Figure 3.5: Evolution of sharpness for all possible spectral breaks. Before \( t_i \) and after \( t_f \), \( s \) resumes its standard BM and ST values. The sharpness of the break around \( \nu_m1 \) shows the most complex pattern, by declining initially during the transrelativistic regime and then rising again to meet its ST asymptote. The value of \( s_{m1} \) at \( t_{NR} \) is \( \sim 0.18 \) smaller than the BM value.

\[
t_{NR} \sim \frac{8^{\frac{k-4}{3-k}}}{4-k} A_{NR}^{\frac{1}{3-k}} (1 + z),
\]

where \( A_{NR} = \left( \frac{17}{8 \pi m_p} \right) c^{k-5} E_{52} n_0^{-1} \).

Including the numerical calibration the expression for \( t_{NR} \) takes the form

\[
t_{NR} = 10^{13.66} \frac{8^{\frac{k-4}{3-k}}}{4-k} A_{NR}^{\frac{1}{3-k}} (1 + z) \text{ days.}
\]

Like all other timescales describing a transition from BM to ST, \( t_{NR} \) scales as \( (E_{52}/n_0)^{1/(3-k)} (1 + z) \), as we expect from dimensional analysis (van Eerten & MacFadyen 2012b). The same holds for \( t_0 \) appearing in eq. (3.27) for all critical spectrum parameters. However, the actual value of \( t_0 \) for every parameter is influenced by the flux-calibrating polynomials \( C_{pol} \) (eq. 3.19) and is therefore in general unique.
3.5 Using the prescriptions

The equations, tables and plots of Sections 3.3 and 3.4 carry all the information necessary to construct a spectrum at any given time, based on given values for the relevant physical parameters that we have discussed in this work. In the following Section we demonstrate how these results can be used to calculate the observed flux at any given frequency and time.

3.5 Using the prescriptions

In this Section we focus on the practical side of this work which is to construct spectra at any given observer time based on values for the physical parameters of a burst. These parameters we repeat here for clarity: $E_{52}$ (isotropic blast-wave energy in units of $10^{52}$ erg), $n_0$ (number density at $10^{17}$ cm), $p$ (index of the electron power-law distribution), $k$ (index of the density distribution of the matter surrounding the burster), $\xi$ (fraction of accelerated electrons), $\epsilon_e$ and $\epsilon_B$ (fractions of internal energy assigned to the relativistic electrons and magnetic field, respectively), $v_{\text{obs}}$ and $t_{\text{obs}}$ (frequency and time of observation), $d_{28}$ and $z$ (luminosity distance in units of $10^{28}$ cm and redshift).

The task of constructing a spectrum out of the presented formulas can be divided in four parts. The first is to obtain the values of the critical frequencies and the maximum flux at a given observer time. The next step is to determine the shape of the spectrum and its general characteristics, i.e. values of the flux away from the breaks, for each of the two possible spectra. The third step is to assign the appropriate sharpness parameters to all spectral breaks. The fourth and final step is to use eq. (3.32) and (3.37) in order to calculate the observed flux at a given frequency.

Each of the two spectra is described by a single equation, at a given observer time. This equation should essentially represent a mathematical formulation of a double-broken power law. One of the ways to achieve that (Granot & Sari 2002) is to use a heuristic formula that combines eq. (3.22) with a factor that assigns a second break at a different frequency. Then the whole spectrum can be described by the following expression

$$F_v(v_{\text{obs}}) = A \left[ \left( \frac{v_{\text{obs}}}{v_0} \right)^{-a_1 s} + \left( \frac{v_{\text{obs}}}{v_1} \right)^{-a_2 s} \right]^{-1/s} \times \left[ 1 + \left( \frac{v_{\text{obs}}}{v_1} \right)^{h(a_2-a_3)} \right]^{-1/h} \quad (3.32)$$

The first two terms on the right-hand side of the above equation describe the first break as usual, while the third term describes the second break. We have introduced $v_1$ the frequency of the second break, $a_3$ the slope of the third power-law segment and $h$ the sharpness of the second break.
3 Practical flux prescriptions for gamma-ray burst afterglows, from early to late times

Calculate \( F_m, \nu_m, \nu_a \) (Section 3.5.1)

Work out the critical parameters for each type of spectrum (Section 3.5.2)

Calculate the appropriate sharpness for each spectral break (Section 3.5.3)

Use eq. (3.32) and (3.37) to calculate the flux (Section 3.5.2)

Figure 3.6: Flowchart showing the basic steps of the presented method. References are given to specific Sections of the paper where the steps are described in more detail.

Equation (3.32) is exact only when the two breaks of the spectrum are sufficiently away from each other so that the power law connecting them is apparent, even for a small range of frequencies. When this is not the case, this equation provides an approximation to the real spectrum (Granot & Sari 2002).

In Fig. 3.6 we present a flowchart of the basic steps towards creating a spectrum of the emitted radiation at any given observer time. The details of each step can be found in the following subsections of the text.

3.5.1 Values of \( F_m, \nu_m \) and \( \nu_a \)

In order to decide which of the two possible synchrotron spectra is valid (see Fig. 3.1 and 3.2) one needs to calculate all \( \nu_{a1}, \nu_{a2}, \nu_{m1} \) and \( \nu_{m2} \). While all these frequencies can in principle be calculated independently, we opt for a different method. Requiring that the flux at both ends of the spectrum (in the power-laws \( \nu^2 \) and \( \nu^{1-p/2} \)) be the same regardless of the type of spectrum, one can show that only three of the four frequencies are independent. The relation between them is

\[
\nu_{a2} = \nu_{a1}^{\frac{10}{3(4+p)}} \nu_{m1}^{\frac{3p-1}{3(4+p)}} \nu_{m2}^{\frac{1}{4+p}}.
\]  

We have expressed \( \nu_{a2} \) in terms of the others because its transrelativistic profile
is the one deviating more from the broken power-law approach. This way inconsistencies that may arise during a spectral transition, due to the over-specification of the evolution of the spectrum, are avoided and the flux in the leftmost and rightmost power-laws is independent of the ordering of the critical frequencies.

Equation (3.28) should be used to calculate $\nu_{a1}$, $\nu_{m1}$ and $\nu_{m2}$ for a given set of physical parameters and for the same observer time, before applying eq. (3.33) to obtain $\nu_{a2}$. $\Phi_{BM}(t_{\text{obs}})$ and $\Phi_{ST}(t_{\text{obs}})$ assume the corresponding forms of the critical frequencies, as those are expressed in eq. (3.26) and Tables 3.9, 3.10, 3.11 and 3.12. The value of $s_t$ is given by eq. (3.29) and the relevant entries of Table 3.13.

Equation (3.28) should also be used to calculate $F_m$ at the same observer time as the critical frequencies. The asymptotic expressions are presented in eq. (3.24) and (3.25) and are to be evaluated using eq. (3.19) and Table 3.8. Having found the values of all 5 critical parameters at the same observer time, we can now start constructing the spectrum.

### 3.5.2 Shape and flux normalisation of the spectrum

In the case of spectrum 1 the characteristic synchrotron frequency $\nu_{m1}$ lies on the optically thin part of the spectrum. Consequently, its value is affected by radiation from the whole blast-wave. In the case of spectrum 2 self-absorption allows only for the front to contribute to the flux close to $\nu_{m2}$. In practice the values of $\nu_{m1}$ and $\nu_{m2}$ are always close to each other (within the same order of magnitude). Therefore, we conclude that the location of $\nu_m$ on the spectrum is mostly determined by the conditions at the front and is only slightly affected by what the optical depth of the blast-wave is at that frequency.

On the other hand, values of $\nu_{a1}$ and $\nu_{a2}$ can differ substantially with respect to each other. However, in most cases they will both be either smaller or bigger than $\nu_{m1}$ and $\nu_{m2}$. We will be referring to these cases as definite ordering, whereas all other cases will be referred to as indefinite ordering. When $\nu_{a1}, \nu_{a2} < \nu_{m1}, \nu_{m2}$ the spectrum will have the form of Fig. 3.1 (spectrum 1), while if $\nu_{m1}, \nu_{m2} < \nu_{a1}, \nu_{a2}$ that of Fig. 3.2 (spectrum 2). In the case of indefinite ordering, the actual positions of the two critical frequencies on the spectrum are very close to each other signaling a spectral transition, typically from spectrum 1 to spectrum 2. In terms of observer time, the time-span of this transition is relatively small. Let $t_{tr}$ be the observer time when indefinite ordering sets in. The duration of this transition (time-span of indefinite ordering) is typically a fraction of $t_{tr}$. During that time the choice of critical frequencies affects the flux across the spectrum by factors of order unity.

In practice, it is preferable to always take the values suggested by both spectra into account, through a consistent weighing method. This way glitches that may appear in the produced light curves when switching from one spectrum to another are
avoided. Instead, the light curves’ behaviour smoothly progresses from the early-time spectrum 1 configuration to the late-time spectrum 2. The weight of each spectrum is represented by a power-law dependence in time that contains a characteristic timescale \( t_{\text{flip}} \) related to the observer time at which the spectrum is transitioning from spectrum 1 to spectrum 2. This characteristic timescale can be estimated numerically by solving eq. (3.27) for both \( t_{T1} \) (the time at which \( \nu_{m1} = \nu_{a1} \)) and \( t_{T2} \) (the time at which \( \nu_{m2} = \nu_{a2} \)), and defining

\[
t_{\text{flip}} = f_{\text{flip}} \cdot \max(t_{T1}, t_{T2}).
\]  

(3.34)

We have found that the value of \( f_{\text{flip}} \) that results in smaller deviations across the parameter space of \([p, k]\) is 1.6.

The weights of spectrum 1 and 2 can be written as

\[
W_1 = \frac{(t_{\text{obs}}/t_{\text{flip}})^{-1}}{(t_{\text{obs}}/t_{\text{flip}})^{-1} + (t_{\text{obs}}/t_{\text{flip}})}, \quad (3.35)
\]

\[
W_2 = \frac{(t_{\text{obs}}/t_{\text{flip}})}{(t_{\text{obs}}/t_{\text{flip}})^{-1} + (t_{\text{obs}}/t_{\text{flip}})}. \quad (3.36)
\]

The flux at a given frequency will then be

\[
\log F = W_1 \cdot \log F_1 + W_2 \cdot \log F_2, \quad (3.37)
\]

where \( F_1 \) and \( F_2 \) are the fluxes calculated at that frequency through spectrum 1 and 2 (see subsections 3.5.2 and 3.5.2), respectively.

**Spectrum 1: \( \nu_a < \nu_m \)**

This is the asymptotic case where both \( \nu_{a1} \) and \( \nu_{a2} \) are smaller than \( \nu_{m1} \) and \( \nu_{m2} \). Consequently the positions of the critical frequencies are given by \( \nu_a = \nu_{a1} \) and \( \nu_m = \nu_{m1} \). The parameters of eq. (3.32) get the following values:

1. \( \nu_0 = \nu_{a1} \)
2. \( \nu_1 = \nu_{m1} \)
3. \( A = F_m (\nu_0 / \nu_1)^{1/3} \)
4. \( a_1 = 2 \)
5. \( a_2 = 1/3 \)
6. \( a_3 = (1 - p) / 2 \)
3.5 Using the prescriptions

**Spectrum 2:** \( \nu_m < \nu_a \)

For spectrum 2 in the asymptotic limit both \( \nu_{a1} \) and \( \nu_{a2} \) are bigger than \( \nu_{m1} \) and \( \nu_{m2} \). Thus, the positions of the critical frequencies are given by \( \nu_m = \nu_{m2} \) and \( \nu_a = \nu_{a2} \). However, in spectrum 2 the flux at \( \nu_m \) is not \( F_m \); that would be the case if it were not for absorption. We can use this fact to first calculate the flux at \( \nu_a \). Although in the present spectrum-configuration the actual position of \( \nu_m \) is given by \( \nu_{m2} \), it is \( \nu_{m1} \) that we should use for obtaining the flux at \( \nu_a \). The variables become:

1. \( \nu_0 = \nu_{m2} \)
2. \( \nu_1 = \nu_{a2} \)
3. \( A = F_m \left( \frac{\nu_{a2}}{\nu_{m1}} \right)^{1-p}/2 \left( \frac{\nu_{m2}}{\nu_{a2}} \right)^{2.5} \)
4. \( a_1 = 2 \)
5. \( a_2 = 2.5 \)
6. \( a_3 = (1-p) / 2 \)

3.5.3 Sharpness parameters of spectral breaks

The only two parameters left to specify in eq. (3.32) are \( s \) and \( h \), the sharpness of the first and the second break in the spectrum, respectively. In order to assign the proper sharpness to each break we first have to compare \( t_{\text{obs}} \) to \( t_{\text{NR}} \) (see eq. 3.31). There are four distinct cases, as illustrated in Fig. 3.5:

- **\( t_{\text{obs}} < t_i \)**
  
  In this case all sharpness parameters attain their BM values as these are given in Table 3.6.

- **\( t_{\text{obs}} > t_f \)**
  
  In this case all sharpness parameters attain their ST values as these are given in Table 3.7.

- **\( t_i < t_{\text{obs}} < t_{\text{NR}} \)**
  
  In this case breaks \( \nu_{m2} \) and \( \nu_{a2} \) retain their BM sharpness. The other two exhibit some evolution towards the corresponding ST values. Namely, the sharpness around \( \nu_{m1} \) will be
\[ s_{m1} = 0.09 \log \left( \frac{t_i}{t_{\text{obs}}} \right) + s_i, \]  

(3.38)

where \( s_i \) is the sharpness of the particular break in the BM regime.

The sharpness around \( \nu_{a1} \) will be

\[ s_{a1} = \frac{(s_f - s_i)}{3} \log \left( \frac{t_{\text{obs}}}{t_i} \right) + s_i, \]  

(3.39)

where \( s_i \) and \( s_f \) are the sharpness parameters at the BM and ST phases, respectively.

\( t_{\text{NR}} < t_{\text{obs}} < t_f \)

In this final case all breaks exhibit a sharpness evolving towards its ST value. For \( \nu_{m1} \) the sharpness will be given by

\[ s_{m1} = (s_f - s_i + 0.18) \log \left( \frac{t_{\text{obs}}}{t_f} \right) + s_f, \]  

(3.40)

while for \( \nu_{a1} \) the value of the sharpness is still given by eq. (3.39).

The sharpness around \( \nu_{a2} \) will be given by

\[ s_{a2} = (s_f - s_i) \log \left( \frac{t_{\text{obs}}}{t_{\text{NR}}} \right) + s_i, \]  

(3.41)

while for \( \nu_{m2} \) we find a similar result

\[ s_{m2} = (s_f - s_i) \log \left( \frac{t_{\text{obs}}}{t_{\text{NR}}} \right) + s_i. \]  

(3.42)

### 3.5.4 Examples of results

We have described a practical implementation of our results to construct spectra at any given time, based on values for the physical quantities characterising the burst. We now show comparisons between simulation-generated spectra and spectra that have been constructed using the provided flux prescriptions.

In Fig. 3.7 a comparison between a simulation-based spectrum and an analytic one is shown. In all power-law segments and the linking breaks, the flux prediction is never more than 10% off compared to the simulation-based data. We can translate these deviations into relative errors for the values of the physical parameters. This we do by adjusting their values so that those deviations vanish in particular regimes of the spectrum. For the blast-wave energy \((E_{52})\) the error ranges from 3% up to 15% depending on which power-law segment (or spectral break) one uses for the
3.5 Using the prescriptions

Figure 3.7: A typically good match between an analytically constructed spectrum and one based on a simulation. Both are taken at 100 days. $\nu_a$ lies at $\sim 10^7$ Hz and $\nu_m$ at $\sim 10^{12}$ Hz. Model parameters for both spectra are: $E_{52} = 1$, $n_0 = 1$, $p = 2.3$, $k = 0$, $\xi = 10^{-2}$, $\epsilon_e = 10^{-1}$, $\epsilon_B = 10^{-2}$, $d_{28} = 1$, $z = 0.56$.

In Fig. 3.8 we present another comparison between a simulation-based spectrum and a constructed one. This one was chosen for exhibiting one of the largest deviations we have encountered. While the self-absorbed part of the spectrum is matched well by the constructed spectrum, flux in the $\nu^{1-p/2}$ segment differs by $\sim 25\%$. The corresponding errors in the derivation of values for physical quantities are the following: for $E_{52}$ up to 16\%, for $n_0$ 10\%–90\% (flux in the $\nu^{1-p/2}$ segment depends very weekly on $n_0$ for these model parameters), for $\xi$ up to 35\%, for $\epsilon_e$ 15\%, for $\epsilon_B$ 30\%, while for $p$ and $k$ we find differences up to 0.1 and 0.2, respectively.

We stress that these deviations are not with respect to a best-fit value but are indicative of how much every parameter should be tweaked to match fluxes in individual power-law segments of the spectrum. More often than not such a tweak would actually produce a rather bad fit overall. Thus the deviations we have listed may be
Figure 3.8: An example of a constructed spectrum that shows relatively large deviations from the numerical result in the optically thin part of the spectrum. Both spectra are taken at 500 days. $\nu_m$ lies at $\sim 1$ Hz and $\nu_a$ at $\sim 10^7$ Hz. Model parameters are: $E_{52} = 1$, $n_0 = 1$, $p = 2.5$, $k = 0.5$, $\xi = 1$, $\epsilon_e = 10^{-4}$, $\epsilon_B = 10^{-2}$, $d_{28} = 1$, $z = 0.56$.

viewed as an upper limit to what a broadband fit would produce.

3.5.5 Application to mildly relativistic outflows

Recently Nakar & Piran (2011) have discussed the radio-signal following the ejection of spherical, Newtonian or mildly relativistic outflows expected from binary neutron star mergers. They estimate that due to the low initial Lorentz factors of these outflows, their deceleration (and entry to the ST phase) will be manifested at $t_{\text{dec}} \sim 60$ days observer time, for $E_{52} = 0.01$, $n_0 = 1$, $k = 0$, $\beta_i \sim 1$ (initial velocity). This is also the time at which optically thin emission at $\nu_{\text{obs}} = 5$ GHz will peak in the range $0.01 - 0.1$ mJy, for a distance of the source in the range $1 - 3$ Gpc. Here, we test these estimates using the prescriptions presented in this paper.

The model we have developed in this study is based on (and therefore, applicable to) outflows that are initially ultrarelativistic. Thus, it is not obvious that it can be used to model non-relativistic outflows. Order of magnitude calculations in the lab
frame can illustrate the limitations. A relativistic outflow of coasting Lorentz factor $\Gamma_i$ will slow down after sweeping mass $\Gamma_i$ times smaller than the mass of the ejecta (Rees & Mészáros 1992). This will happen at a time

$$t_{BM} = \left( \frac{3E}{4\pi\rho_1 c^5} \right)^{1/3} \Gamma_i^{-2/3}. \quad (3.43)$$

From that point onwards the outflow will decelerate according to the BM solution ($\Gamma \propto t^{-3/2}$) becoming Newtonian ($\Gamma \sim 1$) at

$$t_N = \left( \frac{3E}{4\pi\rho_1 c^5} \right)^{1/3}. \quad (3.44)$$

The corresponding radius is

$$r_N = t_N c. \quad (3.45)$$

Equations (3.44) and (3.45) effectively mark the onset of the ST phase.

In the case of sub- and mildly relativistic outflows the deceleration time (also marking the transition to the ST phase) occurs when the swept mass is comparable to the rest mass of the ejecta

$$t_{dec} = \left( \frac{3E}{2\pi\rho_1 c^5} \right)^{1/3} \beta_i^{-5/3}. \quad (3.46)$$

At $t_{dec}$ the shock is at a radius

$$r_{dec} = \beta_i t_{dec} c. \quad (3.47)$$

From eq. (3.44)-(3.47) it is clear that as $\beta_i \to 1$ the onset of the ST phase for Newtonian outflows approaches that of the relativistic analog with the same energy. This implies that fast ($v \sim c$) outflows (regardless of the Lorentz factor) have no memory of their history from $t_{dec}$ onwards. Therefore we can apply the ST scalings of the flux prescriptions to a mildly relativistic outflow (as the one considered by Nakar & Piran 2011) at observer times $t_{obs} \geq t_{dec}$. In the sub-relativistic case ($\beta_i \ll 1$) eq. (3.46) and (3.47) imply that the outflow will decelerate later and at a greater radius. Nevertheless, the ST scalings of the flux prescriptions apply at observer times $t_{obs} \gg t_{dec}$, i.e. sufficiently later than the deceleration time.

For the application to mildly relativistic outflows we have set $E_{52} = 0.01$, $n_0 = 1$, $k = 0$ for the macroscopic parameters of the blast-wave and its environment and $\xi = 1$, $\epsilon_e = \epsilon_B = 0.1$, for the microphysics, while $\nu_{obs} = 5$ GHz and $t_{obs} = 60$ days. The electron spectral index $p$ is varied within the range $2.1 - 3.0$, while for the distance we have taken the two extreme values of 1 and 3 Gpc.
In accordance with Nakar & Piran (2011) we find that $\nu_{\text{obs}}$ is in the optically thin part of the spectrum for all cases. In this regime the flux increases monotonically for an increasing $p$ and for $p = 3.0$ it is about four times higher than the $p = 2.1$ case. For $d_{28} = 0.31$ ($\sim 1$ Gpc) the flux at 5 GHz lies in the range $0.015 - 0.06$ mJy, depending on the value of $p$. At $d_{28} = 0.93$ ($\sim 3$ Gpc) we find the flux to be always below 0.01 mJy, albeit marginally for relatively high values of $p$. This is illustrated in Fig. 3.9 where two spectra are shown corresponding to distances of 1 and 3 Gpc. They are both taken at $t_{\text{obs}} = 60$ days, for a characteristic value of $p = 2.5$.

Binary neutron star mergers are believed to be the progenitors of short GRBs (see Nakar 2007 and references therein) and are observed in a variety of environments, like elliptical, spiral and irregular galaxies (Berger 2009). A considerable fraction of them, however, appear to be host-less, occuring in the intergalactic medium and thus surrounded by a much more tenuous gas than that commonly found inside galaxies (Berger 2009). We have repeated the calculation of the spectrum from a spherical
outflow resulting from a NS-NS merger for a surrounding medium of density \( n_0 = 10^{-3}\text{ cm}^{-3} \), where the lower density results in a later onset of the ST phase at \( \sim 400 \text{ days} \). Keeping all other parameters constant we find that the radio signal at 5 GHz will be detectable by the EVLA up to a distance of \( \sim 100 \text{ Mpc} \).

The implication of these results is that moderately energetic outflows (\( 10^{50} \text{ erg} \)) expected to accompany NS-NS mergers (Rezzolla et al. 2011) can produce synchrotron radiation detectable by the EVLA from distances up to \( \sim 1 \text{ Gpc} \), larger than the detection horizon of the upcoming versions of gravitational-wave detectors (Nakar & Piran 2011). This assumes that the density of the matter surrounding the merger is of the order \( 1\text{ cm}^{-3} \). These radio signals will peak at timescales of the order of a few months, if the corresponding outflows have initial velocities close to the speed of light. In the case of more tenuous circumburst media the ST timescale grows and the detection horizon of the EM signal drops accordingly. The presented flux prescriptions are applicable throughout the ST phase of these outflows.

3.6 Discussion

We present analytic flux prescriptions, for broadband synchrotron spectra originating from GRB outflows, suitable for fast and detailed modelling of the afterglow phase. They are applicable throughout the evolution of observed afterglows, during which external shocks are the dominant source of particle acceleration, and account for the exact shape of the synchrotron spectrum, including self-absorption, but ignoring cooling. These prescriptions are based on high-resolution, one-dimensional, hydrodynamic simulations performed using the adaptive mesh refinement code AMRVAC. To obtain spectra we have employed a radiation code that solves the equation of radiative transfer through the evolving blast-wave as this is determined by the simulations. The presented formulas carry two components. The first is derived analytically and expresses the dependence of the flux on relevant physical parameters \( (E_{52}, n_0, p, k, \xi, \epsilon_e, \epsilon_B, \nu_{\text{obs}}, t_{\text{obs}}, d_{28}, z) \), while the second component reflects the calibration that the results of simulations have introduced to the flux levels.

For each asymptotic dynamical regime (BM and ST) we provide prescriptions for the flux at every power-law segment but also at frequencies close to spectral breaks. These are modelled as smoothly broken power-laws with the sharpness of the break given in terms of the structure of the surrounding medium \( (k) \) and the electron distribution \( (p) \). During the transrelativistic regime we find that the values of critical frequencies \( (\nu_m, \nu_a) \) and peak flux \( (F_m) \) of the synchrotron spectrum show a gradual transition from the asymptotic power-law behaviour in the BM phase to the corresponding one in the ST phase. This fact has allowed us to model their temporal profiles as smoothly broken power-laws. For every parameter \( (F_m, \nu_m \text{ and } \nu_a) \) we
provide formulas describing the sharpness of these breaks in terms of $p$ and $k$. In order to model the evolution of a spectral break’s sharpness, we have recognized the unique pattern that each break exhibits. We have introduced $t_{NR}$ whose derivation is based on considerations of the outflow dynamics. The result is a set of analytic expressions that extend the applicability of the flux prescriptions to any given observer time.

An element of this study worth emphasising is the inclusion of $k$ (representing the structure of the circumburst medium) as a fitting parameter. This is motivated by the fact that environments of stars with variable mass-loss rates (such as massive stars, prime candidates for long GRB progenitors) can have structures more complex than the usually assumed $k = 0$ or $2$ (Ramirez-Ruiz et al. 2005) and fits using $k$ as a free parameter do not exclude such a possibility (Yost et al., 2003; Curran et al., 2009). As can be seen in Tables 3.3 and 3.5 the impact of $k$ on flux values is modest and varies smoothly across a plausible range of $k$-values $[0, 2]$. Nevertheless, its effect is measurable in light of the provided formulas, contributing an extra tool to afterglow fitting and addressing the nature of GRB progenitors.

Beyond the context of GRBs, the provided prescriptions are useful for modelling synchrotron emission from spherical adiabatic blast-waves of arbitrary velocity (with the limitations analysed in Section 3.5.5) as long as they have swept up enough mass to be decelerating. Obvious applications include type Ibc supernovae (Soderberg et al. 2010), often associated with GRBs (Woosley & Bloom 2006) and mildly- or sub-relativistic spherical outflows from binary neutron star mergers. The latter are candidates for providing the EM counterpart (peaking at radio frequencies) to a possible signal of gravitational waves (Metzger & Berger 2012). By applying the ST scalings of the presented flux prescriptions on mildly relativistic outflows we show that prospects of detecting such radio signals from within the horizon of gravitational wave detectors, LIGO (Abbott et al. 2009) and Virgo (Acernese et al. 2008), are realistic (Nakar & Piran 2011).

It is interesting to note that apart from the dependence on $p$ and $k$, we have also found that the sharpness of a spectral break can be influenced to some extent by the microphysical parameters $\epsilon_e$, $\epsilon_B$ and $\xi$. This has been particularly seen in spectral breaks that involve absorption. The reason for this is the dependence of the absorption coefficient on the chosen microphysics through eq. (3.15). The microphysical parameters in effect regulate the physical depth of the blast-wave corresponding to a given value of the optical depth. Therefore an increase/decrease of $\alpha'_\nu$ results in a less/more diverse sample of local electron distributions contributing to the flux across a spectral break and thus a sharper/smoothier transition. We have chosen not to include the effect of the microphysics on the sharpness-formulas, as it typically influences $s$ by no more than $10 – 15\%$ (the flux to a lesser extent) and it would greatly
3.6 Discussion

Contrary to the approach on the evolution of a spectral break’s sharpness, where the introduction of $t_{\text{NR}}$ is useful, when describing the temporal evolution of the spectrum’s critical parameters we deliberately choose not to use such a timescale. The reason is that, as it turns out, there is no such thing as a single global timescale applicable to the behaviour of all observable quantities. Instead, every critical parameter of the spectrum is characterised by its own break time, the meeting point of the BM and ST asymptotes. One can verify that by computing $t_0$ of eq. (3.27) for a few critical parameters of the same model, by equating the asymptotic expressions. They will be found to differ by factors up to a few. This happens because at any given observer time all these parameters are affected by contributions of radiation from various parts of the outflow, emitted within a range of lab-frame times. For each parameter the weight of these contributions will differ, leading to the inference of contrasting timescales by an observer. This stresses the need for models that can naturally account for this kind of features, by implementing accurate calculations of the blast-wave dynamics and the shape of the spectrum.

By inspection of Fig. 3.4 one can realise that the broken power-law approach is an approximation to the actual behaviour of any critical spectrum parameter during the transrelativistic phase. The parameter that exhibits the strongest deviation from this description is $\nu_{a2}$. The reason for this can be traced to the behaviour of the flux in the optically thin part of the spectrum. An example of this behaviour is shown in Fig. 3.10, where a feature readily apparent is a smooth bump centered at $\sim 500$ days (this can also be seen in Fig. 10 of van Eerten et al. 2010). This introduces a similar feature in the temporal profile of $\nu_{a2}$. As a result, the actual values of that frequency can deviate as much as $\sim 15\%$ from the fitting function (smooth power-law break) at observer times relatively close to $t_{\text{NR}}$. This can have an effect on constructed light curves if the self-absorbed part of spectrum 2 is used and only during the transrelativistic phase. The impact on flux levels is stronger than the deviations shown in Fig. 3.10 because the flux at the $\nu^2$ and $\nu^{5/2}$ segments of spectrum 2 scales as $\nu_{a2}^{-\frac{p+4}{2}}$. We therefore recommend using eq. (3.33) in all cases as this method provides a more accurate and consistent way of constructing spectra and light curves.

By now there are a number of studies in the literature that present formulas calculating spectra from GRB afterglows. The importance of taking into account the blast-wave structure and the exact shape of the spectrum has been stressed by discrepancies in the derived values of physical parameters between simple (Wijers & Galama 1999) and more elaborate (Granot & Sari, 2002; van Eerten & Wijers, 2009) models. Inclusion of details regarding the shock structure can be done either analytically (in either of the two asymptotic regimes of the dynamics) or through the
Figure 3.10: Simulation-based light curve taken at $10^{20}$ Hz (no cooling taken into account). A bump at $\sim 500$ days is apparent. For comparison we have plotted the constructed light curve based on the presented flux prescriptions. Model parameters are: $E_{52} = 1$, $n_0 = 1$, $p = 2.3$, $k = 0$, $\xi = 1$, $\epsilon_e = 10^{-4}$, $\epsilon_B = 10^{-2}$, $d_{28} = 1$, $z = 0.56$.

performance of simulations, as is done in the present work. The advantage of numerical simulations is that they cover the transrelativistic phase of the outflow and the level of detail they provide in all cases. The disadvantage is the price they come at, both in terms of time and resources.

In this paper we provide an efficient way of utilizing the benefits of simulations, as those are reflected on the presented analytic prescriptions. A similar approach has been taken by van Eerten & MacFadyen (2012b), who are basing their fitting method on the scale-invariance of light-curves. This method naturally accounts for features like sideways spreading of the jet and off-axis observation angles, features that only arise in simulations of at least two dimensions and cannot be captured in the context of this research. However, it requires the use of a large database of light curves which does not yet exist. So far no study of the afterglow radiation using 2D hydrodynamic simulations (Zhang & MacFadyen, 2009; Wygoda et al., 2011; De Colle et al., 2012) has resulted in the derivation of flux prescriptions. In fact this is the first time, even for the simple spherical case, that simulation-based flux prescriptions beyond the BM
3.7 Conclusions

We have used high-resolution 1D hydrodynamic simulations to calibrate flux scalings of synchrotron, self-absorbed radiation for GRB afterglows in the relativistic and Newtonian dynamical phases (BM and ST, respectively). The transition from the former to the latter is well described by approximating the evolution of spectral parameters (maximum flux and positions of critical frequencies) by power-law breaks connecting the two asymptotic behaviours. The properties of these breaks have been modelled in terms of the values of the physical parameters describing the blast-wave. This way we have managed to encapsulate the precision of the performed simulations into a set of analytic formulas that trace the full evolution of GRB afterglows, from the ultrarelativistic to the Newtonian phase. Due to the general nature of the prescriptions, they are applicable to any source characterised by emission of synchrotron phase are presented. The box-fit method of van Eerten et al. (2012) does provide a fitting code but requires the use of a parallel computer network in order to fit data by iterating through a “box” of simulations. Therefore, analytic flux prescriptions based on 1D simulations, as the ones presented here, can be the base for comparisons with future work in that direction based on 2D simulations. Moreover, 1D models are always relevant both at observer times before the jet break (when most parts of the outflow are causally disconnected) and at late times when the outflow is roughly spherical and allow for accurate calorimetry of jetted outflows after the jet-break but well before spherical symmetry has been reached, as long as the observer is not far off-axis (Wygoda et al. 2011).

In all the simulations that we have performed to arrive at the presented flux prescriptions, the microphysical parameters have been kept constant throughout the run and at every part of the outflow. This is by no means guaranteed and therefore introduces an uncertainty in our results. An interesting topic for further study is the implementation of evolving microphysical parameters and their effect on the flux prescriptions. Such an evolution is expected on theoretical grounds for some of those parameters (Granot et al. 2006). A qualitative study of the evolution of ξ at the shock front and the evolution of εB downstream has already been presented in van Eerten et al. (2010). Meanwhile, there is also growing amount of observational evidence for this process taking place in GRB afterglows (Panaitescu, 2006; Kong et al., 2010; Filgas et al., 2011). Incorporating effects like time-dependence of the microphysics into flux prescriptions can extend the predictions of the standard fireball model and thus broaden the theoretical framework within which observations are currently being interpreted.
radiation from an adiabatic blast-wave.

A numerical code containing a practical implementation of the results presented in this paper combined with a fitting code is freely available on request and on-line at http://www.astro.uva.nl/research/cosmics/gamma-ray-bursts/software/.

3.8 Acknowledgements

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Applying an accurate spherical model to gamma-ray burst afterglow observations


Abstract We present results of model fits to afterglow data sets of GRB 970508, GRB 980703 and GRB 070125, characterized by long and broadband coverage. The model assumes synchrotron radiation (including self-absorption) from a spherical adiabatic blast wave and consists of analytic flux prescriptions based on numerical results. For the first time it combines the accuracy of hydrodynamic simulations through different stages of the outflow dynamics with the flexibility of simple heuristic formulas. The prescriptions are especially geared towards accurate description of the dynamical transition of the outflow from relativistic to Newtonian velocities in an arbitrary power-law density environment. We show that the spherical model can accurately describe the data only in the case of GRB 970508, for which we find a circumburst medium density $n \propto r^{-2}$. We investigate in detail the implied spectra and physical parameters of that burst. For the microphysics we show evidence for equipartition between the fraction of energy density carried by relativistic electrons and magnetic field. We also find that for the blast wave to be adiabatic, the fraction of electrons accelerated at the shock has to be smaller than 1. We present best-fit parameters for the afterglows of all three bursts, including uncertainties in the parameters of GRB 970508, and compare the inferred values to those obtained by different authors.
4 Applying an accurate spherical model to gamma-ray burst afterglow observations

4.1 Introduction

Afterglow observations of gamma-ray bursts (GRBs) have provided important insight into the nature of these events. Some of it has been direct, for example the measurements of redshifts (Metzger et al., 1997), or the association of some bursts with supernova explosions (Hjorth et al., 2003). On the other hand, some has been indirect, accessible only once the available data are interpreted within the context of a physical model. The commonly used fireball model (Rees & Mészáros, 1992; Paczyński & Rhoads, 1993), for instance, is firmly supported by extensive modelling of afterglow observations as synchrotron radiation originating from a decelerating relativistic blast wave (Wijers et al., 1997; Waxman, 1997; Sari et al., 1998; Chevalier & Li, 2000; Panaitescu & Kumar, 2000).

Despite the success of the aforementioned studies in interpreting afterglow observations within a general framework, the values derived by independent groups for the physical parameters of individual afterglows are often substantially different. Such is the case for the well-studied afterglows of GRB 970508 and GRB 980703 for which large differences can be found in the derived values for blast-wave energy, density of the circumburst medium (CBM) and microphysics parameters from different authors (Wijers et al., 1997; Granot & Sari, 2002; Panaitescu & Kumar, 2001, 2002; Frail et al., 2003). The CBM density seems to be especially unconstrained, as differences of many orders of magnitude can be found in the literature. One of the most important parameters of GRB outflows, that directly affects the inferred energetics and rate of these events, is the opening angle of the jet. Specifically, jetted instead of spherical outflows would significantly alleviate the energy requirements and boost the event rate of GRBs. The first strong inference of their presence (Harrison et al., 1999) was perceived as evidence for the ubiquitous role they play in the GRB phenomenon. Accumulating observations, however, have failed to fully confirm this picture, with many afterglows not showing any steepening in the light curves that can be attributed to a jet break (e.g. Racusin et al. 2009), rendering the influence of collimation on GRB outflows for the most part ambiguous. All these uncertainties on the inferred physical parameters of GRB blast-waves have called for refinement and greater precision in the methods that underlie afterglow modelling.

Theoretical afterglow calculations have been continuously improved to include more precise methods of calculating the dynamics and spectra of the source (e.g. Kobayashi et al. 1999; Huang et al. 1999; Rhoads 1999; Granot & Sari 2002; Granot & Piran 2012; Pe’er 2012). Many recent studies (e.g. Meliani et al. 2007; Zhang & MacFadyen 2009; van Eerten et al. 2010; Wygoda et al. 2011; De Colle et al. 2012; van Eerten et al. 2012) are based on high-resolution relativistic hydrodynamic (RHD) simulations which are essential to understand critical aspects of the outflow’s dynamics, like lateral spreading of jets and the transition to the non-relativistic phase.
This allows in principle for accurate determination of spectra and light curves from simulation runs. However, this method is not suitable for iterative fitting of model parameters to observations due to the limitations posed by the necessary performance of numerous time-consuming RHD simulations.

Recently (van Eerten et al. 2012; Leventis et al. 2012; see also van Eerten & MacFadyen 2012b) a new approach has been developed for the calculation of spectra and light curves that retains the accuracy of the numerical techniques, without requiring the long run times of simulations. While the methods of these studies differ, they are common in how they are based on sets of blast-wave simulations that span the parameter space. In the case of van Eerten et al. (2012) dynamical results of 2D simulations have been tabulated allowing the user to perform a straightforward numerical calculation of the afterglow radiation for any combination of the physical parameters within the explored range. Even so, this calculation can be lengthy and is best executed on a parallel computer network.

The method of Leventis et al. (2012) is based on 1D RHD simulations that span the entire range of dynamics, from ultrarelativistic to Newtonian velocities. These simulations, however, do not account for jet features as they rely on the assumption of spherical symmetry. Several runs have been used to calibrate analytically derived scalings of observed synchrotron spectra. The resulting formulas have the unique advantage of combining the accuracy of high-resolution trans-relativistic simulations with the versatility of analytic equations. The fact that they cover a sequence of dynamical phases has motivated us to use them in order to fit model parameters to observational data for afterglows with extensive monitoring. The bursts we are mainly concerned with in this paper are GRB 970508, GRB 980703 and GRB 070125, all monitored in several bands from radio to X-ray frequencies and covering observer times from hours to several months. The two former are among the most studied afterglows with several groups publishing results they have obtained through afterglow modelling.

In this work we present fit results for the afterglows of these bursts and investigate the extent to which a spherical outflow can provide an adequate description of the data. These results also serve as a basis for comparison to model fits based on 2D simulations. Furthermore, the prescriptions of Leventis et al. (2012) enable us to examine the density structure of the burster’s immediate environment, as a continuous range of values for the slope of the CBM density is allowed. The resulting slope can then reveal unusual density distributions of the CBM, or confirm previous claims based on models with only preset structures available, typically constant density or a profile corresponding to a stellar-wind environment ($\propto r^{-2}$).

The paper is organized as follows. A description of the observational data that have been used during fitting is presented in Section 4.2. In Section 4.3 we illustrate
Applying an accurate spherical model to gamma-ray burst afterglow observations

Section 4.2 Data

In this study we focus on three sources: GRB 970508, GRB 980703 and GRB 070125. All three have well-sampled afterglows across the electromagnetic spectrum. In particular they are among the few GRBs that have detections in multiple radio bands at hundreds of days after the initial gamma-ray trigger. This allows us to model the full evolution of the GRB blast wave from the ultrarelativistic to the non-relativistic phase. Another burst with afterglow monitoring spanning almost a decade in the radio is GRB 030329. We have not fit that data set as it is clear from the light curves that a jetted model is needed to interpret the observations (see Van Der Horst et al. 2008 and references therein).

Since the launch of the Swift satellite, it has become clear that the early (10³ – 10⁵ s) afterglow behaviour of many bursts cannot be explained by standard afterglow models (Nousek et al., 2006). Energy injection into the blast wave has been proposed to explain the typically shallow decay that the optical and X-ray light curves show (e.g. Granot & Kumar 2006; Nousek et al. 2006; Zhang et al. 2006; Panaitescu & Vestrand 2011). Other plausible explanations are evolution of the shock microphysics parameters (Granot et al., 2006), or viewing angle effects (Eichler & Granot, 2006). In our sample, GRB 970508 and GRB 070125 display an atypical behaviour, lasting in both cases up to 1.5 days. Especially for GRB 970508, the fast-rising optical light curves before 1.5 days may reveal a refreshed shock, occuring when a slow shell catches up with the afterglow shock at later times (Kumar & Piran, 2000; Granot et al., 2003). After 1.5 days the light curves are compatible with the canonical afterglow decay. Processes like energy injection, refreshed shocks and effects due to off-axis viewing angle cannot be accounted for in the model we are using in this work. For this reason we have excluded data before 1.5 days from the fitted data sets of both GRB 970508 and GRB 070125.

For GRB 970508 radio observations were performed at 1.43, 4.86 and 8.46 GHz (Galama et al., 1998; Frail et al., 2000). Near-infrared and optical data have been published at 6 observing bands (Chary et al., 1998; Galama et al., 1998; Sokolov et al., 1998; Sahu et al., 1997; Garcia et al., 1998). The magnitudes of the underlying host galaxy in the B, V, R c and I c bands have been presented in Zharikov & Sokolov (1999), while the observations in the K and U bands are sufficiently early that they are...
not affected by the host galaxy brightness. We have corrected the observed optical magnitudes for galactic extinction, subtracted the host galaxy flux, and converted them to fluxes. The afterglow was observed in X-rays with BeppoSAX (Piro et al., 1998), for which we have converted the X-ray count rates to fluxes by assuming a spectral index of $-1.1$ over the observing band.

GRB 980703 was observed at the same radio frequencies as GRB 970508 (Berger et al., 2001; Frail et al., 2003). We have used all the near-infrared and optical data in the $H$, $J$, $I$, $R$, $V$ and $B$ bands (Bloom et al., 1998; Castro-Tirado et al., 1999; Vreeswijk et al., 1999). We have corrected the observed magnitudes for galactic extinction, but also for extinction in the host galaxy with $E(B-V) = 0.29$ (Starling et al., 2007; Starling, 2008). The host galaxy of GRB 980703 was bright, not only in the optical (Frail et al., 2003) but also at radio wavelengths (Berger et al., 2001), and we have subtracted the host galaxy flux at all these wavelengths from our measured fluxes. The afterglow has also been detected at X-ray energies (Vreeswijk et al., 1999), for which we have used the same conversion method as in the case of GRB 970508.

For GRB 070125 we have used all the broadband data presented in De Cia et al. (2011). Radio observations were performed at 4.86, 8.46, 15 and 22.5 GHz, while millimetre observations were carried out at 95 and 250 GHz. The data set is supplemented by observations at 12 more bands ranging from the near infrared to X-ray energies, including optical and ultraviolet bands.

To carry out the modelling we have adopted the following cosmology: $\Omega_M = 0.27$, $\Omega_{\Lambda} = 0.73$ and the Hubble-parameter $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$; so for the GRB 970508 redshift of $z = 0.835$ (Metzger et al., 1997) the luminosity distance is $d_L = 1.64 \cdot 10^{28} \text{ cm}$, for GRB 980703 the redshift $z = 0.966$ (Djorgovski et al., 1998) corresponds to $d_L = 1.96 \cdot 10^{28} \text{ cm}$, and for GRB 070125 the redshift $z = 1.547$ (Cenko et al., 2008) implies that $d_L = 3.53 \cdot 10^{28} \text{ cm}$.

During the fit process, no data were excluded based on flux values. However, in the figures we present, data that are not significant at the $2\sigma$ level are depicted as upper limits, for display purposes.

### 4.2.1 Scatter of the data

A noticeable feature of radio data is the high degree of scatter they show, especially compared to the size of the error bars (see Section 4.4). In the case of GRB 970508, notable scatter is also present in near-infrared and optical frequencies. In these bands it is presumably caused by the use of data from various telescopes without the performance of cross-calibration analysis.

In the radio, interstellar scintillation affects the flux levels (Goodman, 1997). Its strength diminishes as the angular size of the source grows and this has been used
Applying an accurate spherical model to gamma-ray burst afterglow observations to infer the radius of GRB outflows (Frail et al., 1997; Taylor et al., 1997; Waxman et al., 1998; Frail et al., 2000; Yost et al., 2003). Various other groups have accounted for the effect of scintillation (Panaitescu & Kumar, 2002; Chandra et al., 2008), especially in early data, by effectively increasing the size of the error bars, more so in early observer times. In our study we have not included the effect of scintillation in the model. One reason is that it does not affect the best-fit values of our results significantly, as the central value of the measurements is not perturbed. Another reason is the lack of detailed measurements for the amount of scattering material off the galactic plane, which makes the effect of scintillation in models of extragalactic sources uncertain (Chandra et al., 2008).

### 4.3 The model

#### 4.3.1 General description

The model we have used is a direct implementation of the method presented in Lev-entis et al. (2012). In that paper we present simulation-calibrated flux prescriptions of synchrotron radiation, including self-absorption, throughout the entire dynamical evolution of GRB afterglows. The model assumes an initially ultrarelativistic spherical blast wave expanding adiabatically inside a medium with a density profile described by a power law: \( n(r) \propto r^{-k} \). The energy distribution of the electrons accelerated at the forward shock is also assumed to be a power law. The minimum Lorentz factor of that distribution is calculated through the energy density and mass density of the shocked gas. The synchrotron spectrum is then determined through the emissivity and absorption coefficient of these relativistic electrons.

In total there are seven free parameters. These are the blast-wave energy \( E_{52} \) in units of \( 10^{52} \) erg, the number density \( n_0 \) at \( 10^{17} \) cm (regardless of the density structure), the index \( p \) of the electron power-law distribution, the index \( k \) of the density distribution of the matter surrounding the GRB, the fraction \( \xi \) of accelerated electrons, and \( \epsilon_e \) and \( \epsilon_B \) denoting the fractions of internal energy carried by the relativistic electrons and magnetic field, respectively. In practice, due to a degeneracy of this model (Eichler & Waxman 2005) a value for one of these parameters has to be assumed in order to uniquely determine the others. In this work we ‘break’ the degeneracy by assuming \( \xi = 1 \) in all runs, unless otherwise stated.

The flux prescriptions are based on analytic calculations of flux scalings during the relativistic (Blandford & McKee 1976) and Newtonian (Sedov 1959; Taylor 1950) phase of the blast-wave dynamics. In these two dynamical regimes the flux at every power-law segment of the spectrum has been calibrated in terms of \( p \) and \( k \). Several hydrodynamic simulations of the afterglow dynamics were run and sub-
4.3 The model

sequently post-processed using a radiative-transfer code (van Eerten & Wijers, 2009; van Eerten et al., 2010). The calibration was carried out by matching analytic expressions for the flux scalings to these numerical results. The sharpness of spectral breaks connecting different power laws of the spectrum is also expressed as a function of $p$ and $k$. The transition from the relativistic to the Newtonian solution is nicely described as a temporal power-law break between the asymptotic behaviour of the critical parameters of the spectrum, namely maximum flux $F_m$, self-absorption frequency $\nu_a$ and synchrotron characteristic frequency of the lowest-energy electrons $\nu_m$. It is worth noting that the characteristics (break time and sharpness) of those temporal breaks are, in general, unique for every parameter of the spectrum. This emphasizes the advantages of simulation-based flux prescriptions compared to simple analytic models for the transrelativistic behaviour of observed afterglows.

4.3.2 The cooling break

A feature of the synchrotron spectrum not covered in the treatment of Leventis et al. (2012) is the cooling break, manifested as a fourth spectral parameter $\nu_c$. Its presence in the spectrum, however, might be important, especially for observations at optical wavelengths and X-ray energies. For that reason all the performed fits have been checked for consistency by calculating the value of $\nu_c$ according to formulas available in the literature (e.g. Granot & Sari 2002; van Eerten & Wijers 2009) and comparing it to the frequencies of the observations. The results of the two aforementioned studies are compatible. We have chosen to use those of van Eerten & Wijers (2009) due to the fact that a general value for $k$ is allowed in their prescriptions. The consistency checks have been performed throughout the range of observer times covered by the data. A value of $\nu_c$ greater than the observing frequencies implies that cooling has not affected the fits and the obtained values for the physical parameters are consistent with the underlying physical model. To the best of our knowledge simulation-based analytic prescriptions for $\nu_c$ beyond the relativistic phase do not exist in the literature. That being the case, we have used formulas applicable in this phase throughout. This extrapolation provides a lower limit on the actual value of $\nu_c$ because its temporal slope in the Newtonian phase is shallower than in the relativistic (van Eerten et al. 2010), which is sufficient when $\nu_c$ is found not to interfere with the observing frequencies.

On the other hand, when the value of $\nu_c$ is found to be lower than – or at about the same levels as – the observing frequencies a different approach is necessary in order to firmly constrain the influence of cooling on the data. Our fitting code has been expanded to include a prescription for the position of $\nu_c$ as a function of time. We have made use of the formulas from van Eerten & Wijers (2009) by calculating $\nu_{c,1}$ of that paper. Formally this expression should only apply in the case of slow
cooling \((v_m < v_c)\). However, it is easy to verify (see also Granot & Sari 2002) that the expression for \(v_c\) in the case of fast cooling gives a similar result within a factor of about 2. A few modifications in the prescriptions are then required in order to account for the influence of cooling in the broadband spectrum. When \(v_a < v_m < v_c\) or \(v_m < v_a < v_c\) the only modification is that of appending another break in the spectrum at the cooling frequency, beyond which the spectrum steepens by a half (Sari et al. 1998). The formula we have used is

\[
F_v(v_{\text{obs}}) = A \left[ \left( \frac{v_{\text{obs}}}{\nu_0} \right)^{-a_1 s} + \left( \frac{v_{\text{obs}}}{\nu_0} \right)^{-a_2 s} \right]^{-1/s} \times \left[ 1 + \left( \frac{v_{\text{obs}}}{\nu_1} \right)^{h(a_2-a_3)} \right]^{-1/h} \times \left[ 1 + \left( \frac{v_{\text{obs}}}{\nu_2} \right)^{r(a_3-a_4)} \right]^{-1/r} \tag{4.1}
\]

The first line in eq. (4.1) describes the first break of the spectrum at the lowest characteristic frequency, while each factor on the second line stands for an extra break at progressively higher frequencies. The parameters \(\nu_0, \nu_1, \nu_2\) and \(s, h, r\) represent the values of the three critical frequencies and the sharpness of the spectral breaks they correspond to, respectively, while \(a_1, a_2, a_3\) and \(a_4\) are the slopes of the four power laws present in a spectrum with three breaks. Finally, \(A\) is the normalising factor of the spectrum derived through modelling of the peak flux \(F_m\).

When \(v_m, v_c < v_a\) the ordering of \(v_m\) and \(v_c\) does not play a role and one retrieves spectrum 3 of Granot & Sari (2002). In that case we have approximated the self-absorption frequency with the values applicable to the no-cooling case. Similarly, when \(v_a < v_c < v_m\) (spectrum 5 of Granot & Sari 2002) we have approximated both \(v_m\) and \(v_a\) with their values in the absence of cooling, while the peak flux is attributed to \(v_c\). Formally, when \(v_a < v_c < v_m\) the self-absorption break is split in two break frequencies with an extra power-law segment between them that has a slope of 11/8.

We have neglected that effect and used only one self-absorption frequency that has the value of \(v_{a1}\) from Leventis et al. (2012). This frequency connects power laws of slope 2 and 1/3. In reality, we have found that most of the time best-fit values of the physical parameters imply that these approximations are not used since \(v_c > v_a, v_m\). However there are instances when this is not the case and we address these in more detail in Section 4.5.

A last issue that needs to be dealt with when cooling influences the fits is the application of the relativistic formulas for \(v_c\) throughout the range of observer times. To assess the validity of this application one needs to estimate the duration of the relativistic phase of the afterglow in the observer frame. In the absence of a detailed description for the transrelativistic behaviour of the cooling frequency, the most general way to do that is by calculating the observer time which corresponds to the transition between the relativistic and Newtonian asymptotes, \(t_{\text{NR}}\) (e.g. Piran 2004; Leventis
et al. 2012). This calculation has been performed for all sets of best-fit parameters and is presented along with our main results in Section 4.4.

### 4.4 Results

For all afterglow data sets, we present three classes of models. Each class corresponds to a different assumption (or the lack thereof) for the value of $k$. We have run fits for $k = 0$ and 2, corresponding to constant density CBM (labelled ISM) and a constant-stellar-wind profile (labelled Wind), respectively, and fits where $k$ is a free parameter. For each class, a range of microphysics settings has been tested. Namely, we have either allowed for both $\epsilon_e$ and $\epsilon_B$ to be free parameters, or connected them through a closure relation that effectively reduces them to one free parameter. Two options for the closure relation have been explored. On the one hand we have imposed equipartition ($\epsilon_e = \epsilon_B$) and on the other the ‘Medvedev’ relation ($\epsilon_e^2 = \epsilon_B$; Medvedev 2006). All other parameters have been kept free at all runs, apart from $\xi$ which, for every run, has taken the value of 1.

#### 4.4.1 GRB 970508

We have performed several fits both to the full data set and to different subsets (radio only, radio and optical only, radio, optical and X-rays) of the afterglow observations of GRB 970508. Radio data alone do not provide enough information to determine simultaneously all the parameters. However, when $k$ is frozen (either in the ISM or the Wind scenario) and a microphysics constraint is used, the results from fitting the radio only, are fairly similar to those from fits to the full data set; all best-fit values of parameters are less than 50% off in the Wind class and less than a factor of 2 off
Figure 4.1: Afterglow of GRB 970508. Best-fit light curves for ISM (solid grey line) and Wind (dashed black line) classes. When $k$ is a free parameter, the Wind scenario is retrieved with high precision. The three radio bands are on top and the rest follow in order of increasing frequency, spanning near-infrared, optical, ultraviolet and X-ray energies. Data points before 1.5 days have been excluded from the fits but appear in grey in the figure. In all bands, data points have 1σ errors. Triangles depict upper limits at the 2σ level.
Table 4.1: Best-fit parameters, with 1σ errors, for GRB 970508 in all classes of models and for all microphysics settings. The fits have been performed to data including radio, near-infrared, optical, ultraviolet and X-rays, but excluding observations made prior to 1.5 days after the gamma-ray trigger (see Section 4.2). The uncertainties in the last row have been calculated for 5 times higher error bars. The third column contains the blast-wave energy in units of 10^{52} erg. The fourth column represents \( n_0 \), the number density at a radius of 10^{17} cm. When \( k = 2 \), \( n_0 \) and \( A_\ast \) (Chevalier & Li, 2000) are related by the formula \( n_0 \approx 30 A_\ast \). For example, the best-fit model (equipartition constraint) of the Wind class has \( A_\ast = 0.243 \) g cm\(^{-1} \). The last column presents the value of \( \chi^2 \) divided by the degrees of freedom (dof).

<table>
<thead>
<tr>
<th>Class</th>
<th>Constraint</th>
<th>( E_{52} )</th>
<th>( n_0 )</th>
<th>( p )</th>
<th>( \epsilon_B )</th>
<th>( \epsilon_e )</th>
<th>( k )</th>
<th>( \chi^2 / \text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>–</td>
<td></td>
<td>1.227^{+0.911}_{-0.360}</td>
<td>15.02^{+25.25}_{-6.62}</td>
<td>2.483^{+0.024}_{-0.022}</td>
<td>(4.2^{+11.3}_{-3.7}) \cdot 10^{-4}</td>
<td>0.702^{+0.298}_{-0.122}</td>
<td>0</td>
<td>29.89</td>
</tr>
<tr>
<td>ISM</td>
<td>Equipartition</td>
<td>0.300^{+0.047}_{-0.033}</td>
<td>0.319^{+0.098}_{-0.084}</td>
<td>2.279^{+0.020}_{-0.019}</td>
<td>0.337^{+0.023}_{-0.026}</td>
<td>0.337^{+0.023}_{-0.026}</td>
<td>0</td>
<td>40.05</td>
</tr>
<tr>
<td></td>
<td>Medvedev</td>
<td>0.333^{+0.024}_{-0.020}</td>
<td>0.784^{+0.111}_{-0.133}</td>
<td>2.307^{+0.013}_{-0.011}</td>
<td>0.130^{+0.009}_{-0.010}</td>
<td>0.361^{+0.013}_{-0.014}</td>
<td>0</td>
<td>35.63</td>
</tr>
<tr>
<td>–</td>
<td></td>
<td>0.131^{+0.006}_{-0.045}</td>
<td>7.516^{+2.934}_{-2.080}</td>
<td>2.277^{+0.017}_{-0.053}</td>
<td>0.551^{+0.449}_{-0.092}</td>
<td>0.589^{+0.411}_{-0.045}</td>
<td>2</td>
<td>28.55</td>
</tr>
<tr>
<td>Wind</td>
<td>Equipartition</td>
<td>0.134^{+0.003}_{-0.007}</td>
<td>7.263^{+0.133}_{-0.258}</td>
<td>2.280^{+0.014}_{-0.013}</td>
<td>0.575^{+0.028}_{-0.014}</td>
<td>0.575^{+0.028}_{-0.014}</td>
<td>2</td>
<td>28.46</td>
</tr>
<tr>
<td></td>
<td>Medvedev</td>
<td>0.121^{+0.009}_{-0.005}</td>
<td>8.858^{+0.150}_{-0.408}</td>
<td>2.259^{+0.015}_{-0.011}</td>
<td>0.448^{+0.038}_{-0.054}</td>
<td>0.669^{+0.028}_{-0.042}</td>
<td>2</td>
<td>28.56</td>
</tr>
<tr>
<td>–</td>
<td></td>
<td>0.131^{+0.857}_{-0.006}</td>
<td>7.465^{+19.890}_{-0.491}</td>
<td>2.277^{+0.240}_{-0.017}</td>
<td>0.555^{+0.402}_{-0.555}</td>
<td>0.595^{+0.405}_{-0.038}</td>
<td>1.983^{+0.046}_{-2.483}</td>
<td>28.64</td>
</tr>
<tr>
<td>kfree</td>
<td>Equipartition</td>
<td>0.133^{+0.003}_{-0.004}</td>
<td>7.207^{+0.267}_{-0.223}</td>
<td>2.280^{+0.015}_{-0.013}</td>
<td>0.580^{+0.016}_{-0.028}</td>
<td>0.580^{+0.016}_{-0.028}</td>
<td>1.983^{+0.047}_{-0.016}</td>
<td>28.54</td>
</tr>
<tr>
<td></td>
<td>Medvedev</td>
<td>0.125^{+0.014}_{-0.003}</td>
<td>8.717^{+0.323}_{-0.374}</td>
<td>2.260^{+0.014}_{-0.013}</td>
<td>0.453^{+0.020}_{-0.073}</td>
<td>0.673^{+0.015}_{-0.057}</td>
<td>1.972^{+0.046}_{-0.036}</td>
<td>28.64</td>
</tr>
<tr>
<td>kfree</td>
<td>Equipartition</td>
<td>0.133^{+0.130}_{-0.065}</td>
<td>7.207^{+14.088}_{-3.156}</td>
<td>2.280^{+0.067}_{-0.243}</td>
<td>0.580^{+0.420}_{-0.237}</td>
<td>0.580^{+0.420}_{-0.237}</td>
<td>1.983^{+0.517}_{-0.389}</td>
<td>1.158^a</td>
</tr>
</tbody>
</table>

^a Error bars of data points are rescaled by a factor of 5.
in the ISM class. Including X-ray data has almost no influence on the inferred values of the physical parameters, as the fits are governed by the combination of radio and optical observations. Nevertheless, we present results and light curves from fits to all bands for completeness.

In Fig. 4.1 we present light curves of best-fit models applied to the full data set. We have found that the spherical model can produce an adequate fit to the data, when $k = 2$. Results for the Wind scenario are almost identical to those from fits where $k$ is a free parameter. Models of the ISM class consistently overpredict late radio flux at 4.86 and 8.46 GHz. On the other hand, Wind models provide a good description at all observer times. In the optical and near-infrared bands, the ISM and Wind cases are practically indistinguishable. One common feature of both is the systematic, albeit minor, underprediction of early (< 10 days) flux, especially in the $R$ and $V$ bands. This is less pronounced in the surrounding $K$, $I$, and $B$ bands. It is worth noting that the X-ray data cannot be fitted by any combination of parameters. Along with the fact that the flux drops sharply after the first two data points, this hints towards a separate origin of the early X-ray flux, for example, inverse Compton (e.g. Sari & Esin 2001). Alternatively, the high X-ray flux at early times could be due to flaring activity, which is not temporally resolved due to the poor coverage.

In the Wind scenario, all critical frequencies lie below the near-infrared. On the other hand, both $\nu_a$ and $\nu_m$ pass through the radio bands. This is in rough agreement with the findings of Chevalier & Li (2000) and Panaitescu & Kumar (2002), although we do not confirm the expectations of the former group regarding the passage of $\nu_c$ from the optical. Instead we find that $\nu_c$ stays below $10^{14}$ Hz during the observations. In the ISM case we find that $\nu_m$ starts off between the optical and radio and crosses $\nu_a$ ($5 \cdot 10^9$ Hz) at ~ 50 days. We also find that $\nu_c$ remains between optical and X-ray energies throughout, contrary to the results of Galama et al. (1998) and Wijers & Galama (1999) who find that $\nu_c$ crosses the optical frequencies early on. Calculation of $t_{\text{NR}}$ yields 145 and 180 days, in the best-fit models of the ISM and Wind class, respectively.

In Table 4.1 we present best-fit parameters, with 1σ errors, of runs to the full data set. A readily apparent feature is the value of $k$ when it is a free parameter, which converges to the Wind scenario. Actually, all best-fit values as well as the $\chi^2$ of these two classes are almost identical, regardless of the chosen microphysics. From the ISM class only the run with no constraints on the microphysics comes close in terms of $\chi^2$, but that model requires a low value for $\epsilon_B$ and high value for $\epsilon_e$ to work. The energy inferred in this case is an order of magnitude higher than the values corresponding to the Wind scenario.

For all classes of models, the best-fit values of $\chi^2$/dof are much higher than 1. This is mainly caused by the notable scatter that data in radio, near-infrared and
optical bands show. The scatter (discussed in Section 4.2.1) is not reflected in the size of the error bars. This is clearly demonstrated in the very small uncertainties that the inferred parameters have, when a microphysics constraint is used. To obtain a better measure for the uncertainties when scatter is accounted for, we have artificially increased the error bars of all the data by a factor of 5 and re-calculated them for the best-fit model of the $k$ free class. The results are presented in the bottom row of Table 4.1. The choice of the factor is motivated by the value of $\chi^2$/dof $\approx 1$ that it results in, producing a statistically ‘good’ fit. However, scatter is not the only reason for the high values of $\chi^2$/dof, there are also systematic deviations from the data (for example in the $R$ and $V$ band during the first 20 days). Therefore, strictly speaking, the method of artificially increasing the error bars should not be applied to the whole data set. Nevertheless, its application results in uncertainties that represent better the parameter range allowed by the data and is not used to draw any conclusions on the quality of the fits.

A discussion of the spectra, dynamics and inferred parameters in the Wind scenario (that produces the best fits) is presented in Section 4.5.

### 4.4.2 GRB 980703

Another well-sampled afterglow that has been extensively modelled in the literature is that of GRB 980703. We have performed fits to the full data set, from radio to X rays, and we have found that no set of parameters can fit the data. The Wind model does better than the ISM, but the best fit is obtained for $k \approx 1.15$.

In Fig. 4.2 we present light curves from best-fit models of all classes. In the radio, the ISM model underperforms compared to the other classes. In the optical and near-infrared none of the models seems to be able to reproduce the data adequately, especially in the low-energy bands. X-ray data, on the other hand, can only be described within the ISM class. From this general picture we can conclude that the physical scenario of synchrotron radiation from a spherical blast wave is not realistic for this source.

For every class of models, we have selected those with the microphysics settings that produced the best fits and present them in Table 4.2. For the ISM and Wind class, the model that performs better is the one with no constraint on the microphysics, whereas when $k$ is free, equipartition produces the best $\chi^2$/dof. Fitting the afterglow of this burst we have allowed for $p$ to range between 2.0 and 4.0 because requiring $p < 3.5$ results in values on the edge of the parameter space. Both the best-fit model of the $k$ free class and the one from the Wind class have very high values for $p (> 3.8)$. Their $\chi^2$/dof values are notably better than those of the ISM class. The values of $t_{NR}$ are 100, 1310 and 880 days for the ISM, Wind and $k$ free class, respectively. Due to the overall-bad fits to the light curves and the extreme best-fit values of $p$, we
Figure 4.2: Afterglow of GRB 980703. Best-fit light curves for ISM (solid grey line), Wind (dotted black line) and k free (dashed black line) classes. Radio bands are shown in the top panel. The lower panel contains near-infrared, optical and X-ray bands. All data were taken into account for the light curves we present. In all bands, data points have 1σ errors. Triangles depict upper limits at the 2σ level.
4.4 Results

Table 4.2: Best-fit parameters of each class for GRB 980703. For column description see Table 4.1.

<table>
<thead>
<tr>
<th>Class</th>
<th>Constraint</th>
<th>$E_{52}$</th>
<th>$n_0$</th>
<th>$p$</th>
<th>$\epsilon_B$</th>
<th>$\epsilon_e$</th>
<th>$k$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISM</td>
<td>–</td>
<td>17.82</td>
<td>760.3</td>
<td>2.538</td>
<td>$10^{-7}$</td>
<td>1.0</td>
<td>0</td>
<td>15.56</td>
</tr>
<tr>
<td>Wind</td>
<td>–</td>
<td>1.771</td>
<td>14.220</td>
<td>3.865</td>
<td>0.0377</td>
<td>0.133</td>
<td>2</td>
<td>9.090</td>
</tr>
<tr>
<td>$k$ free</td>
<td>Equipartition</td>
<td>2.546</td>
<td>4.265</td>
<td>3.933</td>
<td>0.115</td>
<td>0.115</td>
<td>1.154</td>
<td>8.691</td>
</tr>
</tbody>
</table>

consider the values we obtain unreliable. For that reason we have not calculated any errors on the derived parameters for this burst.

It is worth noting the consensus over the outflow geometry of GRB 980703. Several studies infer small opening angles and jet breaks in the timescale of days-weeks (Panaitescu & Kumar, 2001; Yost et al., 2003; Frail et al., 2003). In the spherical model, the very fast decays observed in the $H$, $J$ and $R$ bands, can only be explained by very large values of $p$, that result in steep light-curve profiles. However, a more natural explanation of the observed slopes would be that the edge of the jet has become visible (Rhoads, 1999; Panaitescu, 2005). We therefore regard the results presented in this paper implicit confirmation of the jet geometry in the outflow of GRB 980703.

4.4.3 GRB 070125

The afterglow of the exceptionally luminous GRB 070125 was observed in several bands, lasting more than ten days in X rays and about a year in the radio. We find that Wind-like models provide the best description of the data, but with noticeable outliers and with inferred parameters that are fairly extreme ($E > 10^{53}$ erg, $\epsilon_e = 1$, $\epsilon_B < 10^{-5}$). We consider the results indicative, but by no means conclusive, as additional physics (e.g. jets) may be needed to explain the deviations and special conditions are required to account for the physical parameters we obtain.

In Fig. 4.3 we present light curves of the best-fit models from each class. Results for the Wind and $k$ free classes are similar to each other and differ significantly from the ISM class in radio, millimetre and X-ray bands, where the former perform better. However, late-time behaviour of the data at 4.86, 8, 46 and 22.5 GHz, as well as millimetre observations are hard to explain within any model. In the near-infrared and optical bands all classes produce good fits. In the ultraviolet, there is a slight underestimation of the flux levels. In X rays, only Wind and $k$ free models are able to
Figure 4.3: Afterglow of GRB 070125. Best-fit light curves for ISM (solid grey line), Wind (dotted black line) and k free (dashed black line) classes. Radio and millimetre bands are on top. The lower panel shows near-infrared, optical, ultraviolet and X-ray bands. Data points before 1.5 days have been excluded from the fits but appear in grey colour in the figure. In all bands, data points have 1σ errors.
Table 4.3: Best-fit parameters of each class for GRB 070125. For column description see Table 4.1.

<table>
<thead>
<tr>
<th>Class</th>
<th>Constraint</th>
<th>$E_{52}$</th>
<th>$n_0$</th>
<th>$p$</th>
<th>$\epsilon_B$</th>
<th>$\epsilon_e$</th>
<th>$k$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISM</td>
<td>–</td>
<td>6.968</td>
<td>1.053$\cdot$10$^3$</td>
<td>3.203</td>
<td>1.12$\cdot$10$^{-5}$</td>
<td>1.0</td>
<td>0</td>
<td>14.30</td>
</tr>
<tr>
<td>Wind</td>
<td>–</td>
<td>11.85</td>
<td>1.478$\cdot$10$^3$</td>
<td>2.717</td>
<td>1.59$\cdot$10$^{-6}$</td>
<td>1.0</td>
<td>2</td>
<td>10.80</td>
</tr>
<tr>
<td>$k$ free</td>
<td>–</td>
<td>15.32</td>
<td>3.062$\cdot$10$^3$</td>
<td>2.831</td>
<td>5.19$\cdot$10$^{-7}$</td>
<td>1.0</td>
<td>1.670</td>
<td>10.49</td>
</tr>
</tbody>
</table>

describe the data.

In agreement with De Cia et al. (2011) we find that $\nu_c$ lies between optical and X-ray energies throughout the observations. Chandra et al. (2008), on the other hand, find that they can best explain the data when $\nu_c$ lies below the optical. Calculation of $t_{\text{NR}}$ yields 80 days in the ISM case and $\sim$ 140 days in the other classes. This ensures that the relativistic formula for $\nu_c$ is valid during near-infrared, optical, ultraviolet and X-ray observations, that last up to 10 days after the gamma-ray trigger. In all classes, $\nu_m$ starts off below the optical and overtakes $\nu_a$ at 30 – 80 days. The different temporal evolution of $\nu_a$ makes for the deviations in late radio light curves between the ISM class and the others.

In Table 4.3 we present the values of the inferred parameters for the best-fit models of each class. Deviations between different classes are moderate. Best $\chi^2$/dof values are found when no assumption for the microphysics is made. This is because to explain the data, all models require a high value for $\epsilon_e$ and a very low one for $\epsilon_B$. Values of the parameters when $k$ is free (model with the best $\chi^2$/dof) are closer to those from the Wind scenario, without, however, matching them. The inferred energies are high in all cases, as are the values for $p$. Given the imperfect fits and the extreme parameters we infer, we have not calculated errors for their values.

### 4.5 The curious case of GRB 970508

GRB 970508 is a unique burst in many ways. Its afterglow was only the second ever observed and despite the multi-frequency monitoring, in some bands over the period of several months, the inferred physical parameters vary widely between different authors (Wijers & Galama, 1999; Chevalier & Li, 2000; Frail et al., 2000; Panaitescu & Kumar, 2002; Yost et al., 2003; Berger et al., 2004). From our sample, the fits to GRB 970508 are deemed the most reliable and the most successful, despite the
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higher values for $\chi^2$/dof. There are two basic reasons for this. The first is the overall behaviour of model light curves that successfully reproduce the trends of the data at all well-probed wavelengths. The second reason is the stability and convergence that the fits to GRB 970508 show, especially in the Wind scenario, but also when $k$ is a free parameter and a constraint on the microphysics is imposed. If no constraint is placed on the microphysics and $k$ is a free parameter, we cannot discern between the ISM and the Wind scenario. However, once either equipartition or the Medvedev formula are used, the results clearly favour a wind-type CBM. In this Section we present an analysis of the physics implied by the best-fit parameters we obtain for GRB 970508 and compare those to values inferred by other authors.

4.5.1 Microphysics

At first glance, the best-fit values presented in Table 4.1 reveal an issue concerning the microphysics of the blast-wave, namely, the sum of $\epsilon_B$ and $\epsilon_e$ is greater than 1. In fact, in order for the outflow to be adiabatic, as assumed by the model, at least one of these parameters has to be much smaller than 1. A low value of $\epsilon_e$ ensures that most of the energy remains in the blast wave, even if the electrons radiate efficiently, while a low value of $\epsilon_B$ moderates the energy losses of the electron population. The degeneracy of the theoretical model (Eichler & Waxman, 2005) which has prompted us to freeze $\xi = 1$ during the fit process, can be used to solve this issue. The net effect of this degeneracy is that a set of parameters $E'_52 = f^{-1} E_{52}$, $n'_0 = f^{-1} n_0$, $\epsilon'_e = f \epsilon_e$, $\epsilon'_B = f \epsilon_B$, $\xi' = f \xi$ produce the same spectrum as the unprimed ones, regardless of the value of (the positive number) $f$. Therefore, the inconsistency implied by the high values of $\epsilon_B$ and $\epsilon_e$ may be seen as evidence that $\xi < 1$, which means that not all electrons are accelerated at the shock. Consequently, the values for $E_{52}$ and $n_0$ presented in Table 4.1 should be viewed as lower limits, whereas those for $\epsilon_B$ and $\epsilon_e$ as upper limits.

Another notable feature of the results for the microphysics in the Wind scenario is that we can not conclusively distinguish between the three possibilities (no constraint, equipartition, Medvedev relation). Equipartition settings seem to be favoured marginally by the better $\chi^2$/dof values these models have, but the Medvedev relation cannot be ruled out. The ambiguity of our results is mainly caused by the relatively high values that both $\epsilon_B$ and $\epsilon_e$ have. We have run fits where $\xi$ was frozen at 0.1 and 0.01 and monitored the behaviour of the two former quantities. They were found to be approximately equal to each other and always (as did $E_{52}$ and $n_0$) followed the scalings implied by the degeneracy relations. This confirms energy equipartition between power-law electrons and magnetic field, which is also suggested by $\chi^2$/dof values.
4.5.2 Spectra

In the Wind scenario the synchrotron spectrum starts off at 1.5 days exhibiting fast cooling (Sari et al., 1998) with the critical frequencies having the following values: $\nu_a = 1.2 \cdot 10^{10}$ Hz, $\nu_c = 1.1 \cdot 10^{12}$ Hz, $\nu_m = 1.4 \cdot 10^{13}$ Hz. At 5 days, $\nu_m$ overtakes $\nu_c$, causing the wiggle in the radio light curves of the model (see Fig. 4.1). The flux at the highest-frequency power law of the spectrum (where near-infrared, optical, ultraviolet and X-ray data lie) is independent of the ordering of critical frequencies and therefore no feature is observed in those bands during the spectral transition. After 5 days the spectrum settles into the slow-cooling regime. During the fast-cooling phase (i.e. before 5 days), almost all available data lie above $\nu_m$ and $\nu_c$; there are hardly any significant radio observations during that time. Therefore, our approximations for $\nu_a$ when $\nu_a < \nu_c < \nu_m$ have a negligible effect on the fits. Moreover, given that the values of $\nu_m$ and $\nu_c$ are largely independent of their ordering in the spectrum (Granot & Sari, 2002; van Eerten & Wijers, 2009), the validity of our approach towards optical data is ensured. From 5 days onwards, no approximation is made for the value of any of the critical frequencies and the model assumes its most accurate form.

It is worth noting that the best-fit spectra naturally explain the spectral evolution (at $\sim$ 100 days) depicted in Fig. 5 of Frail et al. (2000), due to the passage of $\nu_m$. In the ISM case $\nu_m$ crosses the radio earlier, at around 45 days, something excluded by the data. Frail et al. (2000) also find $\nu_a = 3$ GHz at seven days, whereas in our best fit $\nu_a = 5.5$ GHz, at the same observer time. We consider the difference negligible, especially considering the strong variation the light curves show around those observer times, due to scintillation. This can be verified by inspection of Fig. 4 of Frail et al. (2000), where the spectral index between 4.86 and 8.46 GHz varies between 0.4 and 1.6 within the first two weeks. We have also checked the claim of Galama et al. (1998) who suggest that $\nu_c$ is observed to pass through the near-infrared bands at $\sim$ 10 days. When all the available data from several different bands ($K, I, R, V$) are taken into account, we find that the spectral index starts off (at $\sim$ 2 days) having values consistent with late time observations, thus showing no evidence of spectral evolution.

In the best-fit model of the Wind scenario, $\nu_c$ lies below the optical bands throughout the duration of optical observations. Therefore, its exact value is important at all observer times. As mentioned in Section 4.4.1, calculation of $t_{NR}$ yields $\sim$ 180 days. This implies that the values of $\nu_c$ during late near-infrared and optical observations (extending up to $\sim$ 200 days in the $R$ band) should be mildly affected by the transition towards the Newtonian dynamical phase. Since $\nu_c$ is not included in the treatment of Leventis et al. (2012) we do not have a description of the transition for this critical frequency, at least not at the level of accuracy that we do for the others. Assuming that the transrelativistic behaviour of $\nu_c$ is similar to those of the other spectral param-
eters (smoothly broken power-law) and that the break is centered around \( t_{NR} \), we have explored various sharpnesses for that transition and found that the fit results remain consistent. The only parameter that changes noticeably is \( p \) which grows from 2.28 to about 2.34 when the transrelativistic evolution of \( \nu_c \) is taken into account. Having established that this evolution does not affect the inferred parameters, the results we present in Table 4.1 are obtained using the relativistic formula for \( \nu_c \) only.

### 4.5.3 Transrelativistic phase

First noticed in van Eerten et al. (2010) and subsequently quantified in Leventis et al. (2012), the duration of the trans-relativistic phase of a spherical outflow in the observer frame can be long (this also holds in the case of a jetted outflow; Zhang & MacFadyen 2009). The near-infrared and optical light curves in Fig. 4.1 show strong deviations from the ultrarelativistic behaviour already at a few tens of days, in observer time. Their progressive steepening is caused entirely by the dynamics slowly adjusting to the Sedov-Taylor solution, as there is no critical frequency crossing these bands. The effect is similar in the radio, but less pronounced due to the simultaneous spectral evolution.

Deviations of the observed radio light curves from the relativistic scalings at timescales of several weeks prompted Waxman et al. (1998) to propose a jetted outflow for GRB 970508. In this paper we demonstrate how accurate modelling of the transrelativistic phase can account for the deviations from the ultrarelativistic scalings at observer times \( \ll t_{NR} \). This implies that a similar trend may hold for at least some other GRB afterglows, the temporal evolution of which has been interpreted as a jet break.

### 4.5.4 Comparison to previous work

Several broadband fits to the afterglow of GRB 970508 have been performed and presented in the literature (Wijers & Galama, 1999; Chevalier & Li, 2000; Yost et al., 2003; Panaitescu & Kumar, 2002). Others have fit only late-time radio data (Frail et al., 2000; Berger et al., 2004), while Starling et al. (2008) have fit only the slopes of light curves and spectra to infer values for \( p \) and \( k \). Most of these studies assume or find that an ISM scenario fits the data better, apart from Chevalier & Li (2000) and Panaitescu & Kumar (2002) who favour the Wind case. In this study we have presented a detailed investigation of both density structures that clearly favours a stellar-wind CBM. In addition we demonstrate how models with no assumption on the slope of the CBM converge to the Wind scenario. Interestingly, Starling et al. (2008) find that, in their sample, four out of five afterglows with well-constrained values for \( k \) suggest the same. In that study the density structure of GRB 970508 is
4.6 Discussion

poorly constrained.

There seems to be more agreement on the geometry of the outflow of GRB 970508. Most studies (also Rhoads 1999) do not need to invoke a jet, while those that do infer a jet geometry, usually find large half-opening angles: 18° (Panaitescu & Kumar, 2002), 30° (Frail et al., 2000), 50° (Yost et al., 2003). We find that the spherical model provides a good description of the data, capturing the trends of the light curves at different wavelengths for more than two orders of magnitude in observer time. We argue that GRB 970508 may have indeed originated from an almost spherical outflow. The energy of the prompt emission is estimated around $5 \cdot 10^{51}$ erg, if isotropic (Bloom et al., 2001). Although on the high side, this value is not unreasonable (e.g. Metzger et al. 2011).

In terms of the whole set of fitted parameters, our results are similar to those of Chevalier & Li (2000) and Panaitescu & Kumar (2002). Given the uncertainties in the last row of Table 4.1, their best-fit values are within, or just outside the allowed range of our results. We find moderately higher values for $\epsilon_e$ and $\epsilon_B$ than both studies, but these values are effectively upper limits. Lowering $\xi$ to 0.3 results in $E_{52} \simeq 0.4$, $A_\ast \simeq 0.73$ g cm$^{-1}$, $\epsilon_e \simeq \epsilon_B \simeq 0.19$, while the value for $p$ remains the same, 2.28. None of these parameters are more than a factor of three off compared to both aforementioned studies (note, however, the inference of a jet from Panaitescu & Kumar 2002). It is worth mentioning that the blast-wave energy inferred through modelling of the afterglow radiation is very similar to the radiative output of the prompt emission. This result holds regardless of the outflow geometry and implies a very high efficiency of the gamma-ray radiation from the main burst. However, given the fast cooling at early times, adiabatic evolution of the blast wave demands $\epsilon_e \ll 1$. For $\xi < 0.17$, both $\epsilon_e$ and $\epsilon_B$ are smaller than 10%. The corresponding blast wave energy becomes $> 8 \cdot 10^{51}$ erg, which reduces the efficiency of the prompt emission below 40%.

4.6 Discussion

In this Section we discuss the implications of our results for the properties of GRB outflows and afterglow fitting.

4.6.1 Collimation of GRB outflows

We have demonstrated how a spherical outflow can account for the observations of GRB 970508 and how it fails in the case of GRB 980703. The former afterglow has often been successfully modelled both with a spherical and a collimated outflow. For GRB 980703 a jet is invariably inferred and in this research, similarly to Frail et al. (2003), we find that a spherical model cannot provide an adequate description to the data under any combination of physical parameters.
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The degree of collimation in the case of GRB 070125 is less clear. On the one hand, there are only a few studies of the afterglow radiation and only one of them performs broadband (radio to X-rays) fitting (Chandra et al., 2008). On the other hand, our results provide a satisfactory description of most of the broadband data, apart from late time behaviour in the radio, when an additional component is observed in the light curves. This component, however, cannot be explained in the jetted model of Chandra et al. (2008) either. Moreover, they propose that the X-ray flux is dominated by inverse Compton, in order to explain what seems to be a chromatic break in the optical and X-ray light curves (at about 4 and 10 days, respectively). However, the claim for a jet-break in the optical is based only on two data points (one in the I and one in the R band), while in the X-rays it is only based on one (see Fig. 4.3). We find that a spherical model offers a similar level of accuracy, without the need to invoke a jet or other radiation mechanisms beyond synchrotron. However, the parameters we obtain are extreme, both on the microphysics side, but also in the total energy budget they imply ($>10^{53}$ erg). We therefore consider it likely that the model we have used lacks some physics, which at least in some bands and observer times ‘drives’ the radiated spectrum. That extra physics could be a jetted outflow, but the evidence from previous studies combined with our findings is not conclusive.

In this study we cannot quantify the opening angle of jets, in cases that one is inferred. We can, however, qualify afterglows as spherical by successfully fitting their broadband data set. This has been the case for GRB 970508 and we consider this a clear demonstration of the diversity in the geometry of GRB outflows. This is in accordance with searches for jet breaks in large samples of afterglow observations that fail to clearly identify a jet break in more than half of the sources (Kocevski & Butler, 2008; Racusin et al., 2009).

Quantifying the distribution of jet opening angles is not an easy task, especially considering the inadequate (for broadband modelling) coverage that a large fraction of afterglows have. On the observational side, Curran et al. (2008) have shown that jet breaks may be misidentified as single power laws, due to data-analysis effects. Moreover, van Eerten et al. (2010) have shown that a moderately off-axis viewing angle (but smaller than the jet semi-opening angle) can ‘mask’ the appearance of a jet-break. If jets are present, observing them off axis should happen more often than not. Therefore, this is an important effect that should be taken into account in the model fits. Another issue that needs to be better understood is the early ($10^3 – 10^5$ s) afterglow behaviour which in a large fraction of bursts suggests some form of energy injection, continuous or irregular (Panaitescu & Vestrand, 2011). This may affect the overall dynamics of the outflow but also result in misinterpreting a potentially coincident jet break (Racusin et al., 2009). Thus, connecting the dynamics of the early afterglow with the more regular behaviour observed at larger timescales.
is essential to uncover evidence for jets that may not be in the form of the canonical achromatic jet break.

### 4.6.2 The immediate environments of gamma-ray bursts

In this work we have treated the density structure of the CBM as a free parameter ($k$), assuming that a constant power law applies. Out of the three data sets we studied, one (GRB 970508) showed convergence to a constant stellar wind, represented by $k = 2$. The best fit to GRB 980703 is obtained for $k = 1.154$. Lastly, for GRB 070125 the best-fit value of $k = 1.67$, which is closer to that of a constant stellar wind than homogeneous CBM. For all data sets, Wind environments produce better fits than the ISM class.

Similarly to the discussion on the geometry of the outflows, GRB 970508 is the only one with reliable results. In both GRB 980703 and GRB 070125 large values for both $p$ and $k$ are needed to best describe the data within the spherical model, the applicability of which is at least doubtful in these cases. For GRB 970508, the value of $A_*$ implies that the inferred density profile corresponds to a constant mass-loss rate of \(2.4 \cdot 10^{-6} \xi^{-1} \, M_\odot/yr\), for a wind velocity of 1,000 km s\(^{-1}\). Interpreting the adiabatic condition as $\xi < 0.17$, we find $\dot{M} > 1.5 \cdot 10^{-5} \, M_\odot/yr$, which implies a relatively massive Wolf-Rayet star towards the end of its life (Chevalier & Li, 1999).

Several studies have fit individual bursts and found or assumed a homogeneous density structure for the CBM. When the fits are compared against those with stellar-wind CBM the results are often ambiguous (e.g. Frail et al. 2003; Chandra et al. 2008), while in some cases the Wind scenario seems to be favoured (Chevalier & Li, 2000; Panaitescu & Kumar, 2002). On the theoretical side, van Eerten & MacFadyen (2012a) have shown that the majority of Swift post jet break slopes are not reconcilable with a constant density CBM, if late energy injection and viewing angle do not significantly affect the observations. Instead, the observed slopes suggest a wind-type environment for the CBM. Starling et al. (2008) have studied a sample of 10 Beppo-SAX afterglows and found that the majority of the data sets that were sufficient to constrain the value of $k$ implied a stellar-wind CBM. However, half of them have error bars that allow for a wide range for $k$. Curran et al. (2009) have performed a similar study using Swift bursts and find a division in the sample between constant and wind-like profiles. It seems, therefore, likely that the density structure of the CBM in GRBs is diverse, similar to the geometric characteristics of their outflows. However, this does not necessarily translate to diversity of the progenitors as well, because in the collapsar model (Woosley, 1993; MacFadyen & Woosley, 1999) the CBM of a large fraction of long GRBs is modified by multiple stellar winds from the neighbouring stars (Mimica & Giannios, 2011).
4.6.3 Model constraints

For all the afterglows we studied, we have found that a multi-frequency data set is more suitable for fitting all the parameters at once. This has led to the expansion of the model with the inclusion of the cooling frequency of the synchrotron spectrum, \( \nu_c \). However, even when radio to X-ray data are fitted and all details of the spectrum are taken into account, setting \( k \) a free parameter results in large uncertainties, if no assumption for the microphysics is made. This is manifested in the large errors for the best-fit values of physical parameters in the case of GRB 970508 (see row 7 of Table 4.1).

When \( k \) is free and no microphysics assumption is made, the number of fitted parameters is six, equal to the maximum number of constraints we can have from the light curves – four from the positions of the critical frequencies and the value of \( F_m \), plus two more from the slopes of spectra and light curves. However, our results imply that not all of these constraints are efficiently used during the fitting process. This means that the effects of two or more of the constraints cannot be separated, leading to a case-specific degeneracy. In the case of GRB 970508, for the best-fit model, both \( \nu_m \) and \( \nu_c \) lie between radio and near-infrared bands for the best part of the observations (\( \nu_m \) stays above the radio bands for about 100 days). Therefore, their positions are not independently constrained by the data, leading to a wide range of possible values when all six parameters are simultaneously fitted.

An interesting feature of the prescriptions we have used is the inclusion of \( \xi \) as a parameter. Due to the degeneracy of the model, the presence of \( \xi \) is not necessary per se. One can imagine a situation where a range in the allowed values for \( \xi \) is reflected in the adjustment of the ranges of the other parameters. For example, by assuming that \( \xi = 1 \) and allowing \( \epsilon_B \) and \( \epsilon_e \) to obtain values \( > 1 \) during fitting, one accounts for the possibility of \( \xi \) being smaller than those two parameters, while all of them are smaller than unity. However, the inclusion of \( \xi \) in the model demonstrates these situations more clearly. In the results we obtain for GRB 970508 it was not initially possible to discern between the Medvedev constraint and the equipartition constraint for the microphysics due to the high values of both \( \epsilon_B \) and \( \epsilon_e \), that, within the uncertainties, extend to the upper limit of the allowed range. By freezing \( \xi \) at values much lower than 1, we have excluded the presence of better fits in which \( \epsilon_B > \xi \) and/or \( \epsilon_e > \xi \), and confirmed that energy equipartition between power-law electrons and magnetic field describes better the afterglow observations of GRB 970508.

4.7 Conclusions

We have performed broadband fits of three afterglow data sets using accurate analytic flux prescriptions applicable to spherical outflows. We have shown that GRB 970508
is successfully fit by a spherical model. The fits fail in the case of GRB 980703 and GRB 070125 at varying degrees, implying that these sources may be indeed related to jetted outflows. This is supported by extensive modelling of the former and the extremely high isotropic energy inferred for the latter.

For GRB 970508 we find that the best-fit value for $k$ is practically 2, strongly suggesting a stellar-wind environment. Fits to GRB 970508 also show evidence for a population of electrons that is not accelerated at the forward shock. The implied values for the microphysics parameters, $\epsilon_e$ and $\epsilon_B$, suggest that power-law electrons and magnetic field are close to energy equipartition.

Modelling of GRB 970508 illustrates how an accurate spherical model accounts for the progressive deviations of light curves from the ultrarelativistic scalings at $t_{\text{obs}} \ll t_{\text{NR}}$. This feature had been previously interpreted as a jet break in the context of simpler models, but emerges naturally from precise calculations of dynamics and spectra in the spherical scenario. Therefore, we consider it possible that similar features in the data sets of other afterglows have been misinterpreted as jet breaks, in the absence of detailed calculations for the spherical case.

4.8 Acknowledgements

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The plateau phase of gamma-ray burst afterglows in the thick-shell scenario

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Abstract  We present analytic calculations of synchrotron radiation from the forward and the reverse shock of gamma-ray burst blast waves, in the thick-shell scenario (i.e. when the reverse shock is relativistic). We show that this scenario can naturally account for the plateau phase, observed early in the afterglows of about half the bursts detected by Swift. We generalize our approach to include power-law luminosity of the central engine and show that when radiation from both regions (forward and reverse shock) is taken into account, a wide range of possibilities emerges, including chromatic and achromatic breaks, frequency-dependent spectral evolution during the injection break and widely varying decay indices in different bands. For both the forward and the reverse shock, we derive scalings for the spectral parameters and the observed flux in different power laws of the spectrum, as a function of observer time. We explore the $F_b - t_b$ relation (between the observed time of the end of the plateau phase and the optical flux at that point) in the framework of the presented model and show that model predictions favour the reverse shock as the dominant source of optical emission. As case studies, we present simultaneous fits to X-ray and UV/IR data of GRB 060729 and GRB 090423, respectively. We identify the end of the plateau phase with the cessation of energy injection and infer the corresponding upper limits to central-engine activity, which are 9 hours for the former and 1.5 hours for the latter. We conclude that smooth energy injection through the
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reverse shock is a plausible explanation for the plateau phase and the transition to the canonical afterglow behaviour of gamma-ray bursts. During the plateau phase, radiation from the reverse shock is likely to be important, or even dominant, and should be taken into account when fitting model parameters to observations.

5.1 Introduction

The afterglow radiation of gamma-ray bursts (GRBs) carries important diagnostics of the physics behind these events. The most recent advances in their study have been initiated by observations of the Swift satellite (Gehrels et al., 2005). Perhaps the most intriguing discovery of Swift has been the afterglow behaviour within the first few hours after the burst (Zhang et al., 2006; Nousek et al., 2006). X-ray and optical light curves during that period, reveal the details of the transition from the prompt emission, responsible for the γ rays, to the canonical afterglow behaviour, typically attributed to synchrotron radiation from an adiabatic relativistic blast-wave (see Mészáros 2006, for a review). This transition, however, has been found to be rich in features that cannot be explained by the classical fireball model. These features include flares, plateaus, chromatic and achromatic breaks and decay indices that vary widely compared to those observed at time-scales of days to weeks.

Regular features in Swift data, flares and plateaus have often been linked to delayed or prolonged engine activity (Panaitescu et al., 2006; Nousek et al., 2006; Zhang et al., 2006). In the case of flares, the energy supply is episodic and, similarly to the origin of the prompt emission, results in internal dissipation of the kinetic energy. In the case of plateaus, the energy supply is gradual, leading to smoother light curves. The energy is ‘injected’ into the blast wave through the reverse shock which propagates inside the ejecta. If long-lasting, energy injection can have important consequences on the dynamics by introducing an intermediate phase during which the deceleration of the blast wave is moderated by the rate of energy supply (Sari & Piran, 1995). This is what is known as the ‘thick-shell’ scenario.

Calculation of the spectral signatures of the reverse shock, usually assuming synchrotron radiation, have been used to predict or interpret deviant behaviour in radio, optical and X-ray frequencies (Panaitescu et al., 1998; Sari & Piran, 1999; Kobayashi & Sari, 2000; Kumar & Panaitescu, 2003; Zhang et al., 2006; Uhm & Beloborodov, 2007; Dall’Osso et al., 2011; Panaitescu & Vestrand, 2012). However, these studies are mostly concerned with non-relativistic reverse shocks, and when the relativistic phase is addressed, radiation from the reverse shock is either ignored or calculated only after the reverse shock has crossed the ejecta. But if the plateau phase of the afterglow indeed reflects energy injection, the inferred duration ($10^3 - 10^5$ s) can easily result in a relativistic reverse shock which may contribute significantly to the
observed radiation, during the entire plateau phase.

Understanding the effects of prolonged energy injection and the resulting radiation from the forward and the reverse shock is important for two main reasons. The first reason is that the profile of the ejecta, probed through afterglow observations, may carry important information on the ejection mechanism of the central object (and thus its identity) and especially on the processes that modify that profile during the main burst. The second reason is that early-afterglow data have blurred the picture regarding the existence and time of jet breaks in GRB afterglows, partially due to their similarity to injection breaks (Racusin et al., 2009). Therefore, separation of the two effects is crucial to infer reliably the jet properties and thus the correct energetics of these events.

In this work we use a simple analytic description of the dynamics of the blast wave while the reverse shock is relativistic and track the radiation both from the forward and the reverse shock self-consistently. Once all the ejected particles have been shocked, the dynamics settles into the adiabatic phase, the transition manifested as a simultaneous break in light curves of different bands. This is the so-called injection break, after which radiation from the reverse shock fades quickly due to expansion and radiative losses. By exploring reasonable values for the physical parameters, we show that a range of possibilities exists for the observed spectra, during the intermediate phase, but also at the time of the injection break.

At this point we consider it useful to define some terms we will be using throughout this paper and briefly discuss the ambiguity in the use of the word “(a)chromatic”, common nomenclature of the field. In Harrison et al. (1999), where for the first time a simultaneous steepening of the afterglow light curves was interpreted as evidence for the presence of a jet in GRB 990510, the optical break is characterised as “achromatic” due to the lack of spectral evolution locally (i.e. in optical bands). However, in most of the subsequent publications (e.g. Granot & Loeb 2001; Panaitescu et al. 2006; Li et al. 2012) “(a)chromatic” is used to mean ‘frequency-(in)dependent’ rather than ‘(not) accompanied by spectral evolution’. In this paper we will be concerned both with temporal breaks occurring at only one frequency and with temporal breaks accompanied by spectral evolution locally, regardless of the possible simultaneity of breaks at other frequencies. We will use the word “chromatic” to refer to light-curve breaks that are not accompanied by breaks at other frequencies, while “achromatic” will be the breaks that occur simultaneously at two or more frequencies, regardless of the temporal indices before and after the break. When the spectrum changes locally (i.e. around an observing frequency) during a temporal break, we will refer to it as “change of the spectral index”, or simply “spectral evolution”.

This paper is structured as follows. In Section 5.2 we present our description of the dynamics, radiation and particle populations of both the forward and the reverse
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... shock, in the case of steady energy injection. We also generalize our approach to energy injection that can be described by a power-law in time. In Section 5.3 we show a direct numerical implementation of the model and discuss the richness of the resulting light curves as a consequence of the interplay between forward and reverse-shock radiation. In Section 5.4 we present scalings for spectral parameters and observed flux for both radiating regions. In Section 5.5 we present eyeball fits of model light curves to both X-ray and optical data of a few example afterglows with a plateau phase. In Section 5.6 we discuss our results in the context of early-afterglow observations and assess the feasibility of the thick-shell scenario as an interpretation of the observations. We conclude by summarizing our main findings in Section 5.7.

5.2 Model

In this Section we present a detailed derivation of the dynamics and spectral parameters during energy injection. For the adiabatic phase that follows after the injection break, see, for example, Sari et al. (1998). The first part of the calculations is concerned with the characteristic time-scales of the model. Such calculations have been presented in the past in various different forms, but we repeat them here for completeness, but also as a reference for the subsequent analysis.

The main assumptions are those of a single-zone approximation both for the shocked ejecta and shocked circumburst medium (CBM), pressure equilibrium at the contact discontinuity separating the two, and a thin-zone approximation, where we neglect differences in the arrival times of photons emitted simultaneously (in the lab frame) but at different depths inside the blast wave. For simplicity, the density distribution of the CBM is taken to be homogeneous.

5.2.1 Dynamics and time-scales of thick shells

We assume a constant ejection of mass from the central source with Lorentz factor \( \eta \). The duration of the ejection is \( \Delta t \) in the lab-frame. The total energy of the ejecta is initially purely kinetic (cold ejecta) and is denoted by \( E \), while the circumburst medium is assumed to have a constant density \( n_0 \). The four physical parameters \( \eta \), \( \Delta t \), \( E \) and \( n_0 \) uniquely determine the dynamical evolution of the blast-wave that is generated as the ejecta plough into the CBM.

There are four regions in this scenario. Region 1 is the unshocked CBM, while region 2 is the CBM that has been shocked by the forward shock. Region 3 contains the ejecta that have been shocked by the reverse shock and region 4 the unshocked ejecta. Regions 2 and 3 are the only ones that produce radiation as a result of converting part of the kinetic energy of the ejecta to thermal energy, via the forward and...
the reverse shock. Subscripts 1, 2, 3 and 4 will be used throughout this Section to specify the region that a quantity refers to.

The pressure in regions 2 (shocked CBM) and 3 (shocked ejecta) can be approximated by (see Beloborodov & Uhm 2006)

\[ P_2 = \frac{e_2}{3} = \frac{4}{3} \gamma_2^2 m_p n_0 c^2 \]  
\[ P_3 = \frac{e_3}{3} = \frac{4}{3} (\gamma_{34}^2 - 1) m_p n'_4 c^2 \]

where \( \gamma_2 \) is the Lorentz factor of region 2 in the lab-frame and \( \gamma_{34} \) is the relative Lorentz factor between regions 3 and 4. The quantity \( n'_4 = n_4 / \eta \) is the number density of particles in region 4 in its comoving frame. Conservation of mass implies

\[ n_4 = \frac{E}{4 \pi m_p c^5 \eta \Delta t} t_{lab}^{-2}. \]

Demanding pressure equilibrium at the contact discontinuity (CD) between regions 2 and 3 we see that the ratio \( f = n'_4 / n_0 = \gamma_2^2 / (\gamma_{34}^2 - 1) \) decreases with time and therefore the initially Newtonian RS becomes faster until it becomes relativistic. From that point on we approximate both the FS and the RS as ultrarelativistic shocks \( (\gamma_2, \gamma_{34} \gg 1) \). This implies that \( \eta \gg \gamma_2 \) and therefore

\[ \gamma_{34} \gamma_2 \approx \frac{\eta}{2}. \]  

Given eq. 5.4, the ultrarelativistic approximation for \( \gamma_2 \) and \( \gamma_{34} \), and the pressure equilibrium at the CD, we find that the RS becomes relativistic when \( f = \eta^2 / 4 \). The lab-frame time at which this occurs is

\[ t_{rel} = \left( \frac{E}{\pi m_p c^5 n_0 \eta^4 \Delta t} \right)^{1/2} \]

This time-scale \( t_{rel} \) should be compared against the lab-frame time it takes the FS to give half the blast-wave energy to the CBM:

\[ t_{CBM} = \left( \frac{3 E}{8 \pi m_p c^5 n_0 \eta^2} \right)^{1/3}. \]

If \( t_{CBM} < t_{rel} \) the outflow will enter the Blandford-McKee self-similar phase (Blandford & McKee, 1976) before the RS becomes relativistic. However, if \( t_{CBM} > t_{rel} \) the RS will first become relativistic. This is, in fact, the condition for the thick-shell scenario (see, for example, Sari & Piran 1995).
Assuming that the lab-frame Lorentz factors of regions 2 and 3 are common during the phase of a relativistic RS we obtain the lab-frame time profile of the two shocks:

$$\gamma_2 = \frac{\eta}{2} \left( \frac{t_{\text{rel}}}{t_{\text{lab}}} \right)^{1/2} \quad (5.7)$$

$$\gamma_{34} = \left( \frac{t_{\text{lab}}}{t_{\text{rel}}} \right)^{1/2} \quad (5.8)$$

This phase will end when the FS becomes non-relativistic, or the RS crosses the shell of ejecta, whichever comes first. The lab-frame time of the FS becoming non-relativistic can be found using eq. (5.7) for a final Lorentz factor of 2.

$$t_{\text{newt}} = \frac{\eta^2 t_{\text{rel}}}{16} \quad (5.9)$$

The lab-frame time at which the RS crosses the shell can be found by simply considering the lab-frame width of the shell and the difference in lab-frame velocity between the unshocked ejecta and the shocked ejecta, once the RS has become relativistic.

$$c \Delta t = \int_0^{t_{\text{cr}}} (v_{ej} - v_2) \, dt_{\text{lab}} \approx \frac{c}{2} \int_0^{t_{\text{cr}}} \left( \frac{1}{\gamma_2^2} - \frac{1}{\eta^2} \right) \, dt_{\text{lab}} \quad (5.10)$$

Using the approximation $\eta \gg \gamma_2$ once again and expressing $\gamma_2$ through eq. (5.7) we find

$$t_{\text{cr}} \approx \eta (t_{\text{rel}} \Delta t)^{1/2} \quad (5.11)$$

A useful consistency check here is to note that we get the same result when calculating the lab-frame time at which the FS has given half of the blast-wave energy to the CBM according to the scaling presented in eq. (5.7).

The observed time-scales can be found by using the well-known formula resulting from the finite speed of light:

$$dt_{\text{obs}} = \frac{dt_{\text{lab}}}{2 \gamma_2^2} \quad (5.12)$$

and integrating through the appropriate time-interval. This way we find:

$$t_{\text{rel,obs}} = \frac{2 t_{\text{rel}}}{\eta^2} \quad (5.13)$$

and

$$t_{\text{cr,obs}} = \Delta t + \frac{t_{\text{rel}}}{\eta^2}. \quad (5.14)$$

If $t_{\text{cr}} < t_{\text{newt}}$ the FS will enter the self-similar phase, described by Blandford & McKee (1976), right after $t_{\text{cr}}$. We can then follow the radiation from the FS until the flow becomes non-relativistic.
5.2 Model

5.2.2 Radiation

To obtain the radiation spectrum from the blast wave, we calculate the number of particles in each region, as a function of lab-frame time. We then proceed to calculate the basic features of the synchrotron spectrum as a function of the thermodynamic quantities in each region.

Particle populations of regions 2 and 3

In order to calculate the radiation emanating from each region (2 and 3) we need to take account of the rate of influx of particles into regions 2 and 3 as a function of time.

For region 2 the number of particles as a function of time is easily found, as the FS is constantly moving at the speed of light and the density of the surrounding material is also constant. Therefore

\[ N_2(t_{\text{lab}}) = \frac{4\pi}{3} c^3 n_0 t_{\text{lab}}^3 \] (5.15)

For region 3 we envision a layer of thickness \( \delta x \) of the unshocked ejecta, upstream from the RS (i.e. at lower radii), moving outwards with Lorentz factor \( \eta \). The layer we are considering will have all been shocked within lab-frame time \( \delta t \). Therefore, the speed at which the RS is moving into the shell, as seen in the lab frame, will be \( \delta x / \delta t \). We can write

\[
\delta x = \int_{t}^{t+\delta t} (v_{\text{ej}} - v_2) \, dt \approx \frac{c}{\eta^2 \tau_{\text{rel}}} \left[ (t + \delta t)^2 - t^2 \right]
\]

\[
= \frac{c}{\eta^2 \tau_{\text{rel}}} \left[ (1 + \delta t)^2 - 1 \right]
\] (5.16)

where \( t \) is a lab-frame time for which \( t_{\text{rel}} < t < t_{\text{cr}}, \tau_{\text{newt}} \) and we have used again the approximation \( \eta \gg \gamma_2 \). For \( \delta t \ll t \) we can then write

\[
\frac{\delta x}{\delta t} \approx \frac{2 c t}{\eta^2 \tau_{\text{rel}}} \equiv U^*.
\] (5.17)

In the lab frame the rate of particles crossing the RS per unit time will be

\[
\frac{dN_3}{dt_{\text{lab}}} = 4\pi r^2 U^* n_4
\] (5.18)

and therefore \( N_3(t_{\text{lab}}) \propto t_{\text{lab}}^2 \).
However, by \( t_{\text{rel}} \) a fraction \( t_{\text{rel}} / (\eta^2 \Delta t) \) of the ejecta have already been shocked. An approximate formula that reconciles the scaling above with the percentage of particles already shocked by \( t_{\text{rel}} \) is

\[
N_3(t_{\text{lab}}) = N_T \left[ \left( 1 - \frac{t_{\text{rel}}}{\eta^2 \Delta t} \right) \frac{t_{\text{lab}}^2 - t_{\text{rel}}^2}{t_{\text{cr}}^2 - t_{\text{rel}}^2} + \frac{t_{\text{rel}}}{\eta^2 \Delta t} \right],
\]

(5.19)

where \( N_T = E / \eta m_p c^2 \) is the total number of ejected particles. The above equation shows the correct asymptotic behaviour at \( t_{\text{rel}} \) and \( t_{\text{cr}} \) and obeys the scaling for \( N_3 \) at \( t_{\text{lab}} \gg t_{\text{rel}} \).

**Synchrotron parameters**

The electrons (primary radiating particles) are assumed to be accelerated to a power-law energy distribution upon passage of the shocks (FS and RS). In the presence of a magnetic field they will radiate via the synchrotron process. The total radiated power of a single electron of energy \( \gamma' e m_e c^2 \) in the comoving frame is (see, for example, Rybicki & Lightman 1986)

\[
P'_{\text{syn}} \approx 2.66 \cdot 10^{-14} \gamma'^2 e U'_B,
\]

(5.20)

where \( U'_B = 4 \epsilon_B \gamma^2 e m_p c^2 n_0 \) is the comoving magnetic energy density. The fraction of internal energy carried by the magnetic field is denoted by \( \epsilon_B \).

The minimum Lorentz factors of the electron distributions that populate regions 2 and 3 (\( \gamma'_{m-2} \) and \( \gamma'_{m-3} \), respectively) will be

\[
\gamma'_{m-2,3} = \gamma'_{2,34} \frac{p - 2}{p - 1} \frac{m_p}{m_e} \epsilon_e,
\]

(5.21)

where \( p \) is the slope of the electron power-law distribution and \( \epsilon_e \) is the fraction of internal energy carried by the power-law electrons (e.g. Sari et al. 1998).

The electron Lorentz factor corresponding to the cooling frequency of the synchrotron spectrum can be estimated by considering those electrons which have radiated a considerable part of their energy, within the available comoving time:

\[
\gamma' c m_e c^2 = P'_{\text{syn}}(\gamma'_c) t'
\]

(5.22)

This can be verified in various ways. One is to consider the energy given to the CBM by the FS at \( t_{\text{rel}} \). It is found to be about a fraction \( t_{\text{rel}} / (\eta^2 \Delta t) \) of the total blast-wave energy. This implies that a similar amount of energy has been subtracted from the unshocked ejecta and since the total energy was initially homogeneously distributed in the ejected mass, the fraction of the width of the shell that has been shocked by \( t_{\text{rel}} \) is of the same order. Another, slightly more complicated way to derive this is by considering events in two reference frames moving with Lorentz factor \( \eta \) with respect to each other.
5.2 Model

The comoving frequencies that these electron energies correspond to can be found using the formula for the synchrotron characteristic frequency:

$$\nu'_{m,c} \approx 2.8 \cdot 10^6 \gamma_{m,c}^2 B'$$, \hspace{1cm} (5.23)

where $B'$ is the magnetic field that can be calculated through the following equation

$$B' = \sqrt{8\pi U'}.$$ \hspace{1cm} (5.24)

The observed values of the characteristic frequencies are Doppler-boosted and are obtained by multiplying the comoving values by the bulk Lorentz factor $\gamma_2(t_{\text{obs}})$. Since regions 2 and 3 are in pressure equilibrium (same internal energy) and comoving, the observed value of the cooling frequency ($\nu_c$) will be the same in both regions, provided that the value of $\epsilon_B$ is common.

The peak flux of the synchrotron spectrum in the observer frame will be at either $\nu'_m$ or $\nu'_c$, whichever is smaller, and will have the value

$$F_m = \gamma_2 P'_{\text{syn}}(\nu'_{m,c}) \frac{N_{\text{beam}}}{d\Omega d^2}.$$ \hspace{1cm} (5.25)

In the above equation $d$ is the distance to the source, $N_{\text{beam}} = N \frac{d\Omega}{4\pi}$ is the total number of particles whose radiation is beamed towards the observer and $d\Omega = 2\pi \left[1 - \cos\left(\frac{\pi}{\gamma_2}\right)\right]$ is the solid angle within which most of the radiation is being emitted.

In the case of a jetted outflow the two effects of $N_{\text{beam}}$ and $d\Omega$ cancel out while $\gamma_2 > 1/\theta_j$, where $\theta_j$ is the half-opening angle of the jet. However, once $\gamma_2 < 1/\theta_j$, $d\Omega$ (the solid angle of the emission) keeps growing, while $N_{\text{beam}} = N/2 \left[1 - \cos(\theta_j)\right]$, which stays constant. In other words, we account for the missing-flux effect after the edges of the jet become visible (Panaitescu et al., 1998), but not for sideways spreading (Rhoads, 1999). The validity of this approach is reinforced by studies based on two-dimensional hydrodynamic simulations that find lateral spreading of the jet to be relatively unimportant, especially while the outflow is still relativistic (Granot et al., 2001; Zhang & MacFadyen, 2009; Wygoda et al., 2011). We will only be concerned with top-hat jets, i.e. possible angular structure is ignored, and with on-axis observations. An off-axis observer will see the jet break at later times and will infer quite different properties for the outflow geometry, if the orientation with respect to the jet axis is not taken into account (e.g. van Eerten et al. 2010).

Because GRBs are often of cosmological origin, redshift corrections on observer times, frequencies and fluxes need to be taken into account when calculating the observed spectra.
5.2.3 Power-law ejection

In this Section we show how the previous results can be generalised to address a source that has a power-law luminosity with time. We will confine our study to shells of ejecta with a uniform Lorentz factor, but variable density, so that at any given radius \( \dot{M} \propto t_{\text{lab}}^q \). The range of \( q \) is \(-1 < q < 2\). For values of \( q \) below \(-1\) the impulsive-explosion scenario is retrieved (Blandford & McKee, 1976), while for \( q \geq 2 \) the RS does not efficiently decelerate the ejecta.

The total mass of the ejecta is \( E / (\eta c^2) \) regardless of \( q \), since the Lorentz factor of all the ejected material is the same. This leads to

\[
\dot{M} = \frac{(q + 1) E}{\eta c^2 \Delta t^{q+1} t_{\text{lab}}^q} \tag{5.26}
\]

It follows that the lab-frame density of the ejected material is given by

\[
n_4(t_{\text{lab}}, t_{\text{ej}}) = \frac{(q + 1) E}{\eta c^2 \Delta t^{q+1} t_{\text{ej}}^q} \left( \frac{t_{\text{lab}} - t_{\text{ej}}}{4\pi m_p c^3} \right)^2 \tag{5.27}
\]

To avoid infinities in the above equations as \( t_{\text{ej}} \to 0 \) we can set \( \dot{M}(0 < t < 1 \text{ s}) = \frac{(q + 1) E}{\eta c^2 \Delta t} \left( \Delta t / 1 \text{ s} \right)^{-q} \).

We proceed to assume that the RS becomes relativistic while shocking material ejected at the front. In other words we expect the RS to be quite close to the CD when it becomes relativistic. Then, by reasoning completely analogous to the case of steady ejection we find

\[
t_{\text{rel,q}} = \left[ \frac{(q + 1) E}{\pi m_p c^5 n_0 \eta^4 \Delta t} \right]^{1/2} \left( \frac{\Delta t}{1 \text{ s}} \right)^{-q} = t_{\text{rel}} \sqrt{\frac{q + 1}{(\Delta t / 1 \text{ s})^q}} \tag{5.28}
\]

The scaling of the Lorentz factor of the shocked gas once the RS has become relativistic is \( \gamma_2^2 \propto t_{\text{lab}}^{q-2} \) (Blandford & McKee, 1976) leading to

\[
\gamma_2(t_{\text{lab}}) = \frac{\eta}{2} \left( \frac{t_{\text{rel,q}}}{t_{\text{lab}}} \right)^{2-q} \tag{5.29}
\]

Knowing the scaling of \( \gamma_2 \) (and therefore also that of \( \gamma_34 \)) we can reconstruct the profile of the density that the RS is shocking as it is moving inside the ejected shell. We find

\[
n_{4-\text{RS}}' = n_0 \frac{\eta^2}{4} \left( \frac{t_{\text{rel,q}}}{t_{\text{lab}}} \right)^{(2-q)/(2+q)} \tag{5.30}
\]
5.2 Model

The speed at which the RS is moving into the shell can be estimated in a manner similar to that in the case of steady ejection.

\[
\delta x = c \int_0^{t+\delta t} \frac{1}{\gamma^2} \, dt = 2c \frac{\gamma_{rel,q}^{q-2}}{\eta^2} \frac{2 + q}{4} \left[ (t + \delta t)^{\frac{4}{2+q}} - t^{\frac{4}{2+q}} \right] 
\]

\[
= 2c \frac{\gamma_{rel,q}^{q-2}}{\eta^2} \frac{2 + q}{4} t^{\frac{4}{2+q}} \left[ \left( 1 + \frac{\delta t}{t} \right)^{\frac{4}{2+q}} - 1 \right] \Rightarrow 
\]

\[
U^* = \frac{2c}{\eta^2} \left( \frac{t_{rel,q}}{t_{lab}} \right)^{\frac{q-2}{2+q}} 
\]

(5.31)

The crossing time of the shell can be calculated in a similar way to the \( q = 0 \) case.

\[
t_{cr,q} = \left( \frac{2}{q + 2} \frac{\gamma_{rel,q}^{q-2}}{\eta} \Delta t \eta^2 \right)^{\frac{q+2}{q+1}} 
\]

(5.32)

We can now derive the scaling for the number of particles in region 3 as a function of time.

\[
\frac{dN_3}{dt} = 4\pi c^2 t^2 U^* n_{4-RS} \Rightarrow N_3 \propto \frac{t_{lab}^{4(q+1)}}{\eta^{2+2}} 
\]

(5.33)

As in the case of steady ejection we construct a formula that obeys this scaling once the RS is relativistic while it takes into account the particles already shocked before \( t_{rel,q} \) and satisfies the conservation of particle number at \( t_{cr,q} \)

\[
N_3(t_{lab}) = N_T \left[ \left( 1 - \frac{N_{rel}}{N_T} \right) \frac{t_{lab}^y - t_{rel,q}^y}{t_{cr,q}^y - t_{rel,q}^y} + \frac{N_{rel}}{N_T} \right], 
\]

(5.34)

where \( N_{rel} = N_3(t_{rel,q}) \) is the number of ejected particles that have been shocked before \( t_{rel,q} \) and \( y = \frac{4(q+1)}{q + 2} \). To get an estimate of \( N_3(t_{rel,q}) \) we adopt the result from steady ejection where the depth of the shell that the RS has shocked at \( t_{rel} \) is \( \Delta x_{RS}(t_{rel}) = c t_{rel} / \eta^2 \). Then

\[
N_3(t_{rel,q}) = \frac{1}{m_p} \int_0^{\Delta x_{RS}} c \dot{M} \, dt = \frac{E}{m_p c^2 \eta \Delta t^{q+1}} \left( \frac{t_{rel,q}}{\eta^2} \right)^q
\]

\[
= N_3(t_{rel}) \left( \frac{t_{rel,q}}{\eta^2 \Delta t} \right)^q 
\]

(5.35)

One can verify that all the expressions for the power-law ejection can be reduced to the ones corresponding to steady ejection when \( q = 0 \).
5.3 Light curves from the forward and reverse shock

In this Section we present results from the numerical implementation of the equations presented in the previous Section. We explore the extent to which emission from the FS and the RS can explain the diversity of observations during the plateau phase. We find that RS light curves are generally different than those of the FS. Therefore, when the RS contribution is significant, a wider range of possibilities exists for the observed decay indices and breaks. We find that the dynamics of the intermediate phase allow for very different decay indices in optical and X-ray frequencies (discussed in Panaitescu & Vestrand 2011), as long as a critical frequency of the spectrum lies between these bands. This result holds both for the FS and the RS. We also show that the interplay between FS and RS can lead to injection breaks accompanied by spectral evolution. Some of these features have been interpreted as evidence for the presence of additional radiation mechanisms. Here we show how they arise naturally, even in the simplest form of the thick-shell scenario.

Construction of spectra as a function of time follows from calculating $F_m$, $\nu_m$ and $\nu_c$ (Sari et al., 1998). Self-absorption of synchrotron photons is ignored, as we are mostly interested in the optical and X-ray behaviour observed in the ‘plateau’ phase, during the early afterglow. The influence of self-absorption on optical frequencies can be excluded by the fact that after the injection break, the slopes of the observed light curves are typically negative. Spectral breaks are assumed to have infinite sharpness, for simplicity. In the case of jetted outflows, the blast-wave energy $E$ stands for the isotropic equivalent of the ‘real’ energy content which is $E \cdot \Omega_j / 4\pi$, with $\Omega_j$ denoting the solid angle of the pair of jets.

5.3.1 Simple light curves with the same temporal index

A simultaneous steepening in the light curves of optical and X-ray data can be interpreted either as a jet break or an ‘injection’ break (Zhang et al., 2006; Racusin et al., 2009). There are cases where the decay indices before and after the break clearly favour the second interpretation. In Fig. 5.1 we present light curves at optical and X-ray frequencies before and after an injection break, calculated using the formulas in the previous Section. The light curves are similar in these two bands because optical and X-ray frequencies lie above $\nu_m$ and $\nu_c$, both in the FS and the RS. Therefore, spectral indices before and after the break remain constant. A notable feature of the RS emission is the rapid decay after the injection break, as expected, due to adiabatic and radiative cooling. The physical parameters for this example (see caption of Fig. 5.1) have been chosen such that the resulting light curves are not complicated by effects like the crossing of critical frequencies, or jet breaks.

The case presented in Fig. 5.1 is one of the simplest possible. However, despite
5.3 Light curves from the forward and reverse shock

Figure 5.1: Optical (top) and X-ray (bottom) light curves before and after the injection break (at approximately $1.5 \cdot 10^4$ s). The optical frequency is $5 \cdot 10^{14}$ Hz, while the X-ray frequency is $2 \cdot 10^{18}$ Hz. The contributions of the forward (dotted line) and reverse shock (dashed line) are depicted for both. Physical parameters are: $E = 10^{51}$ erg, $n_0 = 50 \text{ cm}^{-3}$, $\Delta t = 10^4$ s, $\eta = 600$, $q = 0$, $\epsilon_e = \epsilon_B = 0.1$, $p = 2.3$, $\theta_j = 90^\circ$, $d = 10^{28}$ cm, $z = 0.56$.

5.3.2 Light curves with different temporal indices, chromatic and achromatic breaks

In Fig. 5.2 we present light curves for a set of physical parameters that results in different behaviour in the optical than the X rays. Specifically, the X-ray light curve
The plateau phase of gamma-ray burst afterglows in the thick-shell scenario

Figure 5.2: Similar to Fig. 5.1 but for different physical parameters. Two achromatic breaks are present, the jet break at $10^4$ s and the injection break at $1.5 \cdot 10^5$ s. There is also a break in the X-ray light curve (around $3 \cdot 10^2$ s), due to the crossing of $\nu_c$, with no corresponding optical feature. Physical parameters are: $E = 10^{52}$ erg, $n_0 = 0.04$ cm$^{-3}$, $\Delta t = 10^5$ s, $\eta = 1000$, $q = -0.3$, $\epsilon_e = 0.2$, $\epsilon_B = 0.01$, $p = 2.2$, $\theta_j = 7^\circ$, $d = 10^{28}$ cm, $z = 0.56$.

has one extra break, compared to the optical, due to the passage of $\nu_c$ through the X-ray band both in region 2 and 3. After that chromatic break, there are two achromatic ones, first due to the edges of the jet becoming visible (jet break at $10^4$ s) and then due to the RS crossing the ejecta (injection break at $1.5 \cdot 10^5$ s).

Fig. 5.2 demonstrates clearly how chromatic breaks and different decay indices between optical and X-ray light curves are possible. The break in the X-ray light curve at about $3 \cdot 10^2$ s is caused by the crossing of $\nu_c$. However, such a break is accompanied by a change in the spectral index of the X-ray band, something not always seen during chromatic X-ray breaks (Nousek et al., 2006), pointing to a different origin for the break in those cases. Fig. 5.2 is also an example of light curves in optical and X-ray frequencies having distinct decay indices, their difference in this case bigger than 0.4. For comparison, in the canonical (adiabatic) afterglow phase the difference is 0.25, for $\nu_m < \nu_o < \nu_c < \nu_x$ (Granot & Sari, 2002). This has been previously reported for FS emission (e.g. Zhang et al. 2006; Panaitescu & Vestrand
5.3 Light curves from the forward and reverse shock

Figure 5.3: Similar to Fig. 5.1 but for different physical parameters. Physical parameters are: $E = 10^{52}$ erg, $n_0 = 6 \, \text{cm}^{-3}$, $\Delta t = 10^4 \, \text{s}$, $\eta = 1000$, $q = 0$, $\varepsilon_e = 0.2$, $\varepsilon_B = 0.05$, $p = 2.4$, $\theta_j = 30^\circ$, $d = 10^{28} \, \text{cm}$, $z = 0.56$.

2012). Here, we show how the same feature can also be produced if the RS dominates emission in both bands.

5.3.3 Achromatic breaks with spectral evolution

In Fig. 5.3 we present light curves for another set of parameters. The temporal indices in optical and X-ray bands are the same from $10^3 \, \text{s}$ onwards. Before that, there are two chromatic breaks in the optical light curve due to the passage of $\nu_c$ (both in region 2 and 3) and the passage of $\nu_m$ (only in region 2). Immediately after the injection break at $1.5 \cdot 10^4 \, \text{s}$, emission from the RS fades and the spectrum is dominated by emission from the FS. However, whereas in the X rays the spectral index does not change (assuming that $p$ is the same in both regions), it does in the optical. This happens because the optical frequency ($5 \cdot 10^{14} \, \text{Hz}$) lies in different power laws of the spectrum in the FS and the RS, while the X-ray frequency ($2 \cdot 10^{18} \, \text{Hz}$) does not. Therefore, frequency-dependent behaviour of the spectral index is possible during the injection break, regardless of the decay indices in optical and X-ray light curves.
5.3.4 General remarks

All models presented in this Section have microphysics parameters ($\epsilon_e$, $\epsilon_B$ and $p$) that are the same for the FS and RS. The light curves can become more complicated when this constraint is lifted. In the case of different $\epsilon_B$, the cooling frequency would no longer be the same in regions 2 and 3, resulting in more breaks than in the cases we presented. Different values of $\epsilon_e$ would effectively suppress radiation from the region with the lower efficiency. Different values of $p$ can influence the decay indices, but also the flux levels, and may lead to the inference of spectral evolution during the injection break.

In all the light curves presented in this Section, a common feature is the considerable contribution from the reverse shock, which by the end of the plateau phase dominates the emission. An inevitable consequence in such cases is the characteristic drop of the combined light curve immediately after the injection break, due to the rapid decline of the RS emission. The sharpness of the drop in the combined light curve is dictated by the relative level of the FS emission, which soon after the injection break dominates the emission. We consider this characteristic drop, right after the injection break, to be a strong indication of the RS contributing significantly to the flux detected at the end of the plateau phase. Examples of optical afterglow light curves that clearly exhibit such a behaviour can be seen, for example, in GRB 080928 and GRB 090423 (e.g. Panaitescu & Vestrand 2011).

5.4 Scalings

Calculations of the RS emission have been presented in the past and sometimes have been used to fit model parameters to data (Mészáros & Rees, 1997; Panaitescu et al., 1998; Sari & Piran, 1999; Chevalier & Li, 2000; Kobayashi & Sari, 2000; Kumar & Panaitescu, 2003; Zhang et al., 2006; Uhm, 2011). However, an element that seems to be missing from the literature is flux scalings for the forward and the reverse shock in the intermediate dynamical phase, when the RS is relativistic. In this Section we present scalings for the three spectral parameters ($F_m$, $\nu_m$ and $\nu_c$) introduced in Section 5.2, of both the FS and the RS. We also present the resulting temporal indices for the flux of all optically thin power-law segments in the slow ($\nu_m < \nu_c$) and fast ($\nu_c < \nu_m$) cooling case.

Table 5.1 contains the analytic dependencies of $F_m$, $\nu_m$ and $\nu_c$ on the physical parameters of the model. The values of the spectral parameters are calculated in the observer frame. Under the assumption of pressure equilibrium between regions 2 and 3, the cooling frequency is the same in both regions, provided that they share the same $\epsilon_B$.

The temporal scalings presented in Table 5.1 can be used to calculate the flux at
### Forward shock (region 2)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_m$</td>
<td>$\left(\frac{2}{q+2}\right)^{q+1} (q+1)^{-\frac{2-q}{2}} \epsilon_B^2 E^{-\frac{2-q}{4}} n_0^{\frac{q+1}{2}} \eta^{4q} \Delta t^{\frac{(q+1)(q-2)}{2}} t_{\text{obs}}^{q+1}$</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>$\left(\frac{p-q}{p-1}\right)^2 \left[\frac{2}{(q+2) \sqrt{q+1}}\right]^{\frac{2}{q+2}} \epsilon_c^2 \epsilon_B^2 E^{-\frac{2-q}{4}} n_0^{\frac{q+1}{4}} \eta^{4q} \Delta t^{\frac{(q+1)(q-2)}{4}} t_{\text{obs}}^{-\frac{q+2}{2}}$</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>$\left(\frac{2}{q+2}\right)^{q+2} (q+1)^{-\frac{2-q}{4}} \epsilon_B^{-\frac{3}{2}} E^{\frac{2-q}{4}} n_0^{-\frac{q+4}{4}} \eta^{-2q} \Delta t^{\frac{(q+1)(q-10)}{8}} t_{\text{obs}}^{-\frac{5q+2}{4}}$</td>
</tr>
</tbody>
</table>

### Reverse shock (region 3)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_m$</td>
<td>$\left(\frac{2}{q+2}\right)^{q+2} (q+1)^{\frac{2-q}{8}} \epsilon_B^{-\frac{1}{2}} E^{\frac{10-q}{8}} n_0^{\frac{q+2}{8}} \eta^{q-1} \Delta t^{\frac{(q+1)(q-10)}{8}} t_{\text{obs}}^{-\frac{5q+2}{4}}$</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>$\left(\frac{p-q}{p-1}\right)^2 \epsilon_c^2 \epsilon_B^2 n_0^2 \eta^2$</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>$\left(\frac{2}{q+2}\right)^{q+2} (q+1)^{-\frac{2-q}{4}} \epsilon_B^{-\frac{3}{2}} E^{\frac{2-q}{4}} n_0^{-\frac{q+4}{4}} \eta^{-2q} \Delta t^{\frac{(q+1)(q-2)}{4}} t_{\text{obs}}^{-\frac{q+2}{2}}$</td>
</tr>
</tbody>
</table>

Table 5.1: Scalings of $F_m$, $\nu_m$ and $\nu_c$ in the observer frame, both for the forward and the reverse shock. Results are presented for all the physical parameters of the model.

Every possible power-law segment of the synchrotron spectra that originate from the forward and the reverse shock. The resulting temporal indices are presented in Table 5.2. Notice that in most segments, the light curves are expected to be flat to inverted, for moderate values of $q$ (close to 0). When $\nu < \nu_m$, $\nu_c$ in the FS, the light curves rise more steeply. The fact that $\nu_m$ of region 3 is constant with time, regardless of the value of $q$ (see Table 5.1), implies that the temporal profiles of two consecutive power-law segments (those connected at $\nu_m$) are identical, both in the slow cooling and the fast cooling case. It is easy to check that the temporal indices for the spectral parameters (Table 5.1) and the flux in individual power-law segments of the spectrum (Table 5.3) converge to the impulsive-ejection scenario (thin shell) when $q = -1$ (see, for example, Granot & Sari 2002).
### Table 5.2: Temporal scalings ($F_{\nu} \propto t^x$) for the observed flux in each power-law segment of the spectrum, during the intermediate dynamical phase. Values of $x$ are presented for the forward and the reverse shock, both in the case of slow and fast cooling.

<table>
<thead>
<tr>
<th></th>
<th>FS</th>
<th>RS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slow Cooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu &lt; \nu_m &lt; \nu_c$</td>
<td>$\frac{5q+8}{6}$</td>
<td>$\frac{5q+2}{4}$</td>
</tr>
<tr>
<td>$\nu_m &lt; \nu &lt; \nu_c$</td>
<td>$\frac{3q+pq-2p+6}{4}$</td>
<td>$\frac{5q+2}{4}$</td>
</tr>
<tr>
<td>$\nu_m &lt; \nu_c &lt; \nu$</td>
<td>$\frac{2q+pq-2p+4}{4}$</td>
<td>$q$</td>
</tr>
<tr>
<td><strong>Fast Cooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu &lt; \nu_c &lt; \nu_m$</td>
<td>$\frac{7q+8}{6}$</td>
<td>$\frac{17q+10}{12}$</td>
</tr>
<tr>
<td>$\nu_c &lt; \nu &lt; \nu_m$</td>
<td>$\frac{2+3q}{4}$</td>
<td>$q$</td>
</tr>
<tr>
<td>$\nu_c &lt; \nu_m &lt; \nu$</td>
<td>$\frac{2q+pq-2p+4}{4}$</td>
<td>$q$</td>
</tr>
</tbody>
</table>

#### 5.4.1 The $F_b - t_b$ relation

Accumulating observations of plateau afterglows have allowed for a more systematic study of the behaviour revealed by Swift. Panaitescu & Vestrand (2011) have found that plateau afterglows, 17 in their sample, obey the following relation $F_b \propto t_b^{-1\pm0.5}$, where $F_b$ is the optical flux at the injection break and $t_b$ is the observed time of the break. More recently, Li et al. (2012) studied the $F_b - t_b$ relation for a sample of 39 afterglows with a shallow decay in the optical. They derive a much tighter relation, $F_b \propto t_b^{-0.78\pm0.08}$. This relation falls within the range of Panaitescu & Vestrand (2011) but carries a much smaller uncertainty, by almost an order of magnitude. In this Section we derive model predictions for the $F_b - t_b$ relation and compare them to the aforementioned studies.

Given that the observed time of the injection break (eq. 5.14) is dominated by the time-scale of the ejection (otherwise the intermediate dynamical phase would be very short lived) and ignoring redshift effects, we can approximate $t_b \approx \Delta t$. Using the information of Table 5.1 we analytically derive the scalings of $F_b = F(\Delta t)$ for

The uncertainty is mainly caused by the determination of the break time, rather than the errors of photometric data. In Fig. 5.2 we show an example of how narrow jet angles and long ejection times can result in consecutive achromatic breaks that are ambiguously interpreted.
Table 5.3: Dependence of the observed monochromatic flux at the end of the plateau phase on the duration of ejection ($F_b \propto \Delta t^x$). Values of $x$ are presented for every power-law segment from the forward and the reverse shock, both in the case of slow and fast cooling.

<table>
<thead>
<tr>
<th></th>
<th>FS</th>
<th>RS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slow Cooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu &lt; \nu_m &lt; \nu_c$</td>
<td>$\frac{5q^2+5q+6}{12}$</td>
<td>$\frac{q^2+q-6}{8}$</td>
</tr>
<tr>
<td>$\nu_m &lt; \nu &lt; \nu_c$</td>
<td>$\frac{(p+3)(q^2+q)-6p+6}{8}$</td>
<td>$\frac{q^2+q-6}{8}$</td>
</tr>
<tr>
<td>$\nu_m &lt; \nu_c &lt; \nu$</td>
<td>$\frac{(p+2)(q^2+q)-6p+4}{8}$</td>
<td>$-1$</td>
</tr>
<tr>
<td><strong>Fast Cooling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu &lt; \nu_c &lt; \nu_m$</td>
<td>$\frac{7q^2+7q+2}{12}$</td>
<td>$\frac{5q^2+5q-14}{24}$</td>
</tr>
<tr>
<td>$\nu_c &lt; \nu &lt; \nu_m$</td>
<td>$\frac{3q^2+3q-2}{8}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\nu_c &lt; \nu_m &lt; \nu$</td>
<td>$\frac{(p+2)(q^2+q)-6p+4}{8}$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

the forward and the reverse shock, both in the case of slow and fast cooling and present them in Table 5.3. The relation between the scalings presented in Table 5.3 and the findings of Panaitescu & Vestrand (2011) and Li et al. (2012) can be better understood through Fig. 5.4 and 5.5. In these figures we show how the index of the $F_b - t_b$ relation depends on the value of $q$ for all the power-law segments of the FS and the RS, both for slow and fast cooling. We also present the observationally inferred values of the index, for both studies. The range of $q$ reflects all the possible values for which the presented model holds, i.e. from $-1$ to $2$. However, not all values are equally plausible. The impulsive-ejection scenario is retrieved for $q < -1$ (Blandford & McKee, 1976). Therefore, if the distribution of $q$ is continuous, values close to, but greater than, $-1$ are expected. A value close to $0$ is predicted if the outflow is powered by a millisecond pulsar (Dai & Lu, 1998), while fits to several Swift afterglows seem to indicate $q \sim -0.5$ (Zhang et al., 2006). This has been recently confirmed by Li et al. (2012). Furthermore, Curran et al. (2009) have found that half of the afterglows in their sample are consistent with energy injection, for which $-0.5 < q < 0$ was inferred. Therefore, we consider the range $-1 < q < 0$ to include a dominant fraction of the $q$ distribution, in the case of prolonged ejection.

In Fig. 5.4 we see that model predictions, for some of the power-law segments of
The plateau phase of gamma-ray burst afterglows in the thick-shell scenario

Figure 5.4: Index of $F_b - t_b$ relation as a function of $q$, for all power-law segments of the FS. The lightly shaded region contains the range of values allowed by the scaling of Panaitescu & Vestrand (2011), while the darker region denotes the scaling of Li et al. (2012). The five curves correspond to the five possible power laws, as presented in Table 5.3. From top to bottom: (i) $v < v_m < v_c$, (ii) $v < v_c < v_m$, (iii) $v_c < v < v_m$, (iv) $v_m < v < v_c$ (for $p = 2.5$), (v) $v_m, v_c < v$ (for $p = 2.5$). The two bottom curves that lie partially within the shaded regions are depicted with solid lines.

The FS, lie outside the shaded regions. The curves that, at least partially, lie within the shaded regions, correspond to spectrum configurations where the synchrotron characteristic frequency $v_m$ is smaller than optical frequencies, at the end of the plateau. As long as that condition is fulfilled (which is very likely to be the case, especially for long-lived plateaus), the FS can account for the $F_b - t_b$ relation of Panaitescu & Vestrand (2011), within the range $-1 < q < 0$. However, FS emission cannot reproduce the scaling of Li et al. (2012) for any power-law segment, within the aforementioned $q$ range. The only way this scaling can be reconciled with the predictions of the model for the FS, is for $v_m < v_o < v_c$ and $p \approx 2$ (even for $p = 2.1$ and $q = -0.5$ the predicted value is approximately $-1$, outside the allowed range). In all other cases, the predictions for the FS are at odds with the relation of Li et al. (2012), unless $q \sim 0.5$.

On the other hand, the RS (Fig. 5.5) shows less diversity in the possible values of
the index. In fact, for \( q < 0.3 \) the relation of Panaitescu & Vestrand (2011) is not just possible, but inevitable. The scaling of Li et al. (2012) is achieved if \( \nu_m < \nu_o < \nu_c \), for any value of \( p \). If \( \nu_c < \nu_o \), the predicted relation is slightly steeper with an index of \(-1\), regardless of the value of \( \nu_m \). The findings of both groups (and especially these of Li et al. (2012)) can be better explained if the ejecta shocked by the RS are dominating the emission during the plateau phase. We consider this strong indication of the important role of the RS during energy injection.

### 5.5 Example fits

In this Section we show two examples of simultaneous fits of physical parameters to UV/IR and X-ray data. We stress that the model uses certain approximations (single zone, thin zone, \( \gamma_{34} \gg 1 \), sharp spectral breaks) that mainly affect the relative flux levels of the FS and the RS. Given those approximations, the fits are performed mostly to show how the overall properties of observations, before and after the injection break, can be explained with the presented model. For that reason we have
not attempted a $\chi^2$ minimization to obtain best-fit values of the physical parameters, but we mostly focus on the duration of the plateau phase and the contribution of the reverse shock.

The two afterglows we fit are of GRB 060729 and GRB 090423. In both cases the X-ray fluxes come from the Swift/XRT GRB lightcurve repository (Evans et al., 2007, 2009). For GRB 060729 the Swift/UVOT light curves are taken from Grupe et al. (2007). The magnitudes presented in that paper have been corrected for Galactic extinction and converted to fluxes using the conversion factors presented in Roming et al. (2009). For GRB 090423 the infrared fluxes are from Tanvir et al. (2009).

### 5.5.1 GRB 060729

In Fig. 5.6 and 5.7 we present light curves at 10 keV ($2.418 \cdot 10^{18}$ Hz) and the UVW1 band ($1.15 \cdot 10^{15}$ Hz), respectively, along with model light curves for GRB 060729.
Figure 5.7: Fit to the UV afterglow light curve of GRB 060729. Data and model light curves in the UVW1 band. Data have been corrected for galactic extinction assuming a spectral index of 1.1. Different curves and physical parameters are as described in the caption of Fig. 5.6.

This afterglow can be best fit by interpreting the steepening at \( t_{\text{obs}} \approx 5 \cdot 10^4 \) s as the injection break. This implies that in its own frame \( \Delta t = 3.2 \cdot 10^4 \) s. We interpret the mild achromatic break at \( t_{\text{obs}} \approx 10^6 \) s as the jet break. This yields a semi-opening angle of 24°, which for an isotropic energy \( E = 1.3 \cdot 10^{53} \) erg gives the real energy content of the blast wave \( E_{\text{bw}} = 1.12 \cdot 10^{52} \) erg. The value of \( q \) in the presented fits is \(-0.22\). The contribution of the RS up until the injection break, both in X rays and UV, is modest.

5.5.2 GRB 090423

In Fig. 5.8 and 5.9 we present light curves at 10 keV (2.418 \( \cdot \) 10^{18} Hz) and infrared bands, respectively, along with model light curves for GRB 090423. Infrared data points combine observations in the \( K \) (1.38 \( \cdot \) 10^{14} Hz), \( H \) (1.82 \( \cdot \) 10^{14} Hz) and \( J \) (2.4 \( \cdot \) 10^{14} Hz) bands. The infrared light curve has been calculated for a representative frequency of 1.8 \( \cdot \) 10^{14} Hz. We interpret the break at \( t_{\text{obs}} \approx 5 \cdot 10^4 \) s as the injection break. This is the same observer time as in the case of GRB 060729. However, the
numbers differ in their respective frames due to the large redshift of GRB 090423 for which we find $\Delta t = 4.9 \cdot 10^3$ s. A jet break at $t_{\text{obs}} \approx 2 \cdot 10^5$ s is needed to account for the last data point both in the IR and in X rays. The inferred isotropic energy and jet angle yield $E_{\text{bw}} = 1.12 \cdot 10^{52}$ erg. The value of $q$ for GRB 090423 is $-0.52$. The contribution of the RS is considerable both in X-ray and infrared frequencies, which leads to the characteristic curved shape of the light curves after the injection break.

X-ray observations before $3 \cdot 10^3$ s and around $10^4$ s, observer time, have not been fit. The reason is that both of these periods reveal intense flaring activity superimposed on the baseline flux. These flares probably signal episodic activity of the central engine and cannot be accounted for by the smooth-injection model we are using.

\[ F_{10 \text{ keV}} (\text{mJy}) \]

\[ t (\text{s}) \]

**Figure 5.8:** Fit to the X-ray afterglow light curve of GRB 090423. X-ray flux is presented at 10 keV. Data points with squares have been excluded from the fit, as they are a result of high-latitude emission (before $10^3$ s) and flaring behaviour (around $10^4$ s). Physical parameters are: $E = 6.5 \cdot 10^{53}$ erg, $n_0 = 10 \text{ cm}^{-3}$, $\Delta t = 4.9 \cdot 10^3$ s, $\eta = 340$, $q = -0.52$, $\epsilon_e = 0.25$, $\epsilon_B = 2 \cdot 10^{-5}$, $p = 2.1$, $\theta_j = 11^\circ$, $d = 2.577 \cdot 10^{29}$ cm, $z = 8.1$. 

*The plateau phase of gamma-ray burst afterglows in the thick-shell scenario*
5.6 Discussion

We have presented simple analytic calculations of blast wave dynamics in the thick-shell scenario. Based on those, we have shown how the spectra of the FS and the RS can be constructed self-consistently, at any observer time. The combined output of the two regions can account for a big part of the diversity observed in the plateau phase of GRB afterglows, even in the simple case where the FS and the RS share the same microphysics.

5.6.1 Model assumptions

In the derivation of the presented formulas, several approximations have been made. The most important one is that of a homogeneous layer, both for region 2 and 3. Kobayashi & Sari (2000) have studied numerically the profile of the blast wave in the thick-shell case and found small deviations from homogeneity. These deviations, however, are expected to have a small impact on the flux levels. Due to the pressure gradient (pressure rises monotonically, but slowly from the RS to the FS) the flux
ratio $F_{FS}/F_{RS}$ is probably underestimated in this work. A numerical approach is necessary to assess accurately the relative strength of the two regions.

Another approximation that we adopt from $t_{rel}$ onwards is $\gamma_{34} \gg 1$ which is equivalent to $\eta \gg \gamma_{2}$. Obviously, the weakness of this approximation is at observer times close to the onset of the intermediate dynamical phase ($t_{rel}$). However, in the observer frame, $t_{rel}$ is of the order of $10$ s, for typical parameters ($E = 10^{52}$ erg, $n_{0} = 1.0$ cm$^{-3}$, $\eta = 300$, $\Delta t = 10^{3}$ s, $q = -0.5$). This implies that those approximations are valid at observer time-scales of $10^{3} - 10^{5}$ s, when plateau phases are typically observed.

At a more fundamental level lies the issue of the existence of strong reverse shocks in the first place. The doubts are risen due to the degree of magnetisation of the ejecta, which, if high, is widely expected to hinder the generation of powerful shocks. Matter ejected from the central engines of GRBs carries a significant degree of magnetisation in the scenario of magnetically-driven ejecta (Komissarov & Barkov, 2009), but also in the case of neutrino-driven ejecta (MacFadyen & Woosley, 1999). In general, we expect a modification of the magnetisation during the interactions that give rise to the prompt emission. However, this modification may be small for radiatively efficient internal shocks (Komissarov, 2012). This implies that if at the time the prompt emission is produced the outflow is magnetically dominated, it is likely to remain so. This would lead to a suppression of the RS afterglow emission compared to the flux levels presented in this paper. Conversely, if the magnetic component of the ejecta’s energy is not dominant (kinetic-energy dominated ejecta), a strong RS is expected to form. Mimica et al. (2010) have numerically studied the influence of the ejecta magnetisation on early afterglow spectra and found that radiation from the RS is strongest for a ratio of magnetic to kinetic energy $0.01 - 0.1$. In that case, assuming that the source of the post-shock magnetic field is the shock-amplification of pre-existing fields, region 3 is expected to carry a higher fraction of magnetic energy ($e_{B}$) than region 2, which will make the RS emission even stronger.

5.6.2 Implications

Shallow decay

Regardless of the uncertainties in the relative flux levels of the FS and the RS, our results for the temporal scalings of the observed flux still hold. These scalings predict a shallow decay or smoothly rising light curves for all optically thin power-law segments of the synchrotron spectrum, a defining feature of the plateau phase of GRB afterglows. This result is independent of which region (FS or RS) is dominating the radiation.

During the plateau phase, X-ray and optical light curves can be quite different.
Even if the FS dominates the emission, chromatic breaks (due to the passage of critical frequencies) and different decay indices between two bands are possible. The range of possibilities widens when RS emission becomes considerable. The injection break can be accompanied by a change of the spectral index in one or more frequencies. Additionally, the behaviour of the light curves after the injection break will be affected by the relative contribution of the RS at the end of the plateau phase, which may be different across the spectrum. Furthermore, if the microphysics of region 3 is different than that of region 2, the picture can become even more complicated. In that scenario, different values of $\epsilon_B$ will result in different values of $\nu_c$ which may result in additional breaks. Different values of $p$ may lead to varying decay indices and spectral evolution during achromatic breaks.

All of the possibilities mentioned above have been observed (e.g. Panaitescu & Vestrand 2011; Li et al. 2012). Perhaps the only observed feature of the plateau phase that cannot be easily explained with the proposed model is the existence of a handful of chromatic breaks that are not accompanied by spectral evolution (e.g. Panaitescu et al. 2006). The fact that those breaks are chromatic excludes the possibility of them being injection or jet breaks. The constancy of the spectral index excludes the possibility of a critical frequency causing the break. Since these breaks are mostly observed in X rays, a reasonable suggestion is that they originate from inverse Compton scattering, something which does not exclude the presence of energy injection (see, for example, Panaitescu & Vestrand 2012).

The $F_b - t_b$ relation

Based on scalings for the spectral parameters during the plateau phase, we have derived predictions of the thick-shell scenario for the $F_b - t_b$ relation, both for the FS and the RS. We should stress that these predictions (presented in Table 5.3) are not sensitive to the main approximations of the model (single zone, $\gamma_{34} \gg 1$ etc.), but derive from basic considerations of the dynamics and the jump conditions at both the FS and the RS. Therefore, they are expected to hold also in a more thorough analysis of the presented physical model.

In Fig. 5.4 and 5.5, predictions for the $F_b - t_b$ relation are compared against the observationally inferred scalings of Panaitescu & Vestrand (2011) and Li et al. (2012). Based on those scalings, we cannot firmly exclude any of the regions 2 or 3 as the origin of optical emission at the time of the injection break. However, the findings of both groups are more easily accommodated by the RS. Especially under the tight constraints of the relation of Li et al. (2012), the FS would require special conditions across a number of different afterglows to reproduce the observed scalings. Therefore, we consider emission from the RS to be likely dominant in, at least, a fraction of the observed afterglows. A characteristic feature in RS-dominated
light curves is the short-lived steep decay right after the injection break, which is a result of the sudden termination of energy injection.

Radiation from the RS has been neglected in most studies so far, at least in cases where the RS becomes ultrarelativistic before the injection break. This may have important consequences on the inferred values of blast-wave physical parameters through modelling. Our analysis of the physical model and the observed anticorrelation between $F_b$ and $t_b$ demonstrates that the emission of the RS can be important and should be taken into account. A similar conclusion was reached by Uhm & Beloborodov (2007), only in their analysis the RS is responsible for the entire afterglow emission. We propose that the RS emission is significant during the plateau phase of the afterglow, which is typically followed by the ‘canonical’ afterglow behaviour (Nousek et al., 2006), commonly attributed to the FS. Besides, the majority of afterglows display decay indices compatible with the adiabatic blast-wave model, at sufficiently late times (Racusin et al., 2009).

**Internal shocks and GRB engines**

In the context of internal-shock models for the prompt emission, the observed duration of the GRB roughly corresponds to the lab-frame duration of the central engine (e.g. Sari & Piran 1997a). Therefore, the parameter $\Delta t$ in the thick-shell scenario should also be of the same order. However, there are a few effects that can alter this simplistic picture. We discuss them below.

First of all, the observed duration of the burst, as expressed in $T_{90}$ is a lower limit to the duration of ejection, since collisions between individual shells (leading to internal shocks) may occur without resulting in significant detections. Secondly, the velocity profile of the ejected matter can be inhomogeneous, causing contraction or expansion of the ejected shell by the time the internal shocks occur (Rees & Meszaros, 1998). Furthermore, the collisions of shells during the prompt emission modify the profile of the ejecta, long before the radiation due to external shocks becomes detectable. Mimica et al. (2007) have numerically studied collisions of magnetized shells and find that the post-shock thickness of the ejecta is of similar order to the sum of the widths of the pre-shock shells, with higher magnetisation producing thicker ejecta. However, once the internal shocks have crossed the colliding shells, rarefaction waves result in expansion of the ejecta and cooling of the constituent particles.

Under the extreme assumption that expansion of the post-shock shell (at a velocity of the order of the speed of light) continues until the RS crosses the ejecta, the ratio $t_{cr}/\Delta t \approx \eta \sqrt{t_{rel}/\Delta t} \sim \Delta t^{-3/4}$ determines the importance of expansion on the width of ejecta that the RS encounters. If $t_{cr}/\Delta t < 1$, then that effect is negligible, while if $t_{cr}/\Delta t \gg 1$, the ejecta will spread substantially before the RS crosses the en-
tire region, prolonging the duration of energy injection, and therefore, the observed plateau phase. Based on these considerations we conclude that the duration of the plateau phase, reflecting the thickness of the ejecta after the internal shocks, will be at least as long as the prompt emission, while in extreme cases it may last orders of magnitude longer. Similar conclusions hold for the relation between the duration of the plateau phase and the operation of the central engine. Due to the potentially significant expansion of the ejecta after the internal shocks take place, the thick-shell scenario may also be relevant for short bursts with shorter accretion time-scales (Aloy et al., 2005) than those of the long-burst progenitors (Woosley, 1993).

5.7 Conclusions

We have used a simple analytical method to calculate self-consistently synchrotron emission from the forward and the reverse shock in the thick-shell scenario. The approach is generalised to energy injection with arbitrary power, as long as it can be described by a power-law in lab-frame time. The resulting light curves exhibit shallow, or even inverted temporal decays, observed during the plateau phase in early-afterglow observations. Especially when the contribution from the reverse shock is included, a wide range of possibilities emerges for the behaviour of the light curves during the plateau phase and the injection break. This range includes chromatic and achromatic breaks, widely varying decay indices between different frequencies, injection breaks accompanied by spectral evolution and frequency-dependent behaviour right after the injection break, depending on the relative contribution of the reverse shock at the end of the plateau phase. The picture can become more complex if a different set of microphysics parameters is allowed for the forward and the reverse shock, but also if the jet-break time occurs at time-scales comparable to the injection break, for collimated outflows.

We present scalings for the critical parameters of the spectrum \((F_m, \nu_m, \nu_c)\) during the plateau phase and derive temporal scalings of the flux for every power-law segment of the spectrum, both for the forward and the reverse shock. Using the analytical dependence of \(F_m, \nu_m\) and \(\nu_c\) on the thickness of the shell \((c\Delta t)\), we have derived predictions for the \(F_b - t_b\) relation for all power-law segments of the forward and the reverse shock. When compared against the observationally inferred relation of Panaitescu & Vestrand (2011) the predictions are invariably (for all power-law segments) in agreement with the observations for the reverse shock, while they are consistent for the forward shock, as long as \(\nu_m < \nu_0\). When compared against the much tighter relation of Li et al. (2012), drawn from a bigger sample, emission from the reverse shock is favoured throughout a plausible range of values for \(q\). We consider this strong evidence that the reverse shock may, at least, contribute significantly
to the observed flux during the plateau phase in the thick-shell scenario and should therefore be taken into account during modelling. We also present applications of the presented model on two-frequency data sets (X rays and UV/IR) of GRB 060729 and GRB 090423 that cover the plateau phase, the injection break, and the canonical decay of GRB afterglows. The inferred values for $\Delta t$ lie close to $10^4$ s (within factors of a few) while those of $q$ are approximately $-0.2$ and $-0.5$. For GRB 060729 we find that the forward shock dominates emission in all bands during the plateau phase and the injection break. For GRB 090423 we find that the reverse shock emission at $t_b$ is slightly higher than that of the forward shock, leading to a characteristic drop in the light curves after the injection break takes place.

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Gammaflitsen (GRB’s, naar het Engelse gamma-ray bursts) zijn de krachtigste bronnen van electromagnetische straling in het heelal. Ze zijn het resultaat van fysische processen die in korte tijd gigantische hoeveelheden energie (een aanmerkelijk deel van de rustenergie van de zon) omzetten in gammastraling. Wekelijks worden er een aantal gammaflitsen waargenomen, met een duur van enkele milliseconden tot enkele minuten. Deze uitbarstingen van energie vinden plaats op kosmologische afstanden, zijn erg zeldzaam en herhalen zich, voor zover wij weten, niet.

De extreme waarden van alle natuurkundige grootheden die we van een gammaflits kunnen meten (fotonenergie, duur, lichtkracht en totale energie) duiden erop dat de waargenomen straling iets onthult over bijzondere plaatsen in het heelal. De gammaflitsen zelf zijn nooit ruimtelijk opgelost en hun plaats aan de hemel is niet altijd goed bepaald. Hun oorsprong kan dus alleen ontrafeld worden uit het licht van de puntbron die we aan de hemel waarnemen. We denken nu dat deze gammastraling ontstaat in een gecollimeerde magnetohydrodynamische gasstroom, ofwel een jet. Deze gasstroom wordt op zijn beurt weer veroorzaakt door een catastrofale gebeurtenis waarin een zwart gat of een magnetar wordt gevormd. Snelle accretie (het invangen van materiaal) op dit zojuist gevormde compacte object leidt tot de uitstoot van materie en energie met relativistische snelheden. Een deel van de beschikbare kinetische energie wordt daarbij omgezet in gammastraling die we vervolgens als gammaflits waarnemen, terwijl een ander deel het materiaal in de omgeving verhit en ioniseert, wat leidt tot een zogenaamde nagloeier. Deze nagloeiers worden waargenomen op langere golflengtes, en zijn vaak nog waarnembaar lang nadat de oorspronkelijke gammaflits al is uitgedoofd. De ontdekking en verdere studie van deze nagloeiers heeft ons geholpen gammaflitsen beter te begrijpen.

Door hun extreme gedrag en uiterlijk zijn de gammaflitsen zelf de moeite waard om te bestuderen. Maar ze kunnen ook gezien worden als hulpmiddelen om andere fenomenen in het heelal te bestuderen. Zoals vaker het geval is in de sterrenkunde,
kunnen we met behulp van gammaflitsen fundamenteel onderzoek en wetenschappelijke experimenten doen die we in laboratoria op aarde nooit zullen kunnen uitvoeren. Denk daarbij aan het creëren van zwarte gaten - objecten met de sterkste zwaartekrachtsvelden in het heelal; het lanceren van een van de snelste stromen materiaal in het hele universum; en het vormen van krachtige schokken – waarschijnlijk plaatsen waar deeltjes versneld worden tot de meeste energetische kosmische straling. Behalve als laboratoria kunnen we gammaflitsen ook gebruiken als zaklampen die delen van het universum doorlichten die gewoonlijk voor de waarnemer op aarde verborgen blijven. Op die manier kunnen we een glimp opvangen van het interstellaire en intergalactische medium op verschillende tijdstippen na het ontstaan van het heelal.

Nagloeiers

Nagloeiers van gammaflitsen worden van energie voorzien door de overgebleven kinetische energie van de gasstroom, nadat een deel daarvan reeds is omgezet in gammastraling. Het omzetten van die energie vindt plaats in een krachtige schok die de gasstroom veroorzaakt in het omringende materiaal: het CBM (circumburst medium). Deze schok verhit de deeltjes die hij tegenkomt. Deze zenden vervolgens synchrotronstraling uit onder invloed van de aanwezige magnetische velden.

Door de interactie met het omringende materiaal remt de gasstroom geleidelijk aan af, waardoor het materiaal in de gasstroom verschillende dynamische stadia doorkoopt, waarbij het telkens meer energie afstaat aan het omringende materiaal. Het is gebruikelijk om de eigenschappen van verschillende gammaflitsen te bepalen door theoretische modellen van deze gasstromen met waarnemingen te vergelijken. Hierdoor verkrijgen we inzicht in de voorlopers van gammaflitsen en de natuurkundige processen die zich tijdens de flits afspelen.

Dit proefschrift

In dit proefschrift bestuderen we de dynamica en bijbehorende straling in verschillende stadia van de nagloeier. In de eerste drie hoofdstukken richten we ons vooral op de late stadia die we waarnemen, wanneer de schokgolf zo ver is afgeremd dat deze niet meer ultrarelativistisch is. In het laatste hoofdstuk kijken we naar een eerder stadium, waarin de kinetische energie van de gasstroom wordt overgedragen aan de schokgolf in het omringende materiaal. Het volgende gedeelte is een korte samenvatting van de vier wetenschappelijke hoofdstukken uit dit proefschrift.
Dynamica en spectra van transrelativistische jets

In hoofdstuk 2 presenteren we de resultaten van eendimensionale, relativistisch hydrodynamische simulaties van jets van gammaflitsen die relativistisch beginnen, maar afremmen naar niet-relativistische snelheden. We bestuderen synchrotronstraling van deze jets, waarbij we voortborduren op methodes die nauwkeurig de spectra en lichtkrommes berekenen op verschillende tijdstippen in de simulatie. De motivatie voor dit onderzoek is dat nagloeiers honderden dagen zichtbaar blijven, en de verwachting is dat op deze tijdschaal de relativistische gasstroom overgaat in een niet-relativistische, de zogenaamde transrelativistische overgang.

Jets van gammaflitsen gaan, vergelijkbaar met die van supernovae, door verschillende dynamisch stadia van versnelling, \textit{coasting} met constante snelheid en vertraging. De details van deze twee typen jets verschillen in de eerdere stadia, voornamelijk door onzekerheden in het versnellingsmechanisme, maar ook omdat de jets in gammaflitsen relativistisch zijn. Nadat die beginnen te vertragen, verandert met verloop van tijd de Lorentzfactor. De Blanford-McKee oplossing geeft een redelijk goede beschrijving van de dynamica tijdens deze vertraging, zolang de Lorentzfactor groter is dan 1 (dus bij hoge snelheden). De Sedov-Taylor oplossing is van toepassing bij jets met veel lagere snelheden. Het grootste probleem zit in het koppelen van deze twee oplossingen, en het begrijpen van de waarneembare aspecten van deze overgang, zoals de tijdsduur.

Wij hebben een reeks aan effecten onderzocht die men kan verwachten bij deze transrelativistische overgang. Een voorbeeld is het veranderen van de adiabatische index van het geschokte gas, wat belangrijk is in de eerste stadia, waar deze afwijkt van de waarde $4/3$ die geldt voor ultrarelativistische snelheden. In tegenstelling tot de gammaflits zelf, kunnen de latere stadia van de nagloeier in zeldzame gevallen wel ruimtelijk opgelost worden. Wij laten zien hoe van deze latere stadia van de jets ruimtelijk opgeloste afbeeldingen kunnen worden berekend met behulp van stralingstransport, en hoe deze afbeeldingen veranderen als functie van de golflengte waarop ze worden waargenomen. We laten ook zien dat de spectra in dit overgangsstadium een langzame overgang van relativistisch naar niet-relativistisch tonen.

Fluxrecepten op alle waarneemtijdstippen

In hoofdstuk 3 gebruiken we het model ontwikkeld in het vorige hoofdstuk om een set formules af te leiden waarmee we voor een gegeven set fysische parameters gedetailleerde spectra en lichtkrommes van gammaflitsen kunnen berekenen: zogenaamde fluxrecepten. We bereiken dit door de analytische oplossingen voor de straling tijdens de ultra- en niet-relativistische stadia te normeren met behulp van numerieke
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berekeningen, en zo de twee stadia met praktische formules aan elkaar te verbinden.

We laten zien dat deze methode een goede beschrijving geeft van de numerieke resultaten, op alle mogelijke tijdstippen die we van gammaflitsen kunnen waarnemen. We concluderen ook dat dit overgangsstadion er op verschillende plekken in het spectrum anders uitziet, wat belangrijke gevolgen heeft voor het interpreteren van waarnemingen met behulp van zulke nagloeiermodellen. Een belangrijk kenmerk van deze methode is dat deze het mogelijk maakt het omringende materiaal van de gammaflits te onderzoeken, wat weer informatie bevat over de voorganger van de gammaflits.

Naast de formules die we presenteren, maken we ook de software beschikbaar die de uitkomsten van deze formules direct met waarnemingen kan vergelijken. Wij hopen dat waarnemers deze eenvoudige code zullen gaan gebruiken om fysische parameters direct uit de waarnemingen af te leiden. Dit is de eerste studie die numerieke simulaties gebruikt om spectra te berekenen op alle mogelijke tijdstippen na de gammaflits. In de toekomst zal het mogelijk zijn om vergelijkbare modellen (al dan niet analytisch) te maken voor tweedimensionale gasstromen, waarmee andere interessante onderwerpen, zoals zogenaamde jet-breaks, direct kunnen worden onderzocht.

Fluxrecepten toegepast op waarnemingen van nagloeiers

In hoofdstuk 4 gebruiken we de fluxrecepten uit voorgaande hoofdstukken om fysische parameters af te leiden uit goed waargenomen nagloeiers. Naast het vinden van de best passende parameters, zijn we ook geïnteresseerd in hoe goed een bolvormig model de waargenomen lichtkrommes kan verklaren. Van de onderzochte uitbarstingen kan er een, GRB 970508, volledig verklaard worden met een bolvormig model, terwijl de structuur van het omringende materiaal wijst op een voorganger met een sterrenwind. Voor de andere uitbarstingen die we hebben onderzocht kunnen we de waarnemingen redelijk verklaren, maar niet altijd perfect en soms ook met vrij onwaarschijnlijke parameters, hetgeen waarschijnlijk betekent dat deze uitbarstingen niet bolvormig zijn.

De parameters die we afleiden voor de schokgolf van GRB 970508 lijken aan-namelijk. De totale energie in de schokgolf is ongeveer $10^{51}$ erg, vergelijkbaar met de energie van de gammaflits zelf, aangenomen dat die ook bolvormig is ($\sim 5 \cdot 10^{51}$ erg). We ontdekken ook dat er evenveel energie in versnelde elektronen zit als in het magnetische veld, alsook bewijs voor een niet-versnelde populatie elektronen.
Thick shells als oorsprong van de vroege en langzame afname van de lichtkromme

In het laatste hoofdstuk van dit proefschrift richten we ons op de vroege stadia van nagloeiers in het tijdperk van de ruimtetelescoop Swift. In het bijzonder richten we ons op de waar te nemen kenmerken van het zogenaamde “thick-shell” (dikke bolschil) scenario. We onderzoeken of dit scenario een goede verklaring biedt voor de vroege en langzame afname van lichtkrommes in het optisch en in röntgenstraling. Een thick shell staat voor een scenario waar het uitstoten van materiaal tijdens de uitbarsting niet instantaan is, maar langer duurt dan gebruikelijk, of langer dan voorheen werd aangenomen.

De gevolgen van deze andere uitstoot op de dynamica in de straalstroom kunnen erg belangrijk zijn. Er kan een terugwaartse schok ontstaan, die zich voortplant binnen het uitgestoten materiaal, en zich beweegt met relativistische snelheden nog voor dat alle energie wordt overgedragen aan de schokgolf in het omringende materiaal. Dit kan leiden tot een extra dynamisch stadium, tussen het coasting- en het canonieke afremstadium, waarbij de dynamica bepaald wordt door energie-uitwisseling tussen de verschillende schokken. Dit is een populaire, maar nog niet volledig onderzochte, manier om energie in de gasstroom te blijven injecteren.

Wij presenteren eenvoudige analytische berekeningen van de dynamica, de deeltjespopulatie en de thermodynamica van de schokgolf en straalstroom als functie van tijd, tijdens het tussenliggende dynamische stadium en de overgang naar het stadium waarin de gammaflits uitdooft. Hieruit kunnen we eenvoudig de waar te nemen kenmerken van dit scenario afleiden. We maken een semi-analytische toepassing van dit model, en gebruiken die om de mogelijke diversiteit aan lichtkrommes tijdens deze fase van energie-injectie te laten zien. We onderzoeken de voorspelling van dit model door de normering van de optische flux aan het einde van dit stadium van energie-injectie af te leiden, waaruit blijkt dat de terugwaartse schok de waarnemingen beter verklaart. We passen dit model toe op waarnemingen van de nagloeiers GRB 060729 en GRB 090423, en leiden de fysische parameters af die de energie-injectie in de schokgolf beschrijven.
Acknowledgements

Having been a member of the API for six and a half years now, I see this section as an opportunity to reflect on the people, events and circumstances that led me into this path, culminating now with the finish of my PhD research. Can’t promise laughs and name listing, but the text that follows is a sincere, albeit still hot-blooded assessment of this important part of my life.

It all started about 9 years ago, when I made my first trip to Amsterdam. I was a proper tourist, even did the Rijksmuseum and Rondvaart tour, which were arguably the lowlights of an uncultured 21-year-old’s trip. That trip, however, would prove illuminating on how a modern city should look and feel, in my opinion. Perhaps this is also a matter of taste, but anybody that has visited Athens (for example) at some point understands what I mean. Lots of things have been said the last few years about the socioeconomic state of Greece, the sudden negative popularity being a result of the default that led to the bailout(s) and the crisis we have all heard about. From my point of view, the most discouraging thing back home is that lots of people, despite their disagreement with the established mentality, will do little, if anything, to change it. I was one of them, and two years after my first trip to Amsterdam, had it not been for the push I got from people close to me (but even random ones) I wouldn’t have even thought of the possibility of doing a Master degree elsewhere than the University of Athens. After all, why would I leave a nice job (barman), nice weather (it turns out there is a point in this one), friends and family behind?

Well, lets start from the family. My parents, having had themselves the experience of living and studying abroad, made sure I got the message loud and clear: “It is a VERY good idea to go and study abroad at this stage of your life”. Once I saw the possibility, I jumped on it, head first, but the spark wasn’t there before these two people insisted on the benefits of such an undertaking. I am still realising today in how many ways they were right. Thank you both for that. This Thesis is a result of your encouragement and support.
Back then in 2004-2005, my ignorance and carelessness on the issue of scientific
career could be brilliantly summarised by a question I asked Nektarios Vlahakis,
my Bachelor-Thesis supervisor, during one of our slightly off-topic conversations:
“You mean people get paid to do theoretical research in astrophysics?!” I was quite
surprised by his affirmative answer. Nektarios, incidentally (?) working on GRBs,
explained how the academia works and most importantly, helped me get a taste of
what research is like. He, like many other Greeks with international experience in
education and research, strongly supported my application to the UvA. Nektarios
even wrote a reference letter! After all these years I can safely say, living abroad
always has a lot to offer in many levels of a person’s development, but the academic
aspect alone has been one of the greatest rewards.

Which brings us to the API. An institute that despite the constant reshuffling
of people manages to feel like a family. I haven’t been the most social person around
(apparently I’m the only one who finds 8-hour-long Sinterklaas parties of 30 people
a bit dull), and, against tradition, I won’t start remembering stories from times we
got drunk and did something embarrassing, mainly because I can’t remember much.
But I can say I feel very lucky to have been part of this family. Working, discussing,
playing, traveling, going out together with so many different people from all over
the world is a unique and irreplaceable experience, more important than the degrees
and jobs we get on the way. And what makes the API special is the warmth that
everyone has, from the youngest bachelor students who visit occasionally, to the most
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to my arrival and growth within the API. Ralph Wijers, Master coordinator back
in 2006, for some reason thought it a good idea to accept me, a guy with low-ish
grades (but with a letter from Nektarios, lets not forget), as a Master student. Two
years later he offered me a PhD position under his supervision and the rest is history.
Hasn’t been a very straightforward story, though. I’ve had my ups and downs in terms
of productivity and motivation, and as if an unreliable student was not enough, for
Ralph this period has been quite eventful. However, this story has a happy ending,
and it is all down to Ralph’s perseverance. Sometimes sticking to plan is a good
idea and the reward in the end only becomes greater if there are difficulties on the
way. This is one of the most important things I have learned over these years. Ralph,
thanks for not giving up on me, and this project as a whole.

Here is where my valiant effort to not list people ends, because, really, this section
would be incomplete otherwise. During my Master years, I really enjoyed working,
learning and hanging out with Ben, Gijs, Mihkel and Sjoert. You guys are top mates. Special thanks also goes to Sera for having a nice Master project for me, which was the passage to the PhD. Not only did I learn a lot with you that year, I also made a good friend on the way. During the PhD years I got the chance to come close to more people, in no particular order Samia, Gerrit, Yuri, Maithili, Alessandro, Atish, Montse, Danai, Tulio, Evert, Eduardo, Theo, Thijs (× 2), Caroline, Pieter, Daniela, Ken, Daan, Joel, Salomé, Frank, Olga, Martin and many others that is impossible to list here. All of you have been there at good and bad times and I can’t even imagine how boring the experience would have been otherwise.

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Last, but most of all, I’d like to thank you Thetis for standing by me throughout this period. You have shown me the true meaning of “we”.

Konstantinos Leventis,
Amsterdam, 12 January 2013