Probing exoplanetary materials using sublimating dust

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Rik van Lieshout

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Promotiecommissie:

Promotor: prof. dr. C. Dominik Universiteit van Amsterdam
Copromotor: prof. dr. C.U. Keller Universiteit Leiden

Overige leden: dr. J.-C. Augereau Université Joseph Fourier / CNRS
prof. dr. W. van Westrenen Vrije Universiteit Amsterdam
prof. dr. L.B.F.M. Waters Universiteit van Amsterdam
prof. dr. M. van der Klis Universiteit van Amsterdam
prof. dr. A. de Koter Universiteit van Amsterdam
dr. J.-M. Desért University of Colorado Boulder

Faculteit der Natuurwetenschappen, Wiskunde en Informatica

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You can always learn something from crumbs.

—John Kennedy Toole
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When we think of the Solar System, the picture that usually comes to mind is the Sun and its eight planets. But there are many more objects orbiting the Sun besides the planets, such as asteroids, comets, and interplanetary dust grains. Many of these bodies can be thought of as leftover material that was never incorporated into planets during the process of planet formation. Others were once part of a planet, but somehow became detached and sent on their own journey through space. By studying these leftovers and fragments, we can learn many things about the origins and evolution of the Solar System.

Circumstellar material has also been found around other mature stars. The discovery of dust around the star Vega in 1984 provided the first concrete evidence that (at least the initial stage of) planet formation also operates around other stars. Since then, many more stars were found to be accompanied by belts of dust, which are now known as debris disks. In addition, from the 1990s to today an abundance of planets around other stars has gradually been uncovered. Unexpectedly, the configurations in which these extrasolar planets (exoplanets) are found are often very different from the architecture of the Solar System.

These findings have made it clear that planet formation is a very common process with wildly diverse outcomes. They have also raised the questions of how common or unique the Earth is and whether other planets exist in the Milky Way galaxy on which intelligent life may have arisen. Advancing towards an answer to these questions requires furthering the knowledge of the formation and evolution of planetary systems in general, as well as the characterisation of particular exoplanets.

The study of faraway systems is hindered by their staggering distance and has a very different character from the study of our Solar System. While planets and other large bodies
in the Solar System can be resolved as worlds of their own and have been visited by space probes, planets in other systems at most appear as unresolved point sources even through our largest telescopes, if they can be distinguished at all. Most exoplanets that we know today are detected and characterised indirectly, through the effects they have on their host star. Intermediate-sized bodies such as asteroids can at present not be detected around other stars, because they are too small and too few to have significant effects even collectively. Small dust grains, however, often occur in large quantities, making their collective cross-section enormous. This allows us to detect circumstellar dust populations directly through their own thermal emission or through scattered starlight. In many cases, their spatial structure can also be resolved using advanced telescopes.

In debris disks and planetary systems, small dust grains are destroyed or removed from the system on relatively short timescales, implying that they must be replenished by larger bodies that constitute more stable mass reservoirs. The composition of the dust therefore reflects that of the bodies from which it originated and its spatial distribution follows to some extent its release location. For this reason, circumstellar dust can act as a valuable probe of the larger objects from which it originates. To retrieve information about these larger bodies, however, it is vital to have a detailed understanding of the creation and subsequent evolution of the dust grains. Building such understanding and using it to infer the properties of larger bodies in distant systems is the main goal of this thesis.

While the innermost part of the Solar System is relatively empty, many other planetary systems that have recently been discovered are very different. Mercury, the innermost planet in the Solar System, orbits at a distance of about 0.4 AU, taking about 88 days to revolve around the Sun. In contrast, several exoplanets have been discovered with orbital periods of less than a single day. At the orbital distances corresponding to such short orbital periods, these planets experience scalding stellar radiation, resulting in extremely high temperatures. Additionally, some debris disk systems show evidence of circumstellar dust in the close vicinity of the central star, where it can reach equilibrium temperatures exceeding 1000 K. These temperatures are enough to vaporise the rocky or metal-rich material of which small exoplanets and dust grains are made of. Hence, the process of sublimation (the phase transition from solid to gaseous form) can be an important factor in these situations. The special focus of this thesis is investigating how sublimation affects the evolution of circumstellar dust grains.

### 1.1 Debris disks

In 1984, it became clear that mature stars other than the Sun are also accompanied by orbiting dust grains, when shortly after launch the Infrared Astronomical Satellite (IRAS) observed the star Vega (Aumann et al. 1984). Being one of the brightest stars on the night sky, Vega has been used for more than a century as a photometric standard. When IRAS began operations, opening up new spectral windows in the infrared, it was therefore a logical step to point the new telescope at Vega for calibration measurements. In the far infrared (FIR) wave-
lengths that IRAS was sensitive to, Vega was expected to show a spectrum consistent with the Rayleigh–Jeans tail of the stellar photosphere. Surprisingly, the observations showed clear excess emission, consistent with thermal emission from dust with a temperature of about 85 K, pointing to an orbital distance of about 85 AU. Similar FIR excesses were detected using IRAS for many other main-sequence stars, most notably the bright, nearby stars Fomalhaut, β Pictoris, and ε Eridani.

While infrared excesses had previously been detected for very young stars (i.e., with ages up to about 10 Myr), these are still accompanied by primordial protoplanetary disks of gas and dust. Vega is a 455-Myr-old main-sequence star and the presence of small dust around a mature star cannot be explained as a left-over from the protoplanetary disk phase. Small dust will be removed by radiation pressure forces on time scales that are much shorter than the age of the star. Specifically, direct radiation pressure will remove dust up to sizes of several µm by blowing it out of the system on unbound trajectories. This happens on time scales comparable to the orbital period, which is of the order of a few thousand years at the proposed orbital distance of the dust. Larger dust grains can remain bound to the star, but Poynting–Robertson (P–R) drag will remove them by letting the dust grains spiral towards the star. Given the age of the system and the time scale on which dust grains of different sizes are removed by P–R drag, the dust was estimated to have a typical size of at least 1 mm (Aumann et al. 1984). Since this is much larger than the typical size of interstellar grains, this discovery pointed to substantial growth of dust grains by coagulation, the initial step of planet formation.

Soon after the discovery of this dust, additional FIR photometry revealed that the grains orbiting Vega could not be as large as initially suggested (Harper et al. 1984). Smaller grain sizes imply shorter survival time scales against radiation forces, and the fact that the dust grains have lifetimes that are shorter than the age of the star means that they must be replenished continuously. This led to the idea that the small dust is the result of catastrophic collisions between larger bodies (Harper et al. 1984). These larger bodies are not removed by radiation forces and their gradual destruction by collisions produces debris that can be observed as small dust.

Further evidence of the link between the FIR excess and planet formation came from coronagraphic visible-light imaging of β Pictoris (Smith & Terrile 1984). Due to its proximity and the sheer quantity of dust around this star, the observations allowed the dust around β Pictoris to be clearly resolved, and showed that it was concentrated along one position angle, rather than symmetrically around the star. This is consistent with dust located in an edge-on disk (as opposed to a shell), tallying with the architecture of the Solar System, in which all planets and most other material is concentrated around the ecliptic plane. Present-day technology produces images with far greater angular resolution and contrast (see Fig. 1.1). These images show that the name debris disk may sometimes be a misnomer, because in some systems the dust is located in a narrow belt rather than an extended disk.
1 Introduction

Figure 1.1: The debris disk of Fomalhaut, imaged in three different wavelengths by three different telescopes. The ALMA data cover only the northwestern part of the debris belt (the right-hand side in this image). It is superimposed on the Hubble image, to show the complete ring. Note that the central component seen in the Herschel and ALMA image is not (only) the stellar flux, but includes an inner, unresolved debris belt. In Chapter 3, we zoom in on the inner region that houses this component. Image credits, from left to right: Kalas et al. (2008), Acke et al. (2012), Boley et al. (2012).

1.2 Hot exozodiacal dust

Most debris belts are found at distances of several tens to hundreds of AUs from their host stars, but there is also evidence of dust closer to the star. The dust found in the inner regions of extrasolar systems is called exozodiacal dust, after the Solar System’s zodiacal light. The latter is a band of light in the night sky that follows the ecliptic (i.e., the zodiac). It is caused by light of the Sun that is scattered off dust particles located preferentially in the ecliptic plane, where they are produced. It can only be observed on very dark nights and is best visible where scattering angles are small (i.e., close to the horizon in the direction of the Sun, just after dusk or before dawn). The population of dust that is responsible for the zodiacal light is very tenuous. Any exozodiacal dust populations that are detectable with current technology are necessarily much denser than their Solar System equivalent.

Exozodiacal dust is often subdivided into different types, depending on the wavelength at which the dust is detected and the temperature the dust is inferred to have. Mid-infrared (MIR) excess emanating from the inner regions of a debris disk is called “warm” exozodiacal dust. It has temperatures of several hundred K, and is located at orbital distances of the order of 1 AU. Excess near-infrared (NIR) emission is referred to as “hot” exozodiacal dust. It is inferred to have temperatures of 1500 to 2000 K, which puts it at a few stellar radii from the star, close to the sublimation radius. Both types contrast with the outer debris located at tens or hundreds of AUs, which in this context is sometimes referred to as “cold” dust. While the division between the two types of exozodiacal dust is mostly motivated by the observational methods (there may in fact be a continuum of dust temperatures), there are also indications (e.g., differences in occurrence rate) that the two types of excess emission reflect different
Because exozodiacal dust is much warmer than the outer belts, its thermal emission spectrum peaks at shorter wavelengths, where the stellar photosphere is still much stronger. Therefore, the fractional excess emission (i.e., relative to the stellar emission) of exozodiacal dust is much weaker than that of a cold, far-out debris belt. For this reason, the detection of exozodiacal dust (in particular the hot variant) relies mostly on spatially resolving the dust from the star, which can currently only be done using interferometry.

The main idea behind the interferometric detection of exozodiacal dust is that the star can be resolved at very long baselines, while the extended emission is already resolved at shorter baselines, at which the star remains nearly unresolved (see Fig. 1.2). The contribution of the star to the short-baseline visibilities can be modelled using the long-baseline observations, thereby isolating the excess emission. Alternatively, photometric estimates of the stellar radius can be used to predict the stellar contribution to the short-baseline visibilities, because uncertainties on the stellar radius only have a small effect on the visibilities at short baselines.

The detection of excess NIR emission does not necessarily point to hot dust. Other possible explanations include the presence of a close stellar or sub-stellar companion. By using different position angles, as well as closure phases, the degree of asymmetry of the excess emission can be established. This, in addition to follow-up observations (high-resolution imaging and radial velocity measurements), can be used to rule out that the excess emission emanates from a stellar or sub-stellar companion.

As with the discovery of cold debris, the first firm detection of hot exozodiacal dust was for Vega (Absil et al. 2006). Soon after this discovery, hot dust was also found around several other bright stars: τ Ceti (di Folco et al. 2007), ζ Aquilae (Absil et al. 2008), β Leonis (Ake-
son et al. 2009), Fomalhaut (Absil et al. 2009), and β Pictoris (Defrère et al. 2012). For all these stars, the observed NIR excess is of the order of 1% of the star’s photospheric emission. This is at the limit of what can be detected using today’s technology, and it is conceivable that many stars have fainter populations of hot dust just beyond the present detection limit.

More recently, surveys using the NIR interferometric instruments CHARA/FLUOR and VLTI/PIONIER have found many more stars exhibiting NIR excess (Absil et al. 2013; Ertel et al. 2014). These surveys have established that the hot excess emission can be detected around an astounding 10% to 30% of main-sequence stars. The phenomenon seems to be more prevalent amongst A-type stars. There are no significant trends with stellar age or rotation rate, and neither with the presence of detectable outer belts or close-in planets. In contrast, warm exozodiacal dust is seen around less than 1% of main-sequence stars (at detectable levels), and it is more common around younger stars (Kennedy & Wyatt 2013).

Once the NIR excess emission is established to come from circumstellar dust, there are several obvious next steps in research: (1) characterising the dust (i.e., its spatial distribution, the typical grain sizes, and possibly its composition) and (2) understanding its origin and what the dust tells us about the system in which it is found. The former can be done by gathering all the available observations that constrain the hot dust population of a particular star and trying to explain those observations using a morphological model of the dust. This is the subject of the first half of Chapter 3. The origin of hot dust around other stars is currently still a mystery. Chapter 2 presents an in-depth investigation of one possible explanation, and the second half of Chapter 3 reviews a range of possible processes using simple analytical arguments.

1.3 Evaporating rocky exoplanets

Hot exozodiacal dust is not the only material found in the innermost regions of exoplanetary systems. One of the early findings of exoplanet research is that some exoplanets orbit surprisingly close to their host star. For example, the first exoplanet to be discovered around a main-sequence star, 51 Pegasi b, was found to have an orbital period of only 4.23 days (Mayor & Queloz 1995). While there are obvious observational biases towards finding close-in planets, statistical studies have revealed that there is a significant population of planets with orbital periods of only a few days. Specifically, small planets in periods of a few days were found to be relatively common (Borucki et al. 2011); hot Jupiters are quite rare (found around only 0.5% to 1.0% of Sun-like stars; Wright et al. 2012).

One of the questions raised by these findings is, “How close to their host star can planets exist?” In extremely tight orbits, planets will experience effects that usually do not have to be taken into account. Gas giants cannot survive very close in, because strong tidal forces from the star can cause their orbits to decay (Rasio & Ford 1996). Additional limitations are presented by Roche-lobe overflow (Gu et al. 2003) and atmospheric evaporation (Murray-Clay et al. 2009). Small rocky planets can survive closer to their host star, but ultimately they
also have an inner boundary. The stellar tidal forces can rip such planets apart (Rappaport et al. 2013) and the extreme radiation field at a few stellar radii can lead to temperatures of several thousand K, enough to vaporise most planetary materials.

Observationally, the population of ultra-short-period (i.e., less than one day) exoplanets was investigated in detail by Sanchis-Ojeda et al. (2014) through a specialised survey of the Kepler data. As part of this exercise, an unusual transit signal was found in the light curve of the star KIC 12557548 (see Fig. 1.3; Rappaport et al. 2012). The signal has a highly stable period of about 0.65 days, but the transit depths vary erratically from orbit to orbit between depths of around 1.4% to non-detections (not visible in the segment shown in Fig. 1.3). After phase-folding over the orbital period, the transit profile is found to have a highly asymmetric profile, with a sharp ingress and a gradual egress, as well as a small but significant brightening just before the ingress. Both properties are in stark contrast to the usual symmetric transit signals produced by spherical planets and call for an explanation.

Several scenarios were investigated to explain the light curve of KIC 12557548. The stability of the period of the signal suggests that it is caused by an orbital companion, and the main challenge is explaining the variations in transit depth. The maximum transit depth of 1.4% requires an object with an effective extinction cross-section comparable to Jupiter’s (Rappaport et al. 2012), while the non-detections in other parts of the light curve put an upper limit on cross-section corresponding to an Earth-sized body (Brogi et al. 2012).

One explanation for the varying transit depth that was considered by Rappaport et al. (2012) is the presence of two planets that gravitationally perturb each other. In this scenario, one of the bodies would transit the star, but with a varying impact parameter (i.e., alternating

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1 The standard Kepler pipeline is not intended to work for orbital periods less than about 12 hours.
between fully in front of the stellar disk, grazing, and fully out of the line of sight), yielding different transit depths. However, in this case there would be periodicity in the variations, while none is observed. Also, it would lead to transit timing variations, inconsistent with the highly stable period of the signal. Finally, it does not explain the unusual shape of the light curve. Another scenario that was investigated is the presence of an eclipsing binary in which one component is a compact object with an accretion disk. In this case, variations in the luminosity of the accretion disk would account for the varying transit depths, but this does not explain why the variations would be confined to the transit part of the orbit.

The explanation that was eventually settled upon is that the occulting object is a cloud of dust grains, emanating from a small planet that is evaporating due to the extreme stellar radiation. The dust trails the planet in a comet-like tail (see Fig. 1.4), and the resulting asymmetry is responsible for the peculiar transit profile. Variability in the dust emission rate of the planet can explain the variable transit depth. The first part of Chapter 5 serves as a further introduction into this subject.

Since the discovery of the first evaporating rocky exoplanet, two more candidates of such objects have been discovered (Rappaport et al. 2014; Sanchis-Ojeda et al. 2015). These objects present the first few examples of a new class of exoplanets that may be very useful in the quest for characterising exoplanetary materials. In particular, because the dust originates in the evaporating planet, it can be used to probe the composition of the parent body. The details of how this can be accomplished are the subject of Chapters 4 and 5.

Figure 1.4: Artist’s impression of the evaporating planet KIC 12557548b. Image credit: C. U. Keller. (2012).
1.4 This thesis

The work presented in this thesis revolves around dust sublimation as a probe for exoplanetary materials. We make in-depth theoretical investigations of this subject using analytical and numerical techniques. This either leads to predictions that are compared to observational data, or provides tools to facilitate the physical interpretation of observations. The thesis can roughly be split up into two parts: Chapters 2 and 3 concern debris disks and the problem of hot exozodiacal dust; Chapters 4 and 5 are about the dusty tails of evaporating exoplanets. We now briefly summarise the main goals and findings of each of the chapters individually.

In Chapter 2, we test whether the hot-exozodiacal-dust phenomenon can be explained by a particular combination of processes, namely the inward transport of material by P–R drag, together with dust pile-up caused by the interplay of dust sublimation and radiation pressure. We first investigate this using simple analytical expressions, and subsequently with a numerical debris-disk model that self-consistently handles destructive collisions, P–R drag, and sublimation. Both methods give (mutually consistent) predictions for the distribution of dust in the inner regions of a debris disk and from these we compute the thermal emission spectra of the dust. Comparing the predicted dust fluxes with observations shows conclusively that the proposed mechanism cannot explain the observed quantities of hot exozodiacal dust. The main reasons are that P–R drag does not provide enough material and the pile-up of dust is insignificant. As part of the numerical analysis, we also infer the size distribution of material that migrates inward from a parent belt due to P–R drag and find that it is strongly dominated by barely bound ($\beta \approx 0.5$) grains.

Chapter 3 presents a detailed investigation of the inner part of the debris disk of the star Fomalhaut. We characterise the precise location and properties of dust close to the star by fitting a set of interferometric and photometric observations with a morphological debris-disk model, which parametrises the spatial and size distributions of dust around the star, as well as its composition. In this model, special attention is given to sublimation as a time-dependent dust removal mechanism to set the size-dependent inner radius at which the spatial distribution of dust is truncated. Using simple analytical arguments, we then survey a range of processes that have been proposed to explain the presence of dust so close to the star. Several of the proposed scenarios can be excluded firmly, and none adequately explains the observed amount dust, leaving the hot-exozodiacal-dust phenomenon a mystery. The main difficulty in explaining hot exozodiacal dust is the very short lifetime of the dust grains against sublimation and removal by radiation pressure. Compensating this requires an extremely high rate of mass supply, which none of the tested mechanisms can reach. Alternatively, our understanding of the dust grains and their emission may be incomplete, or some as-of-yet-unknown trapping mechanism lengthens the lifetime of the dust, or both.

In Chapter 4, we switch our attention to the phenomenon of evaporating rocky exoplanets, which are accompanied by comet-like tails of dust that trail the planet. We demonstrate analytically that the presence and length of such a tail can be used as a probe for the chemical composition of the dust in the tail. In short, the length of the tail is determined by the rate at
which dust grains drift away from the planet due to radiation pressure and the time it takes for them to disappear completely as a result of sublimation. In particular the sublimation rate of the dust grains is highly sensitive to the dust composition: refractory species result in long tails, while volatile materials yield short tails. Observationally, the tail length can be inferred from the duration of the egress part of the transit light curve, created when the evaporating planet occults its host star. We take previously derived tail lengths of two evaporating exoplanets and apply our analytical expression to test a number of dust composition for both objects. Our analysis leads to the conclusion that the dust in the tails could consist of corundum (i.e., crystalline aluminium oxide) or iron-rich silicates, while a range of other dust species can be excluded. The analysis also yields estimates of the rates at which the evaporating planets lose mass in dust. Determining the mass loss rate is important because it can be used to find the mass of the planet, and sets the evaporation lifetime of the planet, which in turn is needed to compute the occurrence rate of these objects.

Chapter 5 builds upon the insights into evaporating rocky exoplanets gained in Chapter 4, refining the method of determining the composition of dust in the tails of these objects. We work out how to utilise the detailed shape of the entire transit light curve to constrain the dust composition, rather than just a one-dimensional tail length. Specifically, we develop a numerical model that predicts the shape of the tail and the resulting light curve for a given set of dust properties. By exploring a large parameter space using a Markov chain Monte Carlo technique, we are able to put constraints on the material properties of the dust in the tail of the prototypical evaporating exoplanet KIC 1255b. These are compared to the parameters of a set of real dust species, confirming that corundum is a possible composition and ruling out a number of other candidate dust materials. Simultaneously, our model puts rough constraints on the typical size of the dust grains and the mass loss rate of the planet.
Near-infrared emission from sublimating dust in collisionally active debris disks

R. van Lieshout, C. Dominik, M. Kama, and M. Min

*Astronomy & Astrophysics, 2014, 571, A51*

**Abstract**

*Context.* Hot exozodiacal dust is thought to be responsible for excess near-infrared (NIR) emission emanating from the innermost parts of some debris disks. The origin of this dust, however, is still a matter of debate.

*Aims.* We test whether hot exozodiacal dust can be supplied from an exterior parent belt by Poynting–Robertson (P–R) drag, paying special attention to the pile-up of dust that occurs owing to the interplay of P–R drag and dust sublimation. Specifically, we investigate whether pile-ups still occur when collisions are taken into account, and if they can explain the observed NIR excess.

*Methods.* We computed the steady-state distribution of dust in the inner disk by solving the continuity equation. First, we derived an analytical solution under a number of simplifying assumptions. Second, we developed a numerical debris disk model that for the first time treats the complex interaction of collisions, P–R drag, and sublimation in a self-consistent way. From the resulting dust distributions, we generated thermal emission spectra and compare these to observed excess NIR fluxes.

*Results.* We confirm that P–R drag always supplies a small amount of dust to the sublimation zone, but find that a fully consistent treatment yields a maximum amount of dust that is about
7 times lower than that given by analytical estimates. The NIR excess due to this material is much less ($\lesssim 10^{-3}$ for A-type stars with parent belts at $\gtrsim 1$ AU) than the values derived from interferometric observations ($\sim 10^{-2}$). Pile-up of dust still occurs when collisions are considered, but its effect on the NIR flux is insignificant. Finally, the cross-section in the innermost regions is clearly dominated by barely bound grains.

2.1 Introduction

Circumstellar dust in debris disks reveals the location and dynamical state of larger bodies and thus sheds light on the architecture of planetary systems in the aftermath of planet formation (see Wyatt 2008 for a review). The dust can be studied by observing its infrared and (sub-)millimeter emission, as well as the stellar radiation it scatters, and is usually found at large distances from the star (tens of AU, Carpenter et al. 2009). Recently, interferometric observations have found excess near-infrared (NIR) emission emanating from the innermost parts of several debris disks, which has been interpreted as thermal emission from hot ($> 1000$ K) dust (Ciardi et al. 2001; Absil et al. 2006, 2008, 2009; di Folco et al. 2007; Akeson et al. 2009; Defrère et al. 2011b, 2012, see Table 2.1 for an overview). This material is known as hot exozodiacal dust. Its origin, hence what it can tell us about planet formation, is still unclear. In this work, we investigate one possible scenario for explaining hot exozodiacal dust.

Dust grains in debris disks have relatively short lifetimes because of their destruction by collisions and removal by radiation forces. To detect these grains around mature stars therefore implies the existence of a mechanism that continuously replenishes them. Cold dust populations at large distances from the star can be maintained by a collisional cascade grinding down much larger bodies that act as a reservoir of mass (Backman & Paresce 1993). Closer to the star, however, the pace at which material is processed by collisions is much higher, so that the lifetime of a debris belt in collisional equilibrium is much shorter there (Dominik & Decin 2003; Wyatt et al. 2007a). For this reason, hot exozodiacal dust cannot be explained by in situ planetesimal belts (Wyatt et al. 2007a; Lebreton et al. 2013), and a different mechanism is needed to replenish it, the lifetime of the dust needs to be extended by some process, or both.

Many of the systems that exhibit NIR excess also feature a debris belt at a large distance from the star (see Tbl. 2.1). Inward transport of material from an outer belt may therefore be a natural explanation for the existence of hot exozodiacal dust. A possible transportation mechanism is Poynting–Robertson (P–R) drag (see, e.g., Burns et al. 1979). Because P–R drag acts on a timescale that is much longer than that of collisions, it is sometimes not favored as a possible mechanism for maintaining exozodiacal dust (e.g., Absil et al. 2006). However,

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1 Kennedy & Wyatt (2013) find that “warm” exozodiacal dust around solar-type stars can be explained by in-situ planetesimal belts. This type of exozodiacal dust is detected at mid-infrared wavelengths and has a typical temperature of a few hundred K, placing it around 1 AU from the star.
Table 2.1: NIR interferometric detections of hot exozodiacal dust, together with associated outer debris belt locations

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<td>10–14, 80</td>
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<td>10–14, 80</td>
<td>D00, S13</td>
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<td>A08, A13</td>
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<td>A08, P09</td>
</tr>
<tr>
<td>β Leo A3V</td>
<td></td>
<td>$K$</td>
<td>0.94 ± 0.26</td>
<td>4</td>
<td>CHARA/FLUOR</td>
<td>Ak09, A13</td>
<td>none detectable</td>
<td>C06</td>
</tr>
<tr>
<td>λ Gem A3V</td>
<td></td>
<td>$K$</td>
<td>0.74 ± 0.17</td>
<td>12</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>none detectable</td>
<td>M09, G13</td>
</tr>
<tr>
<td>Fomalhaut A4V</td>
<td></td>
<td>$K$</td>
<td>0.88 ± 0.12</td>
<td>6</td>
<td>VLTI/VINCI</td>
<td>Ab09</td>
<td>2, 8–11, 133</td>
<td>K05, L13, S13</td>
</tr>
<tr>
<td>β Pic c A6V</td>
<td></td>
<td>$H$</td>
<td>1.37 ± 0.16</td>
<td>4</td>
<td>VLTI/PIONIER</td>
<td>D12</td>
<td>10–40$^d$</td>
<td>L94, P97</td>
</tr>
<tr>
<td>β Pic c A6V</td>
<td></td>
<td>$K$</td>
<td>0.76 ± 0.49</td>
<td>1.3</td>
<td>VLTI/VINCI</td>
<td>D04, D12</td>
<td>10–40$^d$</td>
<td>L94, P97</td>
</tr>
<tr>
<td>α Aql A7IV</td>
<td></td>
<td>$K$</td>
<td>3.07 ± 0.24</td>
<td>2</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>none detectable</td>
<td>A13</td>
</tr>
<tr>
<td>α Cep A7IV</td>
<td></td>
<td>$K$</td>
<td>0.87 ± 0.18</td>
<td>6</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>none detectable</td>
<td>C05</td>
</tr>
<tr>
<td>η Lep e F1V</td>
<td></td>
<td>$K$</td>
<td>0.89 ± 0.21</td>
<td>6</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>1–16, 18</td>
<td>L09, E13</td>
</tr>
<tr>
<td>110 Her e F6V</td>
<td></td>
<td>$K$</td>
<td>0.94 ± 0.25</td>
<td>8</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>70–500</td>
<td>M13</td>
</tr>
<tr>
<td>10 Tau e F9V</td>
<td></td>
<td>$K$</td>
<td>1.21 ± 0.11</td>
<td>6</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>&gt;5.8</td>
<td>T08</td>
</tr>
<tr>
<td>ξ Boo e G8V</td>
<td></td>
<td>$K$</td>
<td>0.74 ± 0.20</td>
<td>3</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>none detectable</td>
<td>A13</td>
</tr>
<tr>
<td>τ Cet G8V</td>
<td></td>
<td>$K$</td>
<td>0.98 ± 0.18</td>
<td>1.5</td>
<td>CHARA/FLUOR</td>
<td>D07, A13</td>
<td>10–55</td>
<td>G04</td>
</tr>
<tr>
<td>κ CrB e K11V</td>
<td></td>
<td>$K$</td>
<td>1.18 ± 0.20</td>
<td>12</td>
<td>CHARA/FLUOR</td>
<td>A13</td>
<td>20, 41</td>
<td>B13</td>
</tr>
</tbody>
</table>
Table 2.1: (Continued)

Notes.
(a) FOV denotes the approximate linear field-of-view radius at half maximum.
(b) For the outer belt distance ($r_0$ in our models) we list literature estimates of the radial distance to (the inner edge of) “cold” and “warm” outer belts, derived from SED fitting and/or resolved imaging.
(c) The NIR excess of β Pic contains a significant contribution from stellar light scattered by the outer belt (Defrère et al. 2012).
(d) The debris disk around β Pic is seen edge-on, making it hard to determine the parent belt location. The values given mark the radial range in which the particle density is derived to decrease.
(e) For η Lep, 110 Her, 10 Tau, ξ Boo, and κ CrB, the possibility that the observed NIR excess is due to a low-mass companion within the field-of-view cannot be excluded (Absil et al. 2013).

References. (A06) Absil et al. (2006); (A08) Absil et al. (2008); (Ab09) Absil et al. (2009); (Ak09) Akeson et al. (2009); (A13) Absil et al. (2013); (B13) Bonsor et al. (2013); (C01) Ciardi et al. (2001); (C05) Chen et al. (2005); (C06) Chen et al. (2006); (D00) Dent et al. (2000); (D04) di Folco et al. (2004); (D07) di Folco et al. (2007); (D11) Defrère et al. (2011b); (D12) Defrère et al. (2012); (E13) Eiroa et al. (2013); (G04) Greaves et al. (2004); (G13) Gáspár et al. (2013); (K05) Kalas et al. (2005); (L94) Lagage & Pantin (1994); (L09) Lawler et al. (2009); (L13) Lebreton et al. (2013); (M09) Morales et al. (2009); (M13) Marshall et al. (2013); (P97) Pantin et al. (1997); (P09) Plavchan et al. (2009); (S13) Su et al. (2013); (T08) Trilling et al. (2008).

as long as there are no mechanisms that prevent inward migration, a small amount of dust is always transported to the innermost part of the disk (Wyatt 2005), where it produces a NIR signal.

Morphological models of exozodiacal dust disks, constrained by the NIR observations, indicate that the hot dust is concentrated in a sharply peaked ring, whose inner boundary is determined by dust sublimation (Defrère et al. 2011b; Mennesson et al. 2013; Lebreton et al. 2013). The process of dust sublimation may therefore play an important role in shaping exozodiacal clouds. Kobayashi et al. (2009) find that the interplay between P–R drag and dust sublimation can lead to a local enhancement of dust in the sublimation zone, leading to radial distributions of dust reminiscent of those found by the morphological models. However, they only investigate this pile-up effect for drag-dominated systems, where collisions are unimportant, and it is unclear what happens to the phenomenon if collisions are taken into account.

In this work, we examine whether it is possible to maintain a pile-up of dust in the sublimation zone of a collisionally active debris disk and whether such a pile-up could explain the exozodiacal NIR emission observed very close to some stars. To do this, we compute the steady-state distribution of dust in the inner parts of debris disks by solving the continuity equation, considering collisions, P–R drag, and sublimation. First, we find an analytical solution, using a number of simplifying assumptions (Sect. 2.2). Subsequently, we solve the continuity equation numerically using a debris disk model that for the first time treats...
2.2 Analytical constraints

In this section we analytically investigate the distribution of dust in the inner regions of debris disks. We focus on the radial distribution of material, by assuming (1) that the disk is axisymmetric and (2) that all particles have the same size. Throughout this work, radial distributions are expressed in terms of vertical geometrical optical depth, which is defined as the surface density of cross-section.\(^2\) Under the two assumptions listed above, it is given by

\[
\tau_{\text{geo}}(r) = \frac{\sigma n(r)}{2\pi r}.
\]

(2.1)

Here, \(r\) is the radial distance from the central star, \(\sigma\) the cross-section of a particle, and \(n(r)\) the one-dimensional number density (i.e., the particle number density integrated over disk height and azimuth).

We consider three processes affecting the evolution of dust particles in debris disks: collisions, P–R drag, and sublimation. The strategy for our analytical estimates is as follows. First, we review the balance between P–R drag and collisions (without considering sublimation) and calculate the inward flux of material due to these two effects (Sect. 2.2.1). Subsequently, we consider the interplay of P–R drag and sublimation (ignoring collisions), which can lead to the pile-up of dust in the sublimation zone (Sect. 2.2.2). Finally, we investigate whether collisions can be neglected in the innermost parts of a debris disk and estimate the radial distribution of dust in a disk where all three processes are in operation (Sect. 2.2.3). At the end of the section, we briefly summarize our findings (Sect. 2.2.4).

2.2.1 Poynting–Robertson drag and collisions

The balance between P–R drag and collisions was studied analytically by Wyatt (2005). Because of its importance to the present study, we summarize the main arguments of this work here. The model assumes that (1) there is a source of dust at distance \(r_0\) with a geometrical optical depth of \(\tau_{\text{geo}}(r_0)\), (2) dust particles follow circular orbits, (3) collisions are always destructive, and (4) all dust grains have the same size. These assumptions lead to simple expressions for the timescales on which P–R drag and collisions typically act.

\(^2\) The geometrical optical depth corresponds to the true vertical optical depth only for an extinction efficiency of unity \((Q_{\text{ext}} = 1)\) for all particle sizes.
The P–R drag timescale $t_{\text{PR}}$ is defined as the time it takes for a particle on a circular orbit to spiral from a given distance $r$ to the central star. It is given by (e.g., Burns et al. 1979)

$$t_{\text{PR}}(r) = \frac{cr^2}{4GM_*\beta}, \quad (2.2)$$

where $c$ is the speed of light, $G$ the gravitational constant, $M_*$ the stellar mass, and $\beta$ the ratio of the norms of the direct radiation pressure force and the gravitational force on a particle ($\beta = |F_{\text{rp}}/F_g|$).\(^3\)

The collisional timescale $t_{\text{coll}}$ indicates the average time between two collisions for a given particle. Wyatt (2005) finds

$$t_{\text{coll}}(r) = \frac{t_{\text{orb}}(r)}{4\pi\tau_{\text{geo}}(r)}, \quad (2.3)$$

where $t_{\text{orb}}$ is the orbital period of a circular orbit, given by $t_{\text{orb}}(r) = 2\pi \sqrt{r^3/(GM_*)}$. This equation is valid for particles that are on circular orbits and whose most important collisional partners have similar sizes.\(^4\)

The radial distribution due to P–R drag and collisions

Under the assumptions listed above, there is an analytical solution to the continuity equation, balancing the migration of particles due to P–R drag with the destruction of dust by collisions. Wyatt (2005) finds that the steady-state solution is

$$\tau_{\text{geo}}(r) = \frac{\tau_{\text{geo}}(r_0)}{1 + 4\eta_0(1 - \sqrt{r/r_0})}, \quad r \leq r_0, \quad (2.4a)$$

$$\eta_0 = \frac{c\tau_{\text{geo}}(r_0)}{2\beta} \sqrt{\frac{r_0}{GM_*}}, \quad (2.4b)$$

The parameter $\eta_0$ characterizes the density of the parent belt. It is defined such that for $\eta_0 = 1$ the collisional and P–R drag timescales are equal at $r_0$. Disks with $\eta_0 > 1$ are collision-dominated at $r_0$, while disks with $\eta_0 < 1$ are drag-dominated at $r_0$. Most debris disks with observed outer belts have $\eta_0 > 10$ (Wyatt 2005).

The balance between P–R drag and collisions is self-limiting: a denser parent belt produces more dust drifting inwards, but this dust also suffers more mutual collisions that eliminate grains on their way in. This results in a maximum geometrical optical depth profile for $\eta_0 \gg 1$ (i.e., a very dense parent belt) of

$$\max[\tau_{\text{geo}}(r)] = \frac{\sqrt{GM_*\beta}}{2c\left(\sqrt{r_0} - \sqrt{r}\right)}, \quad r \leq r_0. \quad (2.5)$$

\(^3\) Assuming circular orbits is valid for particles with low $\beta$ ratios, while the small particles most relevant to our study have high $\beta$ ratios. We relax this simplifying assumption in our numerical model (Sect. 2.3).

\(^4\) Equation 2.3 ignores a factor $\sqrt{1/(1-\beta)}$ in the orbital period of radiation pressure affected particles, but this only changes the collisional timescale by a factor of about 0.7 for $\beta = 0.5$. 
2.2 Analytical constraints

Figure 2.1: The maximum geometrical optical depth as a function of distance to the star (Eq. 2.5). The profiles are derived from the analytical model of Wyatt (2005), in which dust is produced by a source at radius $r_0$ and subsequently migrates inward due to P–R drag, while suffering destruction from mutual collisions. The solid lines correspond to solar-mass stars, the dashed lines to $2 M_\odot$ stars. Profiles for different parent belt locations are shown in different colors. All profiles assume dust grains with $\beta = 0.5$.

Figure 2.1 shows examples of this radial profile for different parent belt locations and host star masses, all for dust grains with $\beta = 0.5$. The value for $\beta$ was set to the blowout limit: particles with $\beta > 0.5$ leave the system on hyperbolic paths after they are released from a large parent body on a circular orbit. Therefore, P–R drag is the most efficient for $\beta = 0.5$, and for a given system, this value corresponds to the maximum $\tau_{\text{geo}}$ profile.$^5$

The inward flux of material

Since dust can pile up close to the star due to sublimation (to be discussed in Sect. 2.2.2), $\tau_{\text{geo}}$ may exceed the upper limit given by Eq. 2.5 in the sublimation zone. The material that piles up, however, is supplied from farther out by P–R drag. To investigate the properties of the pile-up, it is therefore useful to assess the inward flux of material.

In the case of a uniform grain size, the inward particle flux due to P–R drag (i.e., the number of particles passing through a ring at radius $r$ per unit of time) can be expressed as

$$\varphi_{\text{PR}}(r) = -n(r)\dot{r}_{\text{PR}}(r),$$

(2.6)

where $\dot{r}_{\text{PR}}$ is the P–R drag velocity, counted positively towards $r > 0$. If the particle orbits are

$^5$ Debris disks around stars with a strong stellar wind have higher values of $\tau_{\text{geo}}$. We ignore stellar wind in our present analysis, and discuss its effects in Sect. 2.5.2.
2 NIR emission from sublimating dust in debris disks

circular, radial migration due to P–R drag is described by (e.g., Burns et al. 1979)

\[
\dot{r}_{PR}(r) = -\frac{2GM_*\beta}{cr},
\]  
(2.7)

The grain parameter \( \beta \) can be expressed as (e.g., Burns et al. 1979)

\[
\beta = \frac{L_*}{4\pi cGM_* Q_{pr}\sigma} m.
\]  
(2.8)

Here, \( L_* \) is the stellar luminosity, \( m \) the mass of an individual dust grain, and \( Q_{pr} \) the radiation pressure efficiency averaged over the stellar spectrum. Combining Eqs. 2.1, 2.6, 2.7, and 2.8 and multiplying by particle mass \( m \) gives the inward (collision-limited) P–R drag mass flux (cf. Rafikov 2011)

\[
\dot{M}_{PR}(r) = \frac{L_*}{c^2} Q_{pr}\tau_{geo}(r).
\]  
(2.9)

The maximum mass flux can now be found by substituting Eq. 2.5 for \( \tau_{geo}(r) \) in Eq. 2.9, which yields

\[
\max\left[\dot{M}_{PR}(r)\right] = \frac{\sqrt{GM_*} L_* \beta Q_{pr}}{2c^3 \left(\sqrt{r_0} - \sqrt{r}\right)}, \quad r \leq r_0.
\]  
(2.10)

Figure 2.1 shows that \( \max[\tau_{geo}(r)] \) levels off in the innermost part of the system \( (r \ll r_0) \), at the radii where exozodiacal dust is seen. The mass flux corresponding to this plateau is

\[
\max[\dot{M}_{PR}(r = 0)] \approx 5.6 \times 10^{-13} \left(\frac{M_*}{1 M_\odot}\right)^{1/2} \left(\frac{L_*}{1 L_\odot}\right)^{1/2} \left(\frac{r_0}{1 \text{ AU}}\right)^{-1/2} \left(\frac{Q_{pr}}{1}\right) \left(\frac{\beta}{0.5}\right) M_\oplus \text{ yr}^{-1}.
\]  
(2.11)

The maximum mass flux only depends on grain properties through \( Q_{pr} \) and \( \beta \). The radiation pressure efficiency \( Q_{pr} \) must obey \( 0 \leq Q_{pr} \leq 2 \), and particles with \( \beta > 1 \) are always unbound. Therefore, Eq. 2.11 with \( Q_{pr} = 2 \) and \( \beta = 1 \) gives a solid upper limit on the inward mass flux due to P–R drag, unless one of the model assumptions does not hold (e.g., collisions are non-destructive).

2.2.2 P–R drag and sublimation

In the preceding, we found that P–R drag supplies a small but non-zero amount of dust to the innermost parts of a debris disk. As this material approaches the central star, it is heated by stellar radiation. Eventually, the dust grains become so hot that they start sublimating. We now review the evolution of these particles considering P–R drag and sublimation, but ignoring collisions.

\( ^6 \) While larger grains constitute more mass for a given geometrical optical depth profile \( (m/\tau_{geo} \propto m/\sigma \propto s) \), they also migrate more slowly \( (\dot{r}_{PR} \propto \beta \propto s^{-1}) \). These two effects cancel each other out.
2.2 Analytical constraints

**Dust sublimation formalism**

For a spherical dust grain in a gas-free environment, the rate at which the grain radius $s$ changes is given by (e.g., Kobayashi et al. 2008)

$$\frac{ds}{dt} = -\frac{P_v(T)}{\rho_d} \sqrt{\frac{\mu m_u}{2 \pi k_B T}}.$$  \hfill (2.12)

Here, $P_v$ is the phase-equilibrium vapor pressure, $\rho_d$ the bulk density of the dust, $\mu$ the molecular weight of dust molecules, $m_u$ the atomic mass unit, $k_B$ the Boltzmann constant, and $T$ the temperature of the dust. This theoretical sublimation rate is sometimes lowered to comply with experimental results, parameterized in a sticking efficiency or accommodation coefficient. Here, we ignore this weak effect, by assuming a sticking efficiency of unity.

The temperature dependence of $P_v$ is given by (Kobayashi et al. 2009)

$$P_v(T) = P_0 \exp\left(-\frac{\mu m_u H}{k_B T}\right),$$  \hfill (2.13)

where $P_0$ is a normalization constant and $H$ the latent heat of sublimation. By assuming $P_0$ is constant, we neglect a small temperature dependence beyond the exponential.

Sublimation parameters depend on material and can be determined by laboratory measurements. The material we consider in this study is carbonaceous dust. This choice is motivated by the proximity of hot exozodiacal dust to its host star, which suggests a very refractory material, like carbon (Mennesson et al. 2013; Lebreton et al. 2013). Specifically, we use the sublimation parameters of graphite, for which many laboratory measurements are available. For the molecular weight, we use $\mu = 36.03$, reflecting that graphite sublimation typically releases clusters of three carbon atoms at the temperatures and pressures relevant to this work (Abrahamson 1974). The parameters for $C_3$ sublimation are $P_0 = 2.95 \times 10^{14}$ dyn cm$^{-2}$ and $H = 2.15 \times 10^{11}$ erg g$^{-1}$ (Zavitsanos & Carlson 1973). For the bulk density of the material, we use $\rho_d = 1.8$ g cm$^{-3}$.

For our analytical estimates, we approximate the grain temperature $T$ by its black-body temperature

$$T_{bb} = \left(\frac{L_*}{16\pi \sigma_{SB} r^2}\right)^{1/4},$$  \hfill (2.14)

where $\sigma_{SB}$ is the Stefan–Boltzmann constant. In reality, the grain temperature is a function of size. Temperatures are generally higher than $T_{bb}$ for particles that are smaller than the typical wavelength of the stellar radiation, because these grains do not cool efficiently. For simplicity, we ignore this effect here and investigate it further in our numerical calculations, which use realistic grain temperatures (see Sect. 2.3.2).

In the black-body approximation, the sublimation rate $\dot{s}$ becomes independent of grain size. This leads to a simple expression for the sublimation timescale $t_{\text{subl}}$, defined as the time it takes for a spherical dust grain to disappear, which is

$$t_{\text{subl}}(r) = -\frac{s}{\dot{s}(r)}. $$  \hfill (2.15)
This estimate assumes that the grain temperature remains constant (at $T_{bb}$) throughout the sublimation process. As a sublimating particle becomes smaller, the black-body approximation is bound to become inaccurate. For sufficiently large particles, however, the true sublimation time is dominated by the black-body regime, and Eq. 2.15 provides a good approximation.

The pile-up of dust in the sublimation zone

As dust grains become smaller due to sublimation, their $\beta$-ratio changes. As a result of this, the interplay between P–R drag and sublimation can lead to a pile-up of dust in the sublimation zone. This phenomenon was studied in detail by Kobayashi et al. (2008, 2009, 2011). Here, we give a brief explanation of the pile-up mechanism.

When dust grains migrate inward owing to P–R drag, their temperature gradually increases. At some point, the grains are heated to the point where sublimation becomes substantial, and their sizes start decreasing significantly. For particles larger than the peak wavelength of the stellar spectrum $\lambda_*$, the radiation pressure efficiency is roughly constant at $Q_{pr} \approx 1$, and therefore the approximation $\beta \propto s^{-1}$ holds (see Eq. 2.8). As the particles lose mass, radiation pressure therefore becomes more important (relative to stellar gravity), which has the effect of increasing the semi-major axes and eccentricities of their orbits, compensating for the decrease terms due to P–R drag. This effectively slows down the inward migration of the dust grains, leading to an accumulation of dust in the sublimation zone. Eventually, the dust grains either sublimate completely or their $\beta$ ratios increase to the point where they become unbound and are blown out of the system.

Accumulation of dust occurs when the decrease of semi-major axis due to P–R drag is compensated by the increase of semi-major axis due to sublimation. This happens approximately at the radial distance where the timescale of P–R drag equals that of sublimation (Kobayashi et al. 2008). We denote this distance with $r_{\text{pile}}$, and determine its value by solving $t_{\text{PR}}(r_{\text{pile}}) = t_{\text{subl}}(r_{\text{pile}})$. Assuming that the dust in the pile-up has the black-body temperature $T_{\text{pile}} = T_{bb}(r_{\text{pile}})$, which holds for $s \gtrsim \lambda_*$, this equation can be written as

$$\frac{12\sigma_{SB}}{c^2} \frac{Q_{pr}}{P_0} T_{\text{pile}}^4 = \exp \left( -\frac{\mu m_u H}{k_B T_{\text{pile}}} \right) \sqrt{\frac{\mu m_u}{2\pi k_B T_{\text{pile}}}},$$

which shows that $T_{\text{pile}}$ is independent of stellar parameters, the bulk density of the dust, and grain size ($Q_{pr}$ is nearly constant for $s \gtrsim \lambda_*$), and only depends on material properties (cf. Kobayashi et al. 2008, 2009, 2011). Using $Q_{pr} = 1$ and the sublimation parameters of graphite given in Sect. 2.2.2, we numerically solve Eq. 2.16 to find $T_{\text{pile}} \approx 2020$ K, hence

$$r_{\text{pile}} \approx 0.019 \left( \frac{L_*}{1 L_\odot} \right)^{1/2} \left( \frac{T_{\text{pile}}}{2020 \text{ K}} \right)^{-2} \text{AU}.$$

Following Wien’s displacement law, $\lambda_* \approx 0.5 \mu m$ for the stars considered in this research.

More precise estimates of the pile-up distance are given by Kobayashi et al. (2009, 2011). We use this simple approximation to facilitate the estimate of the pile-up magnitude in Sect. 2.2.3.
The pile-up distance is independent of grain size because larger particles take longer to sublime, but also migrate more slowly because of P–R drag. This holds as long as the black-body approximation is valid, so the grain temperature in the pile-up is independent of grain size.

**Conditions for dust pile-up**

Kobayashi et al. (2011) list two conditions for the accumulation of dust to be substantial: (1) sufficiently high values of $\beta$ need to be reached as dust grains sublimate, and (2) the orbital eccentricities of the dust grains need to be low enough when they enter the sublimation zone.

Considering an inward stream of dust grains with a range of sizes, the particles contributing the most to the pile-up are those with the highest $\beta$. These particles have the strongest P–R drag drift rates, so per unit of time more of them arrive in the sublimation zone, where their inward drift is canceled as a result of sublimation. In addition, for the typical size distribution resulting from a collisional cascade, the total cross-section is dominated by the smallest particles. Since dust grains with $\beta > 0.5$ are typically blown out as soon as they are created, particles that are barely bound ($\beta \approx 0.5$) before the onset of substantial sublimation are the most important for the pile-up. A requirement for dust migration to slow down is that $\beta$ increases as the grain size decreases. For a given system, however, $\beta$ reaches a maximum value $\beta_{\text{max}}$ at $s \sim \lambda_s$, because smaller particles have lower $Q_{\text{pr}}$. Considering that relatively high values of $\beta$ are needed for an efficient pile-up, low luminosity stars do not have significant pile-ups. Kobayashi et al. (2009, 2011) set the limit at $\beta_{\text{max}} > 0.5$, and derive a lower limit on the stellar luminosity for significant pile-up, assuming that $Q_{\text{pr}} \approx 1$ holds for $s \gtrsim \lambda_s$. This limit is

$$L_\star \gtrsim 0.5 \left( \frac{M_\star}{1 M_\odot} \right) \left( \frac{T_\star}{5 \times 10^3 \text{ K}} \right)^{-1} \left( \frac{\rho_d}{1.0 \text{ g cm}^{-3}} \right) L_\odot,$$

where $T_\star$ is the effective temperature of the central star.

The pile-up of dust in the sublimation zone is highly dependent on the eccentricity of the dust as it enters the sublimation zone (Kobayashi et al. 2008). For the pile-up mechanism to produce a significant enhancement, the eccentricity of the dust particles in the sublimation zone must be very low ($e \lesssim 10^{-2}$; Kobayashi et al. 2008, 2011). Particles with higher orbital eccentricities do not spend enough time in the sublimation zone before they are blown out (or sublimate completely) to contribute significantly to the dust enhancement.

When dust particles are created in collisions in the parent belt, they are put on eccentric orbits with their periastron in the parent belt. Particles released from circular orbits will acquire orbital eccentricities of

$$e = \frac{\beta}{1 - \beta}.$$  

The particles that are the most important for the pile-up are the ones with $\beta \approx 0.5$. These barely bound particles initially follow very elliptic orbits, with eccentricities close to unity.
The initial eccentricity of the β ≈ 0.5 particles is far too high for any significant pile-up to occur. However, as the dust grains migrate inward owing to P–R drag, their orbits are circularized. The eccentricity evolution of dust particles experiencing P–R drag is coupled to their orbital size evolution according to (Wyatt & Whipple 1950)

\[
a_1(1 - e_1^2) = a_0(1 - e_0^2) \left( \frac{e_1}{e_0} \right)^{4/5},
\]

(2.20)

where \((a_0, e_0)\) are the initial semi-major axis and eccentricity, which evolve into \((a_1, e_1)\). This coupling can be used to place a lower limit on the distance of the source region, if significant pile-up is to occur, using the maximum allowed eccentricity in the sublimation zone for efficient pile-up. Written in terms of periastron distance \(q = a(1 - e)\), Eq. 2.20 becomes

\[
q_1(1 + e_1) = q_0(1 + e_0) \left( \frac{e_1}{e_0} \right)^{4/5}.
\]

(2.21)

Substituting \(r_0\) and \(r_{\text{pile}}\) for \(q_0\) and \(q_1\), respectively, and using \(e_0 \approx 1\) and \(e_1 \lesssim 10^{-2}\), yields a lower limit on the source radius

\[
r_0 \gtrsim 20r_{\text{pile}},
\]

(2.22)

for a significant enhancement in the dust density to be possible. Other mechanisms may help in the circularization of the orbits of small particles, decreasing the limit on \(r_0\). An example is the drag force from small amounts of gas that are present in the disk (Takeuchi & Artymowicz 2001).

### 2.2.3 Pile-up in a collisionally active disk

So far, we have looked separately at the balance between P–R drag and collisions (without considering sublimation) and at the balance between P–R drag and sublimation (without considering collisions). We now investigate under what conditions this pairwise approach is justified, and combine the previous findings to estimate the distribution of dust in a debris disk in which all three processes are operational.

**Do collisions interfere with dust pile-up?**

Since collisions might interfere with the process of dust pile-up, the results of Sect. 2.2.2 are only valid if collisions do not play an important role at distances where sublimation becomes significant. We now investigate whether collisions can indeed be neglected in the inner regions of debris disks by comparing the characteristic timescales of the three processes as a function of radius.
2.2 Analytical constraints

Figure 2.2: The characteristic timescale as function of radial distance for sublimation (Eq. 2.15), P–R drag (Eq. 2.2), and mutual collisions (minimum timescale, Eq. 2.23), for $\beta = 0.5$ particles in a debris disk around a solar-mass star with a very dense ($\eta_0 \gg 1$) parent belt located at 30 AU. The gray vertical lines indicate the radial distances used by the analytical model: $r_{\text{pile}}$ is the radius for which the sublimation timescale equals the P–R drag timescale, $r_{\text{crit}}$ is the radius for which the P–R drag timescale equals the collisional timescale, and $r_0$ is the location of the parent belt. For the sublimation timescale, we assume that the dust grains are solid spheres of graphite (see Sect. 2.2.2 for the values of the sublimation parameters) with a radius of $s = 0.64 \mu m$ (the size corresponding to $\beta = 0.5$).

In the case of a very dense parent belt ($\eta_0 \gg 1$), $\tau_{\text{geo}}(r)$ is given by Eq. 2.5. Putting this in Eq. 2.3 results in the minimum collisional timescale

$$\min [t_{\text{coll}}(r)] = \frac{c r^{3/2}}{\beta G M_\star} \left( \sqrt{r_0} - \sqrt{r} \right), \quad r \leq r_0.$$  \hspace{1cm} (2.23)

In Fig. 2.2 this timescale is compared with the sublimation and P–R drag timescales for a debris disk around a solar-mass star with a dense parent belt at 30 AU, consisting of barely bound ($\beta = 0.5$) dust grains and using the sublimation parameters of graphite. In this example system, the collisional timescale is longer than the other timescales at $r_{\text{pile}}$, implying that P–R drag dominates collisions in the sublimation zone, and collisions do not interfere with dust pile-up. We now check whether this a general result or under which conditions it is the case.

We let $r_{\text{crit}}$ be the radial distance at which collisional and P–R drag timescales are equal. Solving $t_{\text{PR}}(r_{\text{crit}}) = t_{\text{coll}}(r_{\text{crit}})$ for $r_{\text{crit}}$, with $\tau_{\text{geo}}(r)$ given by Eq. 2.4, yields

$$\frac{r_{\text{crit}}}{r_0} = \left( \frac{4 \eta_0 + 1}{5 \eta_0} \right)^2.$$  \hspace{1cm} (2.24)

For $\eta_0 = 1$ we recover $r_{\text{crit}} = r_0$, as expected. From the limiting case $\eta_0 \gg 1$ (i.e., a very dense parent belt), we find

$$r_{\text{crit}} > 0.64 r_0.$$  \hspace{1cm} (2.25)
This shows that far enough inward from the parent belt, P–R drag always dominates collisions. Furthermore, if Eq. 2.22 holds we find \( r_{\text{crit}} \gtrsim 12.8 r_{\text{pile}} \), which means that in systems where the parent belt is distant enough for significant dust pile-up to occur, P–R drag dominates collisions in the sublimation zone, and collisions are so infrequent there that they do not interfere with the pile-up process.

The pile-up of dust means that \( \tau_{\text{geo}} \) increases around \( r_{\text{pile}} \), locally decreasing the collisional timescale. However, a significant pile-up requires Eq. 2.22 to hold, in which case the ratio between the minimum collisional timescale and the P–R drag timescale is found to satisfy

\[
\frac{t_{\text{coll}}(r_{\text{pile}})}{t_{\text{PR}}(r_{\text{pile}})} = 4(\sqrt{r_0/r_{\text{pile}}} - 1) \gtrsim 13.9.
\]

To overcome this difference, the pile-up would have to raise \( \tau_{\text{geo}} \) by the same factor. Since the \( \tau_{\text{geo}} \) enhancement factor is never found to be greater than about 10 (Kobayashi et al. 2009), we expect the disk to remain drag (or sublimation) dominated inside \( r_{\text{crit}} \).

### Estimating the pile-up magnitude

The efficiency of dust pile-up was studied in detail by Kobayashi et al. (2009, 2011), who give formulae for the resulting enhancement in particle number density and geometrical optical depth. Here, we present a simple order-of-magnitude estimate of the maximum amount of material in the pile-up, which can be used to assess whether pile-ups can explain the observed NIR excess emission.

Dust grains reside in the pile-up for roughly one sublimation timescale, after which they are either completely sublimated or their size is reduced so much that they are blown out of the system. Since the pile-up occurs roughly at \( r_{\text{pile}} \), defined such that \( t_{\text{subl}}(r_{\text{pile}}) = t_{\text{PR}}(r_{\text{pile}}) \), the dust stays in the pile-up for about one P–R drag timescale. Given this, the total number of particles in the pile-up is

\[
N_{\text{pile}} = \varphi_{\text{PR}}(r_{\text{pile}}) t_{\text{PR}}(r_{\text{pile}}).
\]  

To describe the radial profile in terms of geometrical optical depth, it is necessary to specify the radial width of the pile-up \( \Delta r_{\text{pile}} \). In reality, \( \Delta r_{\text{pile}} \) depends on the orbital eccentricities of the particles in the pile-up and on differences in pile-up distance for particles of different sizes that contribute. Since this is beyond the scope of this work, we keep the relative pile-up width \( \Delta r_{\text{pile}}/r_{\text{pile}} \) as a free parameter.

Combining Eqs. 2.1, 2.6, and 2.26, we find that the geometrical optical depth due to the material in the pile-up is given by

\[
\tau_{\text{geo, pile}} = \frac{\sigma}{2\pi r_{\text{pile}} \Delta r_{\text{pile}}} N_{\text{pile}} = \frac{r_{\text{pile}}}{2\Delta r_{\text{pile}}} \tau_{\text{geo, base}}(r_{\text{pile}}),
\]

where \( \tau_{\text{geo, base}}(r) \) denotes the base level of dust due to P–R drag and collisions given by Eq. 2.4. Kobayashi et al. (2009) find that sublimating dust particles slowly move outward. Therefore, we assume that the pile-up extends from \( r_{\text{pile}} \) outward, overlapping with the inward migrating material. The complete geometrical optical depth profile, which includes the effects
of collisions, P–R drag, and sublimation, can then be formally described by (cf. Kobayashi et al. 2011)

\[
\tau_{\text{geo}}(r) = \begin{cases} 
0 & \text{for } r < r_{\text{pile}} \\
\tau_{\text{geo, base}}(r) + \tau_{\text{geo, pile}} & \text{for } r_{\text{pile}} \leq r \leq r_{\text{pile}} + \Delta r_{\text{pile}} \\
\tau_{\text{geo, base}}(r) & \text{for } r_{\text{pile}} + \Delta r_{\text{pile}} < r \leq r_0
\end{cases}
\] (2.28)

Using Eq. 2.5 for \(\tau_{\text{geo, base}}(r)\) gives the maximum profile. Figure 2.3 shows this maximum profile for different values of \(\Delta r_{\text{pile}}/r_{\text{pile}}\).

We can make a rough estimate of the geometrical optical depth enhancement factor \(f_{\tau_{\text{geo}}}\) of the pile-up (i.e., how much higher \(\tau_{\text{geo}}\) in the pile-up is compared to the base level of the inner disk), as a function of the radial width of the pile-up \(\Delta r_{\text{pile}}\):

\[
f_{\tau_{\text{geo}}} = \frac{\tau_{\text{geo, pile}} + \tau_{\text{geo, base}}}{\tau_{\text{geo, base}}} = \frac{r_{\text{pile}}}{2\Delta r_{\text{pile}}} + 1.\] (2.29)

For a pile-up width of \(\Delta r_{\text{pile}} \approx 0.05r_{\text{pile}}\) (Kobayashi et al. 2011), this gives \(f_{\tau_{\text{geo}}} \approx 11\), which is comparable to the highest values found by Kobayashi et al. (2009, see their Fig. 6) for carbonaceous dust grains around early F-type stars.

For comparison with observations, it suffices to assume that all the pile-up material is located at \(r_{\text{pile}}\) (i.e., \(\Delta r_{\text{pile}}/r_{\text{pile}} \ll 1\)). This gives the highest possible temperature to all pile-
up particles, and therefore results in the maximum NIR flux. In the $\tau_{geo}$ profile, however, it would give a singularity at $r = r_{pile}$. To avoid this, we instead compute the fractional luminosity of the pile-up. Fractional luminosity is defined as the ratio of the infrared luminosity of the dust $L_D$ to the stellar luminosity. Assuming the disk is radially optically thin and the dust grains have unity absorption and emission efficiencies at all wavelengths (i.e., assuming black-body grains), it can be approximated by the fraction of the star that is covered by dust

$$\frac{L_D}{L_*} = \int \frac{\sigma n(r)}{4\pi r^2} \, dr.$$  \hspace{1cm} (2.30)

Evaluating this with $n(r) \, dr = N_{pile}$, and using the maximum geometrical optical depth (Eq. 2.5), we find that the maximum fractional luminosity due to material in the pile-up is

$$\max \left[ \left( \frac{L_D}{L_*} \right)_{pile} \right] = \frac{\sqrt{GM_\star \beta}}{8c} \left( \frac{\sqrt{r_0} - \sqrt{r_{pile}}}{} \right).$$  \hspace{1cm} (2.31)

In the limit of $r_0 \gg r_{pile}$, this becomes

$$\max \left[ \left( \frac{L_D}{L_*} \right)_{pile} \right] \approx 6.2 \times 10^{-6} \left( \frac{M_\star}{1 \, M_\odot} \right)^{1/2} \left( \frac{r_0}{1 \, \text{AU}} \right)^{-1/2} \left( \frac{\beta}{0.5} \right).$$  \hspace{1cm} (2.32)

This is only the fractional luminosity due to the dust in the pile-up. Material just beyond $r_{pile}$ is not accounted for, and will increase the total fractional luminosity.

### 2.2.4 Summary of analytical findings

The analytical model presented above yields several tentative conclusions about dust in the inner parts of debris disks:

1. P–R drag gives rise to a small but non-zero inward mass flux of dust in the inner disk, which is self-limited by collisions (Eq. 2.11).

2. A pile-up of sublimating dust occurs, as long as the star is luminous enough (Eq. 2.18), and the parent belt is distant enough (Eq. 2.22).

3. P–R drag dominates collisions in the inner parts of the disk (Eq. 2.25), so collisions do not interfere with dust pile-up.

4. Given that the pile-up of dust occurs around the radial distance where the sublimation timescale equals the P–R drag timescale and that sublimating dust resides in the pile-up for about one sublimation timescale, there is a maximum fractional luminosity that this dust can provide (Eq. 2.32).
2.3 Numerical modeling

The analytical approach used in Sect. 2.2 contains several simplifying assumptions. Most importantly, we only self-consistently solve the continuity equation for P–R drag and collisions (under the assumptions listed in Sect. 2.2.1), and afterwards estimate the effect of dust pile-up due to sublimation, assuming the grains reside in the sublimation zone for one P–R drag timescale (see Sect. 2.2.3). To test the impact of these assumptions, we now proceed to solve the continuity equation numerically using a debris disk model that self-consistently handles the effects of stellar gravity, direct radiation pressure, P–R drag, sublimation, and destructive collisions. Our strategy here is to simulate a few specific cases and compare the results to the analytical maximum distributions found in Sect. 2.2, to assess the validity and generality of these simple expressions. A description of our numerical debris disk model is given in Sect. 2.3.1, the runs we performed are detailed in Sect. 2.3.2, and the resulting dust distributions are presented in Sect. 2.3.3.

2.3.1 Model description

Our debris disk model closely follows the method developed by Krivov et al. (2005, 2006) and Löhne (2008). We refer the reader to these publications for a detailed description of the method and only provide a brief outline of its principles here. We focus on the changes we made, which are including a time-dependent treatment of dust sublimation (see Sect. 2.3.1) and implementing additional numerical acceleration techniques (see Appendix 2.A). Our code was tested by comparing its predictions to solutions of the equations of motion and sublimation for individual particles and by benchmarking it against the results of Krivov et al. (2006). This verification is described in Appendix 2.B. Two physical processes that are included by Löhne (2008), but not considered by our present code, are stellar wind drag and erosive (cratering) collisions. We discuss the impact they may have on our results in Sect. 2.5.

Method basics

The method of Krivov et al. (2005) applies the kinetic method of statistical physics to debris disks, simultaneously following the spatial and size distributions of dust and planetesimals in a phase space of orbital elements and particle masses. Using orbital element instead of radial distance to follow the spatial distribution makes it possible to account for particles on eccentric orbits, whose orbits can span a wide range of radial distances. The continuity equation is solved in this phase space, with processes that affect the evolution of a particle’s phase-space coordinates in a continuous fashion (P–R drag and sublimation) as diffusion terms, and processes that abruptly change phase-space coordinates (collisions) as source and sink terms. Formally, this is described by (cf. Krivov et al. 2005; Löhne 2008)

\[
\frac{dn}{dt}(m, k, t) = \left( \frac{dn}{dt} \right)_{\text{source}} - \left( \frac{dn}{dt} \right)_{\text{sink}} - \text{div}\left( n \frac{d[m, k]}{dt} \right),
\]

(2.33)
where \( n(m, k, t) \) is the phase-space number density at time \( t \) (i.e., the distribution function that describes the state of the disk), \( k \) the vector of orbital elements, and \( \{m, k\} \) denotes the vector consisting of \( m \) and \( k \). The divergence term represents the diffusion of material in phase space (i.e., transport due to P–R drag and sublimation). For brevity, we omit the arguments \( (m, k, t) \) for all terms on the right-hand side of the equation.

To make the numerical evaluation of the continuity equation manageable with limited computational capacity, the number of phase-space variables needs to be reduced. By assuming the disk is axisymmetric, the distribution function can be averaged over three of the orbital elements: longitude of the ascending node, argument of the periastron, and true anomaly. This implicitly assumes that collisional timescales are much longer than orbital timescales, which generally holds for debris disks (for a more detailed discussion, see Sect. 3.1.3 of Löhne 2008). A further assumption is that the distribution of particles over inclinations is constant, which allows the averaging of the distribution function over inclination. The three remaining phase-space variables are (1) particle mass \( m \), (2) orbital eccentricity \( e \), and (3) an orbital element characterizing the size of the orbit, such as semi-major axis \( a \) or periastron distance \( q = a(1 - e) \).

The final phase-space variable can be chosen to fit the numerical needs of the problem under investigation. For our study of the pile-up of dust due to sublimation, we chose periastron distance. A particle on an eccentric orbit experiences most sublimation around the periastron, owing to the strong temperature dependence of sublimation. Since the periastron distance does not evolve for \( \beta \) changes that happen at the periastron, the orbit’s periastron distance changes much more slowly than its semi-major axis. Using \( q \) instead of \( a \) as phase-space variable therefore has numerical advantages.

In practice, the phase space is divided into a grid of bins, and the distribution function is replaced by a vector listing the number of particles in each bin. Equation 2.33 then becomes a system of ordinary differential equations. The source, sink, and diffusion terms are discretized, and they determine the rates at which the particle numbers evolve, dependent on the population levels of other bins. We now proceed to describe these terms for each of the physical processes considered by our model.

**Poynting–Robertson drag**

P–R drag affects the orbits of particles, circularizing them, and making them smaller. These effects are accounted for in the model by diffusion terms in the continuity equation that move particles to adjacent bins in the phase-space grid. Since the P–R drag timescale is usually longer than the orbital period, we use the orbit-averaged change rates of the orbital elements, given by (e.g., Burns et al. 1979)

\[
\frac{da}{dt}_{PR} = -\frac{\beta GM_*}{ca} \frac{2 + 3e^2}{(1 - e^2)^{3/2}},
\]

\[
\frac{de}{dt}_{PR} = -\frac{5\beta GM_*}{2ca^2} \frac{e}{(1 - e^2)^{1/2}}.
\]
For the rate of change in periastron distance, we find

\[
\left\langle \frac{dq}{dt} \right\rangle_{PR} = \frac{\partial q}{\partial a} \left\langle \frac{da}{dt} \right\rangle_{PR} + \frac{\partial q}{\partial e} \left\langle \frac{de}{dt} \right\rangle_{PR} = -\frac{\beta G M_{*} (4 - e)(1 - e)^3}{2cq (1 - e^2)^{3/2}}.
\]  

(2.37)

Sublimation

The formalism that we use for dust sublimation is described in Sect. 2.2.2. For a spherical dust particle, it gives a mass loss rate of

\[
\frac{dm}{dt} = -P_v(T) s^2 \sqrt{\frac{8\pi \mu m_u}{k_B T}}.
\]  

(2.38)

In our numerical model, we use realistic dust grain temperatures (as opposed to the black-body temperatures used in Sect. 2.2). The method for computing these temperatures is explained in Sect. 2.3.2.

Since the sublimation rate strongly depends on grain temperature, which varies along the path of an eccentric orbit, the mass loss rate needs to be averaged over the orbit. As described qualitatively in Sect. 2.2.2, the change in \(\beta\) ratio associated with mass loss induces changes in orbital elements. Kobayashi et al. (2009) derive the orbit-averaged change rate in orbital elements and mass to be

\[
\left\langle \frac{da}{dt} \right\rangle_{\text{subl}} = -\frac{d \ln \beta}{d \ln m} \left( 1 + e^2 \bar{\psi}_m + \frac{2e}{1 - e^2} \bar{\phi}_m \right) \frac{\beta}{1 - \beta m} \frac{a}{1 - e^2},
\]  

(2.39)

\[
\left\langle \frac{de}{dt} \right\rangle_{\text{subl}} = -\frac{d \ln \beta}{d \ln m} (e \bar{\psi}_m + \bar{\phi}_m) \frac{\beta}{1 - \beta m},
\]  

(2.40)

\[
\left\langle \frac{dm}{dt} \right\rangle_{\text{subl}} = -\bar{\psi}_m,
\]  

(2.41)

with

\[
\bar{\psi}_m = -\frac{1}{2\pi} \int_0^{2\pi} \frac{dm}{dt} \frac{(1 - e^2)^{3/2}}{(1 + e \cos f)^2} df,
\]  

(2.42)

\[
\bar{\phi}_m = -\frac{1}{2\pi} \int_0^{2\pi} \frac{dm}{dt} \cos f \frac{(1 - e^2)^{3/2}}{(1 + e \cos f)^2} df,
\]  

(2.43)

where \(f\) denotes the true anomaly. For the periastron distance, we find

\[
\left\langle \frac{dq}{dt} \right\rangle_{\text{subl}} = \frac{\partial q}{\partial a} \left\langle \frac{da}{dt} \right\rangle_{\text{subl}} + \frac{\partial q}{\partial e} \left\langle \frac{de}{dt} \right\rangle_{\text{subl}} = \frac{d \ln \beta \bar{\psi}_m - \bar{\phi}_m}{d \ln m} \frac{\beta}{1 + e} \frac{q}{1 - \beta m}.
\]  

(2.45)
For each phase-space bin, quantities \( \psi_m \) and \( \phi_m \) are numerically evaluated using the standard Euler method.\(^9\) The change rates of the phase-space variables (Eqs. 2.40, 2.41, and 2.45) are then used in diffusion terms in the continuity equation.

Using orbit-averaged mass loss rates is only correct if the sublimation timescale is longer than the orbital period. For phase-space bins for which this does not hold (small particles close to the star), we compute an equilibrium population of particles from the product of their sublimation timescale and the sum of their gain terms. This implicitly assumes that particles are created on their orbits with a uniform distribution over true anomaly.

**Collisions**

Collisions are different from P–R drag and sublimation in that they cause abrupt rather than smooth changes in phase-space coordinates. In the continuity equation, they are described by sink terms at the phase-space coordinates of targets and projectiles and by source terms at the coordinates of the resulting fragments. Here, we give a summary of the way our model handles collisions. More thorough descriptions of the treatment of collisions, including all relevant equations, are given by Krivov et al. (2006) and Löhne (2008).

For each pair of phase-space bins, we determine collision rates for a range of relative orbit orientations (differences in the longitudes of the periastra of the two orbits). The collision rate is the product of the target and projectile number densities, their relative velocity, the collisional cross-section, and the effective volume of interaction. Krivov et al. (2006) found analytical expressions for these factors in two dimensions as a function of the orbital elements and masses corresponding to both bins and the relative orientation of the orbits. To account for the third dimension, a correction is applied based on the semi-opening angle of the disk, equivalent to the maximum inclinations of the particles. This correction assumes that the disk is relatively flat, consistent with observations of resolved edge-on systems.

To save computational power, we ignore collisions involving unbound particles. This is a valid approximation if the radial geometrical optical depth of the system is much smaller than unity (true for most debris disks), because then the blowout timescale of such particles is much shorter than their collisional timescale.\(^{10}\)

The nature of a collision is determined by the impact energy available per unit of mass. We only consider catastrophic collisions, defined as destructive events in which the largest fragment contains at most half of the mass of the more massive of the two impactors. The threshold for these catastrophic collisions is the critical specific energy for dispersal \( Q_D^* \), which incorporates the fact that fragments may reassemble after destruction, and generally depends on particle size. If the specific energy of a collision is higher than this threshold, the impact destroys both bodies, and their mass is distributed over a swarm of fragments. At specific energies just below \( Q_D^* \), collisions are erosive. Such cratering collisions, however,

---

9 Integrating over true anomaly rather than, e.g., mean anomaly warrants a higher sampling around the periastron, where sublimation rate varies the most.

10 If the radial geometrical optical depth is higher than \( \sim 10^{-2} \), the disk is subject to dust avalanches (Grigorieva et al. 2007).
are not considered in our present model, so if the specific energy of an impact is lower than \(Q_D^\star\), no collision is considered to occur.

A catastrophic collision results in a range of fragments with different masses and orbits. The fragments are distributed over particle masses according to a single power law, up to a maximum fragment mass, which is determined by the kinetic energy of the impact and the material strength of the target. The maximum fragment mass is at most half of the mass of the target and projectile combined, but it can also be less, if the specific energy involved in the collision is more than \(Q_D^\star\). The amount of particles that end up in each mass bin (up to the maximum fragment mass) is computed by integrating the fragment mass distribution. Particles with masses below the lowest mass bin (i.e., that fall off the grid) are considered lost due to immediate blowout or vaporization. For each fragment mass bin, new orbital elements are calculated using the conservation of momentum, and taking into account direct radiation pressure (i.e., the values of \(\beta\) of the fragments), using Eqs. 19 and 20 of Krivov et al. (2006). This assumes that the fragments are not launched away from the collision with any velocity. These orbital elements are rounded to the nearest periastron distance and eccentricity bins for each fragment mass bin.

### 2.3.2 Setup of the model runs

#### Stellar and disk parameters

Hot exozodiacal dust has been detected around stars with spectral types ranging from A to K (see Table 2.1). To focus on this range of stellar types, we did one model run with a solar-mass star and one using a 2 M\(_\odot\) star. Following the mass–luminosity relation for main-sequence stars \(L_\star \propto M_\star^{3.5}\); Allen 1976), we set the stellar luminosities corresponding to these stellar masses to \(L_\star = 1\ L_\odot\) and \(L_\star = 11.31\ L_\odot\), respectively.

Both runs use a parent belt radius of \(r_0 = 30\ AU\). Lower values of \(r_0\) can in principle yield higher dust levels in the innermost regions (see Eq. 2.5), but parent belts closer to the star are generally not dense enough to provide these large amounts of dust, because they do not survive the intense collisional grinding (Dominik & Decin 2003; Wyatt et al. 2007a). In addition, many of the observed outer belts are located at tens of AUs (see Table 2.1). The level of dust in the source region is set to \(\tau_{\text{geo}}(r_0) \approx 5 \times 10^{-5}\), chosen such that (i.e., iterated until) the geometrical optical depth in the inner regions does not become any higher by increasing the level of dust at the source.\(^{11}\) This roughly corresponds to \(\eta_0 \sim 10\) for both stellar mass cases, which is apparently enough to approximate an inner disk \(\tau_{\text{geo}}\) profile corresponding to \(\eta_0 \gg 1\). For the semi-opening angle of the disk we use \(\varepsilon = 8.5^\circ\).

\(^{11}\)The actual input parameter used (which indirectly determines the geometrical optical depth at the source region) is the mass supply term of large dust particles in the source region (see Sect. 2.3.2). It is set to \(10^{-10}\ M_\oplus\ \text{yr}^{-1}\) for the \(M_\star = 1\ M_\odot\) run and \(8 \times 10^{-10}\ M_\oplus\ \text{yr}^{-1}\) for the \(M_\star = 2\ M_\odot\) run. Comparing these mass fluxes to those given by Eq. 2.11 indicates that the vast majority of the material is destroyed in collisions before it reaches the sublimation zone.
Material properties

We consider carbonaceous dust particles with a density of $\rho_d = 1.8 \, \text{g cm}^{-3}$. To compute the optical properties of these grains with different radii, we use the DHS method of Min et al. (2005) with an irregularity parameter of $f_{\text{max}} = 0.8$. This method simulates the properties of irregularly shaped particles. For the material we use amorphous carbon with the refractive index data taken from Preibisch et al. (1993). These optical properties are used to compute $\beta(s)$ and dust temperatures. The dust temperatures are computed by solving the balance between absorption and thermal emission as a function of grain size and distance to the central star. For the sublimation properties, we use those of graphite, given in Sect. 2.2.2.

Modeling collisions requires a prescription for the specific energy threshold for dispersal $Q_{\text{D}}^*$, which is generally found to depend on size. In our numerical simulation, we consider particles with radii up to 1 cm. (see Sect. 2.3.2). For bodies smaller than $s \sim 100 \, \text{m}$, $Q_{\text{D}}^*$ is often described by a power law with a negative exponent (e.g., Benz & Asphaug 1999). However, such a prescription predicts unrealistically high values for the small particles that we consider. Therefore, following Heng & Tremaine (2010), we use the constant value of $Q_{\text{D}}^* = 10^{-7} \, \text{erg g}^{-1}$ found in laboratory experiments with high-velocity collisions of small particles (Flynn & Durda 2004). We follow Krivov et al. (2006) in setting the collisional fragment mass distribution to $n(m) \propto m^{-11/6}$, and in assuming the maximum fragment mass scales with specific impact energy to the power $-1.24$ (Fujiwara et al. 1977).

The phase-space grid

Because this problem is computationally very demanding, great care has to be taken in setting up the phase-space grid. Specifically, resolving the pile-up requires high resolution in the sublimation zone and at small particles sizes, where radiation pressure becomes important. To achieve this with limited computational resources, we designed a non-uniform grid that has a higher resolution where it is required.

The eccentricity grid contains ten logarithmic bins between $e = 0$ and $e = 1$, with the lowest bin at $e = 10^{-4}$. In addition, there are two linearly spaced eccentricity bins between $e = 1$ and $e = 2$ for hyperbolic orbits, as well as two bins between $e = -2$ and $e = -1$ to account for “anomalous” hyperbolic orbits followed by $\beta > 1$ particles (see Krivov et al. 2006).

The periastron distance grid consists of two parts: (1) A high-resolution, linear grid of 21 bins is used to cover the sublimation zone ($0.01 \, \text{AU} < q < 0.03 \, \text{AU}$ for the $M_* = 1 \, \text{M}_\odot$ run, and $0.05 \, \text{AU} < q < 0.1 \, \text{AU}$ for the $M_* = 2 \, \text{M}_\odot$ run). (2) A low-resolution, logarithmic grid covers the rest of the disk out to about $q = 100 \, \text{AU}$, with 60 bins in the $M_* = 1 \, \text{M}_\odot$ run and 50 bins in the $M_* = 2 \, \text{M}_\odot$ run. Care was taken to place one bin exactly at $q = 30 \, \text{AU}$, which was used as the source region.

The mass grid has 48 logarithmically spaced bins in both runs with higher resolution at the smaller sizes ($\beta \gtrsim 0.05$) and a maximum mass corresponding to $s = 1 \, \text{cm}$. For the $M_* = 1 \, \text{M}_\odot$ run, the high-resolution part consists of 30 bins between $s = 0.5 \, \text{\mu m}$ and $s = 10 \, \text{\mu m}$. The
2.3 Numerical modeling

high-resolution part of the \( M_* = 2 \, M_\odot \) run has 36 bins between \( s = 2 \, \mu m \) and \( s = 100 \, \mu m \).

Simulation strategy

Owing to computational limitations, the largest particles we consider have a radius of 1 cm. In reality, the size distribution in the parent belt extends up to planetesimals of tens to hundreds of kilometers. To account for the fragmentation of these larger bodies, we include a source of dust at \( r_0 \). This artificial source term adds particles with sizes between \( s = 1 \, \text{mm} \) and \( s = 1 \, \text{cm} \), following the power-law size distribution \( n(s) \propto s^{-3.5} \). The size of these grains is chosen such that the effect of radiation pressure on them is very small (\( \beta < 10^{-4} \)). Therefore, their eccentricity distribution follows that of the parent bodies. We add the source particles at eccentricities ranging from \( e = 0 \) to \( e = 0.1 \).

The artificial supply of large dust particles is balanced by the loss of material due to blowout and sublimation. Therefore, solving the continuity equation results in a steady-state distribution function. We start the integration without any material in the disk (only the artificial supply is acting) and let the model run until steady state is reached. This is considered to be the case when relative changes in the radial and size distributions between logarithmic (base-10) time steps become less than 1%. Initially, we only consider collisions and P–R drag, and the sublimation module of the code is switched off. At this stage, particles migrate inward due to P–R drag until they reach the inner edge of the grid. Once steady state is reached, sublimation is switched on, and we let the distribution function settle into a new steady state. This procedure is necessary because sublimation forces the time step to become very short. With sublimation switched on from the start, the computation would take unnecessarily long. Additionally, it allows us to isolate the effect of dust pile-up (i.e., the dust in the pile-up can be isolated by subtracting the pre-sublimation state from the final one).

2.3.3 Results

The output of each model run is a steady-state distribution of particles in the phase space of orbital elements and masses. To analyze this output, we convert it into a radial geometrical optical depth profile (Sect. 2.3.3) and size distributions (in terms of cross-section density per unit size decade \( A \)) at different radial locations (Sect. 2.3.3). The conversion from raw model output to these quantities is detailed in Appendix 2.C.

Radial distribution

Figure 2.4 shows the geometrical optical depth profiles derived from the numerical model runs, together with the analytical maxima given by Eq. 2.28, using a pile-up width of

\[ 12 \text{ Formally, steady state is only reached after } \sim 10 \, \text{Gyr} \text{, which is the time it takes for the largest particles we consider to move from the parent belt to the sublimation zone by P–R drag. The barely bound grains that dominate the cross-section, however, already settle into a steady state after } \sim 10 \, \text{Myr} \text{, which is short compared to the typical lifetime of a debris disk.} \]
The geometrical optical depth profiles of debris disks with a dense parent belt at 30 AU around stars of $M_\star \approx 1 M_\odot$ (solid lines) and $M_\star = 2 M_\odot$ (dashed lines). The black lines show the end results of the numerical simulations. In green are the maximum $\tau_{\text{geo}}$ profiles as given by the analytical model (Eq. 2.28, with $\tau_{\text{geo, base}}(r)$ given by Eq. 2.5, $\Delta r_{\text{pile}}/r_{\text{pile}} = 0.15$, $r_0 = 30$ AU, and $\beta = 0.5$).

$\Delta r_{\text{pile}}/r_{\text{pile}} = 0.15$, chosen to match the numerical profile. Generally, there is good correspondence between the numerical results and the analytical maxima, but there are some important differences. The profiles have roughly the same shape, with a slightly steeper slope close to the source region in the numerical results. For both cases of stellar mass, the base level of $\tau_{\text{geo}}$ in the inner disk (i.e., away from the pile-up) is a factor of about 7 lower in the numerical profiles. This discrepancy is a result of the assumption in the analytical model that all orbits are circular. In the numerical model, the small particles that contribute most to the cross-section are released in the parent belt on eccentric orbits. They therefore suffer a higher rate of destruction by collisions.

As predicted, switching on sublimation leads to an accumulation of dust. The pile-ups are located very close to $r_{\text{pile}}$, as determined for graphite grains, using black-body temperatures and $Q_{\text{pr}} = 1$ (Eq. 2.17, which gives $r_{\text{pile}} \approx 0.019$ AU for the $M_\star = 1 M_\odot$ case, and $r_{\text{pile}} \approx 0.064$ AU for the $M_\star = 2 M_\odot$ case). The temperature of the dust (as computed using the full optical properties) at the inner edge of the disk and in the pile-up is between 2000 and 2100 K. In both runs, the pile-up has a geometrical optical depth enhancement factor of about $f_{\tau_{\text{geo}}} \approx 3$. The fractional luminosities due to the pile-ups are $(L_D/L_\star)_{\text{pile}} \approx 7.5 \times 10^{-8}$ for the $M_\star = 1 M_\odot$ case and $(L_D/L_\star)_{\text{pile}} \approx 1.1 \times 10^{-7}$ for the $M_\star = 2 M_\odot$ case. Both are about a factor 15 lower than the maxima given by Eq. 2.32. Given that the base levels of $\tau_{\text{geo}}$ in the numerical profiles are a factor of about 7 lower than the analytical maxima, however, the discrepancy is only about a factor of 2. In short, the pile-up mechanism is found to be somewhat less efficient than predicted by the analytical estimates.
2.3 Numerical modeling

The quantity on the vertical axis, $A$, is the cross-section density per unit size decade (see Appendix 2.C.2), which is horizontal if all sizes contribute equally to the cross-section. The gray band indicates the slope of a size distribution that follows the classical Dohnanyi (1969) power law ($n(s) \propto s^{-3.5}$, hence $A \propto s^{-0.5}$). The vertical lines mark the particle sizes corresponding to relevant values of $\beta$.

The outer disk ($r > 30$ AU) is not the focus of this work. Nevertheless, its radial profile is relevant, since it reflects the status of the balance between collisions and P–R drag. To first order, the geometrical optical depth profile of the outer disk can be characterized by a power law $\tau_{\text{geo}} \propto r^{-\alpha}$. Strubbe & Chiang (2006) derive the theoretical values of $\alpha = 1.5$ and $\alpha = 2.5$ for collision and P–R drag-dominated disks, respectively. We find slopes of $\alpha \approx 2.0$ for both runs, consistent with the outer slope found by Vitense et al. (2010) for the Edgeworth–Kuiper Belt, and interpreted as the sign of a disk that is in between drag and collision-dominated. This indicates that the density of the parent belt (characterized by $\eta_0 \sim 10$) is insufficient to make the outer disk completely collision-dominated.

Size distribution

The size distribution results of the two runs are similar in many ways, so we only discuss the $M_\star = 1$ M$_\odot$ run here. Figure 2.5 shows how the size distribution changes with radial distance when only considering P–R drag and collisions (i.e., before sublimation is switched on in the model). In the parent belt ($r = 30$ AU), it follows the classical Dohnanyi (1969) power law ($n(s) \propto s^{-3.5}$, which is valid for an infinite collisional cascade with self-similar collisions), from the blowout radius upwards. Particles with $\beta > 0.5$ are depleted by about three orders of magnitude in terms of collective cross-section. Superimposed on the power law is a well-known wave pattern related to the discontinuity in the size distribution at the blowout size.
The first bump (i.e., the one at $\beta \approx 0.5$) in the size distribution at $r = 30$ AU does not extend far above the power-law prediction. The reason for this may be that particles with $\beta \gtrsim 0.1$ experience more destructive collisions, because their eccentricities are significantly higher than those of the parent bodies, which are distributed over the range $0 < e < 0.1$ (Sect. 2.3.2, see also Eq. 2.19, and cf. Fig. 5 of Krivov et al. 2006).

Inward from the parent belt, the size distribution seems to become steeper, which is expected from the dependence of the radial profile on $\beta$ (Eq. 2.5). However, this effect is difficult to isolate, because the profile is distorted by the wave pattern, which increases in amplitude and "wavelength" with decreasing radial distance (cf. Fig. 7 of Krivov et al. 2006). The prominent wave pattern indicates that collisions are still important for larger particles in the inner disk (while P–R drag dominates for $\beta \approx 0.5$ particles there, see Fig. 2.2). In the innermost parts of the disk ($r \lesssim 1$ AU), particles with $\beta \approx 0.5$ clearly dominate the cross-section, contributing at least three orders of magnitude more than any other size. At $r = 0.1$ AU, the local slope of the size distribution between the blowout size and $s \approx 10$ $\mu$m is approximately $A \propto s^{-7.5}$, equivalent to $n(s) \propto s^{-10.5}$. This means that not only the cross-section, but also the mass is dominated by barely bound grains in the innermost parts of the disk. Interestingly, such steep size distributions are also invoked to explain NIR interferometric observations of hot exozodiacal dust (Defrère et al. 2011b; Mennesson et al. 2013; Lebreton et al. 2013). The drop in the size distribution from $\beta \approx 0.5$ to $\beta \approx 1$ is also much more pronounced in the inner disk than in the parent belt.

Sublimation only has a significant effect on the size distribution around $r \approx r_{\text{pile}}$ (i.e., in the pile-up). Figure 2.6 shows the size distribution in the pile-up, before and after sublimation is switched on. It clearly demonstrates that sublimation enhances the density of particles with $0.5 \leq \beta < 1$ around $r = r_{\text{pile}}$. This indicates that the pile-up consists mostly of particles that started with $\beta \approx 0.5$ before active sublimation and lost mass due to sublimation, increasing their $\beta$. The rest of the size distribution does not change significantly. Size distributions at larger radial distances are largely unaffected by sublimation.

### 2.4 Comparison with observations

To assess whether the pile-up effect can explain the observed NIR excess, we computed the spectral energy distributions (SEDs) of the dust distributions found in the previous sections. We only calculated the emission spectrum of the dust and ignored the (viewing-angle-dependent) contribution of scattered light, since thermal emission was found to dominate scattered light at the wavelengths in which hot exozodiacal dust is detected (Absil et al. 2006). For the analytical dust distributions, we assumed the dust has a black-body temperature (Eq. 2.14). The flux density of the dust, expressed as a function of wavelength $\lambda$, can
then be computed as

\[
F_{\nu,D}(\lambda) = \frac{2\pi\lambda^2}{cd^2} \int r \tau_{\text{geo}}(r)B_\lambda(T_{\text{bb}}) \, dr,
\]

where \(d\) is the distance to the source, and \(B_\lambda(T)\) the Planck function. For the numerically determined dust distributions, we use realistic dust temperatures and optical properties (see Sect. 2.3.2), which leads to

\[
F_{\nu,D}(\lambda) = \frac{2\pi\lambda^2}{cd^2} \int \int r Q_{\text{abs}}(s) \tau_{\text{geo}}(s, r)B_\lambda[T(s, r)] \, ds \, dr.
\]

Here, \(Q_{\text{abs}}\) is the absorption efficiency of the grains (equal to their emission efficiency) and \(\tau_{\text{geo}}(s, r)\) is the geometrical optical depth profile of dust with grain radius \(s\).

Figure 2.7 shows the debris disk spectra corresponding to dust distributions found from the numerical modeling, as well as the analytical maximal dust profiles and the stellar spectra. The analytical maxima are computed from Eq. 2.28, with \(\tau_{\text{geo,base}}(r)\) given by Eq. 2.5, \(r_0 = 30\ \text{AU}, \beta = 0.5\), and all the pile-up material at \(r_{\text{pile}}\) (i.e., \(\Delta r_{\text{pile}}/r_{\text{pile}} \ll 1\)).\(^{13}\) Singularities at \(r = r_0\) are removed by imposing the condition \(\tau_{\text{geo}} \leq 0.01\), which corresponds to \(\eta_0 \approx 2200\). The numerical dust distributions without pile-up were created by subtracting the isolated pile-up dust from the final profile (see Sect. 2.3.2). The stellar spectra are given by black-body curves, with \(L_\star = 1\ L_\odot, T_\star = 5780\ \text{K}\) for the \(M_\star = 1\ M_\odot\) star, and \(L_\star = 11.31\ L_\odot\),

\(^{13}\) Our approach to computing the SED of a dust profile with pile-up is different from that of Kobayashi et al. (2011), who use \(\tau_{\text{geo,pile}}\) and \(\Delta r_{\text{pile}}\) as independent input variables. In our estimate, the total amount of dust is fixed, and placing it all at \(r_{\text{pile}}\) is the most optimistic configuration for detecting the pile-up.
Figure 2.7: SEDs of stars and their debris disks at a distance of 10 pc. The disk SED is shown for different dust distributions with the numerical results in black and the analytical distributions in green. Dust distributions including the pile-up of dust in the sublimation zone are shown with dashed lines, and distributions in which the pile-up is excluded are shown with solid lines, but the spectra largely overlap. Disk SEDs are calculated according to Eqs. 2.46 and 2.47. The stellar photosphere is indicated by a dotted Planck curve. The vertical gray areas mark the NIR $H$ and $K$ bands in which hot exozodiacal dust is observed.

$T_\star = 7730$ K for the $M_\star = 2 \, M_\odot$ star, following the main-sequence relations $L_\star \propto M_\star^{3.5}$ and $T_\star \propto L_\star^{0.12}$ (Allen 1976). The distance is arbitrarily set to 10 pc.

The synthetic spectra are similar to those described by Wyatt (2005), but are truncated at short wavelengths because there is no material with a higher temperature than the sublimation temperature. Apart from an overall shift in flux, the differences between the analytical and numerical SEDs are minor. In the solar-mass star run, the peak of emission of the numerical result is shifted toward shorter wavelengths with respect to the analytical SED. This is because the grains that dominate the emission in the parent belt are significantly hotter than the black-body temperature.

Only considering thermal emission, the NIR flux originates almost exclusively in the inner 1 AU of the debris disk. The pile-up does not add a significant amount of flux. The theoretical SEDs display NIR flux ratios between the disk and star of about $F_{\nu, D}/F_{\nu, \star} \sim 10^{-4}$. This is much less than the observed flux ratios that indicate hot exozodiacal dust, which are typically on the order of 1% (see Table 2.1). Furthermore, the NIR radiation is accompanied by mid-IR flux at a comparable level, which is incompatible with observed excess spectra (e.g., Akeson et al. 2009). The pile-up does not contain enough material to create a bump in thermal flux in the NIR.

To generalize the above results, we investigate how the NIR flux ratio depends on parent belt distance and stellar type using the analytical model. In Fig. 2.8, we show the analytical maximum NIR flux ratios for four different stellar types. As in the above analysis, the disk flux is calculated from Eq. 2.46, using the analytical maximum dust distribution, and the stars are approximated by black bodies. The M0V, G2V, A5V, and B5V stars correspond to $M_\star = 0.5$, 1, 2, and 4 $M_\odot$, respectively, with $L_\star \propto M_\star^{3.5}$ and $T_\star \propto L_\star^{0.12}$ (Allen 1976).
Comparing the analytical maximum NIR flux ratios to the observed ones demonstrates that P–R drag does not provide enough material to the inner disk to explain the observations.

### 2.5 Discussion

#### 2.5.1 The size distribution in the inner disk

The size distribution in the inner parts of dense debris disks (i.e., inward from the parent belt) has not been studied many times before. Acke et al. (2012) analytically derive a power-law distribution of \( n(s) \propto s^{-3} \), but ignore collisions in the inner disk. In a detailed modeling study of the debris disk around \( \epsilon \) Eri, Reidemeister et al. (2011) find a flat profile \( (n(s) \propto s^{-3} \text{ flat in terms of } A) \) for small sizes, and a steeper one \( (n(s) \propto s^{-3.7}) \) for larger sizes. However, this system is a special case, because \( \epsilon \) Eri is not luminous enough to blow small dust grains out of the system.

With our numerical model, we find a size distribution that deviates significantly from a power law. Specifically, in the innermost regions of the disk \( (r \lesssim 1 \text{ AU}) \), the cross-section is...
completely dominated by barely bound ($\beta \approx 0.5$) particles. This is the result of two effects: (1) Since the efficiency of P–R drag depends on $\beta$, larger particles tend to stay closer to the parent belt (apparent from Eq. 2.5). (2) Owing to the high relative velocities in the inner disk,$^{14}$ the wave in the size distribution is extremely strong. As a consequence, the single size assumption used in our analytical model is a good approximation.

Erosive (cratering) collisions affect the size distribution (Thébault & Augereau 2007). We expect that including this type of collisions in our model would result in a less wavy size distribution, possibly eliminating the second bump in the size distribution (see Sect. 4.3.4 of Löhne 2008). This would mean that $\beta \approx 0.5$ particles dominate the cross-section even more than suggested by Fig. 2.5. Since erosive collisions present an additional mechanism for destroying dust, we expect that including them in our model would only lower the level of dust in the inner disk, so it does not change the conclusions of this work. Erosive collisions do affect the timescale on which debris disks evolve, but since we compute steady-state dust distributions, this does not have any impact on our results.

It may be surprising that a large number of particles are present in the pile-up with $0.5 \lesssim \beta < 1$. Usually (as in Sect. 2.2), particles with $\beta > 0.5$ are assumed to be absent, because they are blown out of the system as soon as they are released from parent bodies on circular orbits. Parent bodies on eccentric orbits can give rise to a population of bound grains with $\beta > 0.5$, but the parent body orbits in our model runs are not eccentric enough to produce bound dust grains with $\beta$ values close to unity. The origin of this high $\beta$ population is the sublimation of barely bound $\beta \approx 0.5$ grains. As these particles migrate inward from the parent belt, their orbital eccentricities are circularized by P–R drag. When they arrive in the sublimation zone, their orbital eccentricities are as low as $e \approx 10^{-4}$. Kobayashi et al. (2009) find that the subsequent evolution of the eccentricities during the active sublimation phase can be described by

$$e = \left( \frac{1 - \beta_1}{1 - \beta} \right)^\kappa e_1,$$

where subscript 1 denotes quantities at the start of substantial sublimation, and the exponent $\kappa$ can be treated as a constant, which mostly depends on the optical and sublimation properties of the material under consideration. For spherical, black-body graphite particles, we find $\kappa \approx 10$. This shows that particles starting with $\beta_1 \approx 0.5$, and $e_1 \approx 10^{-4}$ will only become unbound ($e \geq 1$) when they reach $\beta \gtrsim 0.8$ (corresponding to $s \approx 0.8 \mu$m).

2.5.2 Stellar wind drag

Neither our analytical nor our numerical model includes the effects of stellar wind. For stars with a strong stellar wind and/or low luminosity, the stellar wind equivalent of P–R drag

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$^{14}$ In debris disks relative velocities are mostly due to the eccentricities and mutual inclinations of particle orbits. While orbital eccentricities are diminished in the inner disk by P–R drag induced circularization, inclinations are simply inherited from the parent belt. This is handled correctly in our code, since orbital inclinations are parameterized by the (constant) opening angle of the disk.
shortens the inward migration timescale and thus increases the maximum geometrical optical depth profile. Since stellar wind drag works in the same way as P–R drag, its effect can be accounted for by replacing $\beta$ with (Burns et al. 1979; Minato et al. 2004, 2006)

$$\beta_{PR} = \beta(1 + \gamma), \tag{2.49a}$$

$$\gamma = \frac{M* c^2 Q_{sw}}{L* Q_{pr}}, \tag{2.49b}$$

where $M*$ is the stellar mass loss rate by stellar wind, and $Q_{sw}$ is the stellar wind momentum transfer efficiency. For carbonaceous particles orbiting the Sun, Minato et al. (2004, 2006) find $\gamma \sim 1$.

Kobayashi et al. (2011) argue that the pile-up scenario could work for Vega if the disk is drag-dominated, which requires $\gamma \approx 300$. While this value is consistent with the upper limit on the mass-loss rate of Vega from radio-continuum observations ($M* \lesssim 10^{-10} \, M_\odot \, yr^{-1}$; Hollis et al. 1985), stellar wind models predict much lower mass-loss rates for main-sequence A-type stars ($M* \lesssim 10^{-16} \, M_\odot \, yr^{-1}$; Babel 1995). This theoretical mass-loss rate, together with $L* \approx 10 \, L_\odot$ and $Q_{sw} \approx Q_{pr}$, gives a ratio between stellar wind drag and P–R drag of $\gamma \lesssim 10^{-4}$. We therefore conclude that it is unlikely that stellar wind drag has a significant effect on debris disks around main-sequence A-type stars.

### 2.5.3 Other explanations for hot exozodiacal dust

We find that P–R drag does not provide enough material to the innermost parts of the disk to explain the interferometric detections of NIR excess. There are two additional problems with this scenario, which also serve as clues for solving the hot exozodiacal dust mystery. (1) Hot exozodiacal dust is thought to consist mostly of blowout grains with sizes around 0.01–0.1 $\mu$m (Akeson et al. 2009; Defrère et al. 2011b; Mennesson et al. 2013; Lebreton et al. 2013), while P–R drag only transports bound grains to the sublimation zone, the smallest of which have sizes of about 1 $\mu$m. (2) The dust distribution resulting from the balance between P–R drag and collisions yields an SED with a positive slope in the infrared domain, while observations find negative slopes (e.g., Akeson et al. 2009; Acke et al. 2012), and the pile-up of dust is too inefficient to have an effect on the slope of the SED.  

For these reasons, the origin of hot exozodiacal dust must involve mechanisms that we did not consider in this work.

Our treatment of sublimation assumes spherical dust grains that sublimate uniformly (i.e., layers of material are removed one by one). In reality, dust grains may be aggregates that fall apart during sublimation, abruptly increasing the collective cross-section of the material. To investigate this scenario, the steady-state amount of material can be estimated by multiplying the maximum P–R drag inward mass flux (Eq. 2.11) with an estimate of the lifetime of the fragments. We performed this analysis for Fomalhaut and find that P–R drag from a parent belt at 2 AU still does not supply enough material, unless the lifetime of the fragments is

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15 For the $M* = 1 \, M_\odot$, $r_0 = 30 \, AU$ case, the pile-up efficiency should be at least $\sim 100$ times higher to make the slope of the SED negative.
significantly longer than what can be expected from sublimation and blowout (Lebreton et al. 2013).

Two mechanisms have recently been proposed that can lead to an extended lifetime for small exozodiacal dust particles. Lebreton et al. (2013) investigate the impediment of blowout due to the presence of gas for the hot dust around Fomalhaut, but find that this requires unrealistically high gas densities. Su et al. (2013) propose that charged nanograins can remain trapped in the magnetic field of the star and qualitatively show that this may help explain the NIR excess of Vega.

Another mechanism for the inward transport of material from a cold outer belt is the inward scattering of material by planets. Bonsor et al. (2012) investigated this scenario and find that it is marginally capable of providing mass influxes compatible with observations. However, this requires relatively contrived planetary system architectures, consisting of closely packed chains of low-mass planets. Furthermore, the scenario was investigated for the inward scattering down to 1 AU, and reaching the sublimation zone is likely to be less efficient.

2.6 Conclusions

In this work, we investigated hot dust in the inner regions of debris disks, whose presence is suggested by interferometrically resolved excess NIR emission observed in some debris disk systems (Tbl. 2.1). We tested whether the hot dust can be supplied by P–R drag from a distant parent belt and whether the pile-up of dust in the sublimation zone still occurs if collisions are considered. Our main conclusions follow.

1. As predicted by Wyatt (2005), P–R drag always brings a small amount of dust from an outer debris belt into the sublimation zone. The maximum geometrical optical depth that can be reached in the innermost parts of the disk depends on the mass of the central star and distance to the parent belt (Fig. 2.1). When the production of dust is treated self-consistently, this maximum is found to be a factor of about 7 lower than the analytical estimate (Fig. 2.4). This is because small dust particles, which are efficiently dragged inward by radiation forces, are also put on highly eccentric orbits by those radiation forces and therefore suffer more collisional destruction.

2. Dust that reaches the sublimation zone produces some NIR emission, but this excess flux is insufficient to explain the interferometric observation. While the observed excess ratios are \( \sim 10^{-2} \), the maximum flux ratio due to material supplied by P–R drag is \( \lesssim 10^{-3} \) for A-type stars with parent belts at \( \gtrsim 1 \) AU (Fig. 2.8).

3. The pile-up of dust from the interplay of P–R drag and sublimation still occurs when collisions are considered (Fig. 2.4), as long as the parent belt in which the dust originates is distant enough to allow for sufficient circularization of the orbits, and the central star is luminous enough to blow small dust grains out of the system. Collisions do not interfere with the pile-up process, since in the inner disk, the collisional
timescale is longer than the P–R drag timescale for the barely bound grains that are the most important for the pile-up. The fractional luminosity provided by dust in the pile-up is relatively low, so the pile-up does not influence the disk SED significantly (Fig. 2.7).

4. In the inner parts of dense debris disks, the cross section is clearly dominated by barely bound ($\beta \approx 0.5$) grains, and the size distribution features a prominent wave pattern, related to the discontinuity in the size distribution at the blowout size (Fig. 2.5). In the pile-up, there is an enhancement of particles with $0.5 \lesssim \beta < 1$ (Fig. 2.6). These particles are still bound, because of their almost circular orbits at the start of substantial sublimation.

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2.A Numerical techniques

Our model is computationally very demanding, primarily because of the large number of calculations needed for collisions, which scales with the number of phase-space bins cubed. To ensure that the model can produce results in a reasonable amount of time, an effort was made to reduce the amount of calculations needed per run. To achieve this, we used an integration technique that allows large time-step sizes, which reduces the number of steps needed per run (Appendix 2.A.1). Furthermore, we calculated as many time-independent numerical factors as possible before the actual integration, so these factors do not have to be determined at every time step (Appendix 2.A.2).

2.A.1 Integration method

After discretization, the distribution function $n(m, k, t)$ is replaced by the vector of population levels $\mathbf{n}$, and Eq. 2.33 can be treated as the system of linear equations

$$\frac{d\mathbf{n}}{dt} = A\mathbf{n} + \mathbf{b},$$

(2.50)

where $A$ is a matrix of coefficients, and $\mathbf{b}$ is a constant vector containing the artificial source terms that replenish dust in the parent belt.\textsuperscript{16} This system of equations suffers from stiffness: the population levels of some bins change very rapidly compared to others mainly due to large differences in collisional timescales, and the time-step size is therefore determined by the

\textsuperscript{16} Collision rates are proportional to target and projectile densities, so the entries of matrix $A$ contain terms with elements of $\mathbf{n}$. Because target and projectile bin can be the same, the system of equations is in fact non-linear.
stability of the integration method rather than by its accuracy. When using standard explicit integrators, this leads to an impractically small step size that prevents long integrations.

Following Löhne (2008), we resolve the stiffness by writing the differentials as

$$\frac{dn}{dt} = A' \cdot n + \left. \frac{dn}{dt} \right|_{\text{const}},$$

where $A'$ is a diagonal matrix that contains only the diagonal elements of $A$, while the terms marked “const” contain the off-diagonal parts of $A$, as well as the constant terms $b$. Our integration scheme for the $j^{th}$ component of $n$, for time step $m + 1$, reads as

$$n_{j,m+1} = n_{j,m} \exp (A'_{jk,m} \Delta t) + \hat{n}_{j,m} \left|_{\text{const}} \right. \frac{A'_{jk,m}}{A'_{jk,m}} \left[ \exp (A'_{jk,m} \Delta t) - 1 \right],$$

where $\Delta t$ denotes the time-step size. This scheme is known as the exponential Euler method (see Hochbruck & Ostermann 2010 for an introductory review). It is suitable for time integration of semi-linear problems, which consist of a stiff linear part and a non-stiff non-linear part. In short, the strategy is to solve the linear part exactly and to approximate the non-linear part using an explicit integration scheme.

The time-step size $\Delta t$ is determined dynamically from the condition that population levels can never become negative. We use a scheme similar to that of Krivov et al. (2005), but adapted for the exponential Euler method.

### 2.A.2 Precalculation

Only considering collisions (i.e., ignoring diffusion terms), the evolution of the $j^{th}$ component of $n$ can be written as (Löhne 2008)

$$\dot{n}_j = \sum_{tp} B_{jtp} n_t n_p.$$

Here, $t$ and $p$ are the target and projectile bin indices, respectively, and $B_{jtp}$ is an entry in the time-independent tensor of collisional coefficients $B$. Specifically, coefficient $B_{jtp}$ is the rate at which the population level of bin $j$ changes, per particle in bin $t$, per particle in bin $p$, combining all considered relative orbit orientations. If $j = t$ or $j = p$, this is the loss rate of the target or projectile bin, respectively, caused by collisional destruction (assuming that the mass grid resolution is high enough that fragments do not end up in their parent bin). Otherwise, it is a rate at which fragments are created.

For the phase-space grids we use, the entire tensor $B$ is too large to be stored in memory. However, it is very sparse, because (1) not all orbits that are part of the phase-space grid have mutual overlap, (2) overlap may occur for a limited range in relative orbit orientation, (3) impact velocities are not always high enough to cause catastrophic collisions, and (4) only a fraction of all possible masses are created as fragments.

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17 This corresponds to Eq. 3.107 of Löhne (2008), corrected for a typographical error.
By only storing the non-zero entries of $B$, it becomes possible to keep it in memory. We store the source and sink terms separately. For the source terms, which are by far the most memory intensive, each non-zero entry requires (1) the index of the target bin, (2) the index of the projectile bin, (3) the index of the bin fragment bin, and (4) the rate at which particles are created in the fragment bin, per particle in the target bin, per particle in the projectile bin. By further manual compression, only (3) and (4) need to be stored for each entry separately. The target bin index (1) only needs to be stored once for each possible target bin, along with the number of possible projectile bins for that target bin. Then, the projectile bin index (2) only needs to be stored once for each possible collisional pair of bins, along with the number of possible fragment bins for that pair. A similar scheme is used for the sink terms.

The compressed version of $B$ is still very large (several gigabytes for the runs presented in this work). A disadvantage of the precalculation technique is therefore that the size of the grid that can be used is restricted by the available amount of memory, whereas a code that recalculates (parts of) $B$ at each time step is only limited by CPU power. A cubic dependence of the non-zero entries of $B$ on the number of mass bins (as opposed to quadratic dependencies on the orbital element variables) motivates the choice of a relatively small mass grid, representing only a part of the collisional cascade.

2.B Model verification

We performed several tests to verify our numerical debris disk model. The P–R drag and sublimation modules of the code were tested by comparing the behavior of the model with independent numerical solutions of the equations of motion and sublimation for individual particles. For this purpose, the collisional module of the code was switched off, and a single bin was filled as the initial setup of the model, corresponding to the orbital elements of the particle. For all these tests, the resulting evolution of the dust distribution (not shown here) matches that of the independent solution. The accuracy of the predictions is limited by numerical diffusion and becomes better with higher resolution (i.e., larger phase-space grids).

To test the collisional module of our code, we benchmarked the model against the code of Krivov et al. (2006), by replicating one of their model runs for the debris disk of Vega as accurately as possible. Since the code of Krivov et al. (2006) uses semi-major axes as the distance dimension of the phase-space grid, the benchmark runs were performed with a version of our code that also uses the semi-major axis (as opposed to periastron distance, used in the rest of this paper). These runs do not include the effects of P–R drag or sublimation, so they can be used to separately test the collisional module of our code by switching off P–R drag and sublimation. Of the various runs presented by Krivov et al. (2006), the specific one that was reproduced is characterized by an initial optical depth profile in the outer disk (beyond 120 AU) that scales as $\tau_{\text{geo}}(r) \propto r^{-4}$, an initial eccentricity distribution between 0 and 0.375, and material properties for “rocky” grains. We refer the reader to Krivov et al. (2006) for the specific values used for parameters describing the phase-space grid, the initial
The evolution of the radial and size distribution predicted by the benchmark run are shown in Figs. 2.9 and 2.10, respectively. The corresponding results of Krivov et al. (2006) are their Figs. 10 and 6, respectively. Comparison of the results reveals good agreement between the two codes, and the remaining discrepancies can be accounted for. Relative to the benchmark, our model predicts (1) lower unbound particle populations and (2) shorter evolutionary timescales. Point (1) is to be expected since the unbound particle densities of Krivov et al. (2006) are computed using the product of their production rate and their disk crossing time, rather than from Eqs. 2.59 (T. Löhne, private communication). We attribute point (2) to a mislabeling of the time steps in Figs. 6 and 10 of Krivov et al. (2006). Further discrepancies can be due to small differences in the input parameters and numerical techniques used.

2.C Post-processing of model output

The output of a model run are the population levels $n$ of all bins in the three-dimensional phase space of particle mass and orbital elements, as well as their change rates $\dot{n}$ (which should equal zero once steady state is reached, except for bins corresponding to unbound orbits). While this data is useful for analyzing the orbital characteristics of different classes of particles in the debris disk, it is often more convenient to know about the state of the disk as a function of distance from the star. This is essential, for example, if the results of the model are to be compared with observations. Here, we describe the processing steps that are applied to the model output to derive the quantities used in Sect. 2.3.3.
2.C Post-processing of model output

2.C.1 Conversion from orbital elements to radial distance

To find the radial distribution of matter in the debris disk, the orbital element phase-space distribution function \( n(m, q, e) \) (dimension: \([g^{-1} \text{ cm}^{-1}]\)) needs to be converted to the configuration space distribution function \( N(m, r) \), which denotes the vertically averaged number density of particles with masses \([m, m + dm]\) at distance \(r\) (dimension: \([g^{-1} \text{ cm}^{-3}]\)). This problem was first solved by Haug (1958) for a rotationally symmetric ensemble of particles on Keplerian orbits. Here, we give a brief derivation under the additional assumption that the distribution of inclinations is independent of the distribution of the other orbital elements.

Consider an individual particle on a bound Keplerian orbit that spends \(dt\) time to cross a radial annulus with width \(dr\) at distance \(r\) from the star. The contribution of this particle to \(N(m, r)\) is

\[
N_{\text{part}} = \frac{2dr}{P} \frac{1}{2\pi r dr} \frac{1}{h},
\]

(2.54)

where \(P\) is the particle’s orbital period, and \(h = 2r \sin \varepsilon\) is the disk height, where \(\varepsilon\) is the semi-opening angle of the disk. The explicit factor 2 in the numerator accounts for the fact that the particle passes through this radial annulus twice during each orbit. In terms of orbital elements \(q\) and \(e\), the orbital period \(P\), accounting for direct radiation pressure, is given by

\[
P = 2\pi \sqrt{\frac{q^3}{GM_* (1 - \beta)(1 - e)^3}},
\]

(2.55)
The radial velocity \( \dot{r} \) of the particle is given by
\[
\frac{dr}{dt} = \pm \sqrt{\frac{GM_\star(1-\beta)}{r} \left[ 2 - \frac{r}{q}(1-e) - \frac{q}{r}(1+e) \right]}.
\] (2.56)

Combining Eqs. 2.54, 2.55, and 2.56, and considering all particles on bound orbits, gives (cf. Krivov et al. 2005)
\[
\mathcal{N}(m, r) = \frac{1}{4\pi^2 \sin(\varepsilon)} \frac{1}{r^3} \int_q^r \int_e n(m, q, e) \left[ \frac{r}{q}(1-e) \right]^{3/2} \times \left[ 2 - \frac{r}{q}(1-e) - \frac{q}{r}(1+e) \right]^{-1/2} dq \, de,
\] (2.57)

with the integration domain
\[
\frac{1-e}{1+e} \leq q \leq r, \quad 0 \leq e < 1.
\] (2.58)

The contribution of unbound particles to \( \mathcal{N}(m, r) \) is calculated using their production rate \( \dot{n}(m, q, e) \) and the radial velocity with which they leave the system (T. Löhne, private communication). Assuming all unbound particles are created at the periastron of their orbits, their vertically-averaged particle number density is given by
\[
\mathcal{N}(m, r) = \frac{1}{2\pi rh} \int_q^r \int_e \frac{\dot{n}(m, q, e)}{|\dot{r}(m, q, e, r)|} dq \, de \quad (2.59)
\]

\[
= \frac{1}{4\pi \sin(\varepsilon) \sqrt{GM_\star(1-\beta)r^3}} \int_q^r \int_e \dot{n}(m, q, e) \times \left[ 2 - \frac{r}{q}(1-e) - \frac{q}{r}(1+e) \right]^{-1/2} dq \, de,
\] (2.60)

with the integration domain
\[
q \leq r, \quad -1 \leq e \leq 1.
\] (2.61)

Negative eccentricities correspond to “anomalous” hyperbolic orbits (see Krivov et al. 2006).

In applying this theory to the raw data, we replace the integrals in Eqs. 2.57 and 2.60 with sums; replace \( n(m, q, e) \) and \( \dot{n}(m, q, e) \) with \( n \) and \( \dot{n} \), respectively; and evaluate the resulting equations at discrete points of \( r \). For each bin, we sample the phase space it represents using a Monte Carlo method.

### 2.C.2 Derived quantities

For radial dust distribution profiles, we use the vertical geometrical optical depth \( \tau_{\text{geo}} \), defined as the surface density of collective (i.e., combining particles of all sizes) cross-section. It is
computed from $\mathcal{N}(m, r)$ as

$$
\tau_{\text{geo}}(r) = h \int_m \sigma(m) \mathcal{N}(m, r) \, dm 
$$

$$
= 2\pi \sin(\varepsilon) r \int_m s^2(m) \mathcal{N}(m, r) \, dm. 
$$

To characterize size distributions, we use the quantity $A(s, r)$, defined as the cross-section density per base-10 logarithmic unit of size. It is given by

$$
A(s, r) = \frac{ds}{d\log_{10}(s)} \frac{dm}{ds} \sigma(s) \mathcal{N}(m, r) 
$$

$$
= 4\pi^2 \log(10) \rho_{\text{d}} s^5 \mathcal{N}(m, r). 
$$

This quantity allows for an easy comparison between the relative contributions of particles with different sizes to the total cross-section of the disk. In the size distributions plots (Figs. 2.5 and 2.6), a horizontal line means particles of all sizes contribute equally to the cross-section, and equal areas under the curve correspond to equal contributions to the total cross-section.
An interferometric study of the Fomalhaut inner debris disk
III. Detailed models of the exozodiacal disk and its origin


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Abstract

Context. Debris disks are thought to be extrasolar analogs to the solar system planetesimal belts. The star Fomalhaut harbors a cold debris belt at 140 AU comparable to the Edgeworth–Kuiper belt, as well as evidence of a warm dust component, unresolved by single-dish telescopes, which is suspected of being a bright analog to the solar system’s zodiacal dust.

Aims. Interferometric observations obtained with the VLTI/VINCI instrument and the Keck Interferometer Nuller have identified near- and mid-infrared excesses attributed respectively to hot and warm exozodiacal dust residing in the inner few AU of the Fomalhaut environment. We aim to characterize the properties of this double inner dust belt and to unveil its origin.

Methods. We performed parametric modeling of the exozodiacal disk (“exozodi”) using the GRaTeR radiative transfer code to reproduce the interferometric data, complemented by mid-to far-infrared photometric measurements from Spitzer and Herschel. A detailed treatment of sublimation temperatures was introduced to explore the hot population at the size-dependent sublimation rim. We then used an analytical approach to successively testing several source mechanisms for the dust and suspected parent bodies.
Results. A good fit to the multiwavelength data is found by two distinct dust populations: (1) a population of very small (0.01 to 0.5 \(\mu m\)), hence unbound, hot dust grains confined in a narrow region (~0.1 – 0.3 AU) at the sublimation rim of carbonaceous material; (2) a population of bound grains at ~ 2AU that is protected from sublimation and has a higher mass despite its fainter flux level. We propose that the hot dust is produced by the release of small carbon grains following the disruption of dust aggregates that originate from the warm component. A mechanism, such as gas braking, is required to further confine the small grains for a long enough time. In situ dust production could hardly be ensured for the age of the star, so we conclude that the observed amount of dust is triggered by intense dynamical activity.

Conclusions. Fomalhaut may be representative of exozodis that are currently being surveyed at near and mid-infrared wavelengths worldwide. We propose a framework for reconciling the “hot exozodi phenomenon” with theoretical constraints: the hot component of Fomalhaut is likely the “tip of the iceberg” since it could originate from the more massive, but fainter, warm dust component residing near the ice line. This inner disk exhibits interesting morphology and can be considered a prime target for future exoplanet research.

3.1 Introduction

During the past few years, the increasing number of smaller exoplanets and fainter debris disks have revealed that extrasolar analogs to our solar system may be common, and yet, little is known about the architecture of the very inner parts of planetary systems. A distinguishable feature of the inner solar system is the existence of the zodiacal cloud, composed of small (1 to 100 \(\mu m\)) dust grains (Grün 2007; Rowan-Robinson & May 2013), which are thought to come from the disruption and erosion of comets, asteroids, and Kuiper belt objects (e.g. Nesvorný et al. 2010). Dusty debris disks orbiting other stars than the Sun were first detected by their excess emission in the mid- or far-infrared (IR), and could then be imaged at visible to submillimeter wavelengths. A few warm disks comparable to the zodiacal cloud have been found by space observatories around mature stars via their photometric excess emission at mid-IR wavelengths (Beichman et al. 2005; Lawler et al. 2009; Lisse et al. 2012); but their characterization suffers from insufficient spatial resolution and large photometric uncertainties.

Recent developments in high-angular resolution interferometry have offered powerful tools to characterizing exozodiacal disks (exozodis), which reside in the close environment (typically less than 3 AU) of a large fraction of nearby stars. Large efforts have indeed been made to detect exozodis with near- and mid-IR interferometers worldwide, notably at the VLTI (Absil et al. 2009), the Keck Interferometer (Millan-Gabet et al. 2011) or the CHARA array at Mount-Wilson (Absil et al. 2006; Defrère et al. 2011a). Ongoing surveys in the near-infrared (K < 5) with the CHARA/FLUOR (Coudé du Foresto et al. 2003) and VLTI/PIONIER (Le Bouquin et al. 2011) interferometers indicate that as much as ~ 30% of nearby AFGK main-sequence stars may harbor hot (typically 1000-2000K) circumstellar
dust within a few AU at the 1% level with respect to photospheric emission (Absil et al. 2013; Ertel et al. 2014). Conversely, only \( \sim 12\% \) of the surveyed main sequence stars have been found to harbor mid- infrared excesses (i.e., warm dust) with nulling interferometry (Millan-Gabet et al. 2011).

From a theoretical point of view, the prevalence of these hot excesses around main sequence stars is not understood. Radiative transfer analysis identifies very hot and small refractory grains close to the sublimation limit that should be radiatively blown out over timescales of weeks. Yet they represent typical masses of \( 10^{-8} \) to \( 10^{-10} \, M_\oplus \) that need to be delivered by equivalent masses of dust-producing planetesimals (e.g. Defrère et al. 2011b). Being much more massive than the zodiacal cloud, these hot exozodis are difficult to reconcile with the steady-state collisional evolution of parent body belts (Wyatt et al. 2007a).

One famous example of a dusty planetary system is the one surrounding the nearby (7.7 pc) A3V star Fomalhaut (\( \alpha \) PsA, HD 216956). This young main sequence star (440 \( \pm \) 40 Myr, Mamajek 2012) is well known for its prominent, \( \sim 140 \) AU-wide, debris belt that was first resolved in scattered light with HST/ACS revealing a sharp inner edge and side-to-side brightness asymmetry suggestive of gravitational sculpting by a massive planet (Kalas et al. 2005; Quillen 2006). A point source attributed to the suspected planet was later detected in the optical at the predicted location, moving along its orbit over multiple epochs (Kalas et al. 2008). The absence of detection in the near- and mid- (thermal) infrared range (Janson et al. 2012) resulted in a controversial status for Fomalhaut b. More recent studies confirm the detections of a companion at 118 AU and interpret the various constraints as a large circumplanetary dust disk orbiting a hidden subjovian planet, but do not exclude an isolated dust cloud originating from a recent collision between planetesimals (Currie et al. 2012; Galicher et al. 2013). The cold planetesimal belt is indeed collisionally very active as shown by the recent analysis of resolved images and photometry at far-infrared to submillimeter wavelengths from ALMA (Boley et al. 2012) and Herschel/PACS and SPIRE (Acke et al. 2012). Using radiative transfer modeling, Acke et al. (2012) argue that a dust production rate of \( \sim 3 \times 10^{-5} \, M_\oplus/\text{year} \) is needed to justify the measured amount of blow-out grains, and estimate that the region interior to the cold belt at 140 AU are not devoid of material.

The Herschel/PACS 70 \( \mu \)m, as well as the ALMA image, identify an unresolved excess in the vicinity of the star that was previously reported in the mid-infrared by Stapelfeldt et al. (2004) based on Spitzer/MIPS imaging and IRS spectroscopy. Su et al. (2013) recently analyzed the IRS and PACS data and concluded on the presence of a warm debris belt with a blackbody temperature of \( \sim 170 \) K. However, these facilities lack the spatial resolution and accuracy needed to characterize this warm component.

Near- and mid-infrared long baseline interferometers offer the appropriate tools to study the Fomalhaut exozodi with enough contrast and resolution, free of any modeling assumptions on the stellar spectrum.

The present paper carries out a thorough analysis of the Fomalhaut inner debris disk. It is

\footnote{Kalas et al. (2013) recently announced a fourth epoch detection of Fomalhaut b. They argue in favor of an object larger than a dwarf planet evolving on a very eccentric orbit (see Section 6).}
the last one of a series initiated by Absil et al. (2009) (henceforth Paper I) - who presented the VLTI/VINCI detection of hot excess in the close environment of the star - followed by a mid-infrared characterization using the Keck Interferometer Nuller (KIN) by Mennesson et al. (2013) (henceforth Paper II). In Paper I, we presented the clear K-band (2.18 µm) detection of a short-baseline visibility deficit. It is best explained by circumstellar emission emanating from within 6 AU of the star with a relative flux level of $0.88 \pm 0.12\%$. In Paper II, we presented multiwavelength measurements performed across the N-band (8 to 13 µm) using the technique of nulling interferometry. A small excess is resolved within $\sim 2$ AU from the star, with a mean null depth value of $0.35\% \pm 0.10\%$ 2. Preliminary modeling shows that the near- to mid-infrared excesses can only be explained by two distinct populations of dust emitting thermally; small ($\sim 20$ nm) refractory grains residing at the sublimation distance of carbon are responsible for the near-infrared emission, while $> 1$ µm grains located further than the silicate sublimation limit produce most of the mid-infrared excesses.

The paper is organized as follows. In Sec. 3.2 and 3.3, we present our radiative transfer model of optically thin disks, and introduce a new prescription for calculating the sublimation distance of dust grains in an exozodi. Results of the modeling of the inner Fomalhaut debris disk, based on multiwavelength observations, are detailed in Sec. 3.4. The mechanisms that produce and preserve the hot grains, as well as the connection with the warm and the cold belt are discussed in Sec. 3.5. We discuss further our results and attempt to place the Fomalhaut exozodi in the context of its planetary system in Sec. 3.6. We finally summarize our main findings in Sec. 3.7.

3.2 A schematic exozodiacal disk model

In this section, we elaborate a schematic model of an optically thin exozodiacal dust disk. The model is implemented in the GRaTeR code originally developed by Augereau et al. (1999). It makes hardly any a priori assumptions regarding the nature of the grains and their production. Because of the limited constraints on detected exozodiacal disks, we restrain the model to a 2D geometry. The disk surface density and grain size distribution are parametrized with simple laws to limit the number of free parameters. This allows us to explore a broad range of disk properties using a Bayesian inference method.

3.2.1 Scattered light and thermal emission

We consider a population of dust grains at a distance $r$ from the star and with a differential size distribution $dn(r, a)$, where $a$ is the grain radius. In a self-consistent description of a collisional evolution of debris disks, the spatial and size distribution cannot be formally separated (Augereau et al. 2001; Krivov et al. 2006; Thébault & Augereau 2007). Since we

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2 We note that the true astrophysical excess could be larger because some dust emission may be removed by the destructive fringes of the interferometer. The nuller transmission pattern is accounted for in the models discussed afterwards.
know little about the properties and origin of observed exozodiacal disks, we assume here for simplicity that the dependence of the size distribution on the distance $r$ essentially reflects the size-dependent sublimation distance of the grains. Since exozodiacal grains may reach very high temperatures, sublimation can prevent the smallest grains from surviving in regions where larger ones can remain, thereby truncating the size distribution at its lower end. We therefore write the differential size distribution $dn(r, a)$ as follows:

$$dn(r, a) = H(a - a_{\text{sub}}(r)) \, dn(a)$$

with $\int_{a_{\text{min}}}^{a_{\text{max}}} dn(a) = 1$ (3.1)

where $a_{\text{min}}$ and $a_{\text{max}}$ are the minimum and maximum grain sizes respectively, $H(a - a_{\text{sub}}(r))$ is the Heaviside function (assuming $H(0) = 1$) and $a_{\text{sub}}(r)$ is the sublimation size limit at distance $r$ from the star. Details of the calculation for sublimation will be given in Sec. 3.3.

At wavelength $\lambda$, the dust population thermally emits a flux

$$\Phi_{\text{th}}(\lambda, r) = \int_{a_{\text{min}}}^{a_{\text{max}}} B_\lambda(T_\text{d}(a, r)) \frac{\sigma_{\text{abs}}(\lambda, r, a)}{4d_*^2} dn(r, a)$$

(3.2)

where $d_*$ is the distance of the observer to the star, $T_\text{d}(a, r)$ is the grain temperature and $B_\lambda$ is the Planck function. In the above equation, we implicitly assumed that grains thermally emit isotropically. The absorption cross-section $\sigma_{\text{abs}}(\lambda, r, a)$ reads

$$\sigma_{\text{abs}}(\lambda, r, a) = 4\pi a^2 Q_{\text{abs}} \left( \frac{2\pi a}{\lambda}, \lambda, r \right)$$

(3.3)

where $Q_{\text{abs}}$ is dimensionless absorption/emission coefficient that depends on the size parameter $2\pi a/\lambda$, on $\lambda$ through the wavelength-dependent optical constants, and on the distance to the star as the grain composition may depend on $r$. The grain temperature $T_\text{d}(a, r)$ is obtained by solving in two steps the thermal equilibrium equation of a grain with the star. For any grain size $a$, we first calculate the equilibrium distance $r$ for a broad range of grain temperatures $T_\text{d}$ knowing the $Q_{\text{abs}}$ value

$$r(a, T_\text{d}) = \frac{d_*}{2} \sqrt{\frac{\int_{\lambda} Q_{\text{abs}} F_*(\lambda) d\lambda}{\int_{\lambda} Q_{\text{abs}} \pi B_\lambda(T_\text{d}) d\lambda}}$$

(3.4)

where $F_*(\lambda)$ is the stellar flux at Earth. The $r(a, T_\text{d})$ function is then numerically reversed to get $T_\text{d}(a, r)$.

Assuming isotropic scattering for simplicity, a dust population at distance $r$ from the star scatters a fraction of the stellar flux at wavelength $\lambda$ is given by

$$\Phi_{\text{sc}}(\lambda, r) = F_*(\lambda) \frac{\sigma_{\text{sca}}(\lambda, r)}{4\pi r^2}$$

(3.5)

with $\sigma_{\text{sca}}$ the mean scattering cross section

$$\sigma_{\text{sca}}(\lambda, r) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{sca}} \left( \frac{2\pi a}{\lambda}, \lambda, r \right) dn(r, a)$$

(3.6)
The total flux emitted at wavelength $\lambda$ by the dust population finally reads

$$\Phi(\lambda, r) = \Phi_{\text{sc}}(\lambda, r) + \Phi_{\text{th}}(\lambda, r).$$  \hspace{1cm} (3.7)$$

### 3.2.2 Synthetic observations

We synthesize single aperture photometric observations of non edge-on exozodiacal dust disks at wavelength $\lambda$ by calculating the integral

$$\Phi(\lambda) = \int_{r=0}^{\infty} \Phi(\lambda, r)\Sigma(r) \times 2\pi\langle\text{FOV}\rangle_{\rho}(r) dr$$  \hspace{1cm} (3.8)$$

with

$$\langle\text{FOV}\rangle_{\rho}(r) = \int_{\theta=0}^{2\pi} \text{FOV}(\rho(r, \theta)) \frac{d\theta}{2\pi}$$  \hspace{1cm} (3.9)$$

and

$$\rho(r, \theta) = r\sqrt{1 - \cos^2 \theta \sin^2 i}$$  \hspace{1cm} (3.10)$$

where $r$ and $\theta$ are cylindrical coordinates in the disk plane and $\rho(r, \theta)$ the projected distance to the star in the sky plane. $\Sigma(r)$ is the dust surface number density of the exozodiacal disk, assumed to be axisymmetrical and $i$ is the disk inclination with respect to the sky plane ($i = 0$ for pole-on geometry). In the above equations, we implicitly assumed that the instrument beam profile $\text{FOV}$ only depends upon $\rho$, the projected distance to the star in the sky plane. The $\langle\text{FOV}\rangle_{\rho}(r)$ function gives the azimuthally averaged telescope transmission along circles of radius $r$ in the exozodiacal disk frame.

Most single aperture telescopes such as *Spitzer* or *Herschel*, have sufficiently large beams (or slits in case of spectroscopy), which intercept the entire exozodiacal dust emission. In such a case, the exozodiacal flux emission can be obtained by taking $\langle\text{FOV}\rangle_{\rho}(r) = 1$ in Eq. 3.8. On the other hand, coherent near- and mid-IR interferometric observations, such as those obtained with the VLTI/VINCI and CHARA/FLUOR instruments, have much smaller fields of view (FOV) and the actual transmission profiles of the interferometric instruments on the sky plane is important.

### 3.2.3 Grain properties

In the solar system, the zodiacal dust particles are thought to originate from tails and disruption of comets, or to be produced when asteroids collide. Both interplanetary and cometary dust particles are composed of silicates and carbonaceous material, and zodiacal cloud dust particles are expected to be made of similar material. In this model, we consider mixtures of silicates and carbonaceous material, and calculate their optical properties with the Mie theory valid for hard spheres. The optical index of the mixture is calculated using the Bruggeman mixing rule, given a relative volume fraction $v_C/v_{\text{Si}}$ of carbonaceous grains. In the disk regions where the grain temperature is between the sublimation temperature of silicates and car-

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[^3]: See Fig. I of Paper II for the KIN transmission map
3.3 Dust sublimation model

3.3.1 Sublimation temperatures

We implement a new method of calculating the sublimation temperature $T_{\text{sub}}$ of a dust grain as a function of its size and not only of its composition, based on the method introduced by Kama et al. (2009) to study the inner rim of protoplanetary disks. We first model a grain as an homogeneous sphere of radius $a$ composed of a single material of density $\rho_d$. To ensure the stability of a grain (i.e. no net change of its size), the flux of particles (carbon / silicates monomers) escaping from its surface must equal the flux of particles coming in from the bon, the volume of silicates is replaced by vacuum, mimicking porous carbonaceous grains. The optical constants and grain bulk densities used in this study are summarized in Tab. 3.1.

We adopt a power-law differential size distribution

$$dn(a) = \left( \frac{1 - \kappa}{a_{\text{max}}^{1-\kappa} - a_{\text{min}}^{1-\kappa}} \right) a^{-\kappa} da$$

(3.11)

for grain radii $a$ between $a_{\text{min}}$ and $a_{\text{max}}$. The maximum grain size has no impact at the wavelengths considered and cannot be constrained with the adopted modeling approach. It is thus fixed to $a_{\text{max}} = 1$ mm in the rest of this study.

3.2.4 Fitting strategy

The adopted fitting strategy is based on a Bayesian inference method described in Lebreton et al. (2012). For that purpose, a grid of models is created. For each set of parameters, the goodness of the fit is evaluated with a reduced $\chi^2$ that is transformed into probabilities assuming a Gaussian likelihood function ($\propto e^{-\chi^2/2}$) for Bayesian analysis. Marginalized probability distributions for each free parameter are then obtained by projection of these probabilities onto each dimension of the parameter space.

In order to limit the number of free parameters, we adopt a two power-law radial profile for the surface density

$$\Sigma(r) = \Sigma_0 \sqrt{2} \left[ \left( \frac{r}{r_0} \right)^{-2\alpha_{\text{in}}} + \left( \frac{r}{r_0} \right)^{-2\alpha_{\text{out}}} \right]^{-1/2}. \tag{3.12}$$

This corresponds to a smooth profile, with an inner slope $r^{\alpha_{\text{in}}}$ ($\alpha_{\text{in}} > 0$) that peaks at about $r_0$, and that falls off as $r^{\alpha_{\text{out}}}$ ($\alpha_{\text{out}} < 0$) further to $r_0$.

The parameter space explored here is listed in Tab. 4 of Paper II and recalled in Sec. 3.4.2. Each set of parameters defines an emission spectrum, 2D emission, and scattered light maps that are flux-scaled by searching the optimal $\Sigma_0$ value (surface density at $r = r_0$) that gives the best fit to the panchromatic observations. The range of $a_{\text{min}}$ values includes sizes that are far below the blow-out size limit for grains about Fomalhaut.
ambient medium. From the kinetic theory of gases, the number of particles accreted onto and evaporated from the grain surface per unit time and unit surface reads respectively (Lamy 1974)

\[ F_{\text{accre}} = \frac{P_{\text{gas}}}{\sqrt{2\pi \mu m_u k_B T_{\text{gas}}}} \]  
\[ F_{\text{evap}} = -\frac{P_{\text{eq}}}{\sqrt{2\pi \mu m_u k_B T_{\text{gas}}}} \]  

where \( P_{\text{eq}} \) is the gas saturation partial pressure, \( P_{\text{gas}} \) the partial pressure of the ambient gas medium, \( T_{\text{gas}} \) the gas temperature assumed equal to the dust temperature \( T_d \) here, \( \mu m_u \) the mean molecular weight and \( k_B \) the Boltzmann constant. Introducing an efficiency factor \( \alpha \) (constrained by laboratory experiments), the mass of a grains of radius \( a \) then evolves as

\[ \frac{dm}{dt} = \alpha (F_{\text{accre}} - F_{\text{evap}}) \times \mu m_u 4\pi a^2 \]  

Injecting the ideal gas law \( P = \rho k_B T / \mu m_u \) yields:

\[ \frac{dm}{dt} = \alpha a^2 \sqrt{\frac{8\pi k_B T_d}{\mu m_u}} (\rho_{\text{gas}} - \rho_{\text{eq}}) \]  
\[ \frac{da}{dt} = \frac{\alpha}{\rho_d} \sqrt{\frac{k_B T_d}{2\pi \mu m_u}} (\rho_{\text{gas}} - \rho_{\text{eq}}). \]  

For the purpose of this study, the grains are assumed to lie in empty space (\( \rho_{\text{gas}} = 0 \)) and their temperature equals by definition the sublimation temperature \( T_{\text{sub}} \). The gas density at saturation pressure \( \rho_{\text{eq}} \) is given by the Clausius-Clapeyron equation

\[ \log_{10} \rho_{\text{eq}} = B - \frac{A}{T_{\text{sub}}} - \log_{10} T_{\text{sub}}^{-C} \]  

where the thermodynamical quantities \( A \) and \( B \) are determined from laboratory measurements and \( C = -1 \), as discussed by Kama et al. (2009). Integrating equation 3.17 from the initial grain size \( a \) to 0, under the assumption that the grain temperature \( T_d = T_{\text{sub}} \) does not vary significantly during the sublimation process, yields

\[ \frac{a}{t_{\text{sub}}} = \frac{\alpha}{\rho_d} \sqrt{\frac{k_B T_{\text{sub}}}{2\pi \mu m_u}} 10^{B - \frac{A}{T_{\text{sub}}} - \log_{10} T_{\text{sub}}^{-C}} \]  

where we define \( t_{\text{sub}} \) as the time needed to sublimate an entire grain.

This equation relates the sublimation temperature of a grain to its size and to a sublimation timescale. Although determining this timescale without a time-dependent approach is tricky,
we will see in Sec. 3.3.2 that our parametric approach allows to tackle the issue based on simple assumptions. Beforehand we need to extend the sublimation model to the case of multi-material grains.

We recall that our objective is to interpret real observations of individual debris disks. Fitting the spectral and spatial observables of these disks requires to solve accurately a radiative transfer problem and to model the optical properties of the materials at stake. To achieve this, the GRaTeR code handles grains made of multiple (in particular carbonaceous and silicate) materials by considering homogeneous spheres with optical constants obtained by means of an effective medium theory. Porous grains are represented by a compact sphere in which one of the “materials” is vacuum. When one of the material reaches its sublimation temperature, it is automatically replaced by vacuum. This is in particular the case for the grains that lie very close to the star: then a silicate-carbon mixture becomes a porous carbon grain. This approach proved efficient at reproducing the optical behavior of astronomical dust grains in various situations.

We consider homogeneously distributed mixtures of silicate and carbon characterized by a volume fraction $v_C/v_{Si}$. We anticipate that silicate will sublimate at lower temperatures than carbon whatever the grain size: silicates will have vanished entirely before the carbon starts its sublimation. As a first step the sublimation occurs as if the grains were entirely made of silicates. In a second step, the grains take the form of a carbon matrix filled with cavities. For spheroids, the porosity can be defined as the filling factor of vacuum $P = \frac{V_{\text{vacuum}}}{V_{\text{grain}}}$. We introduce the porosity by correcting the grain density: $\rho_d(P) \rightarrow (1 - P) \rho_d$ in Eq. 3.19 which is then solved using the density of the porous carbonaceous leftover.

Eq. 3.19 is solved using the material properties summarized in Tab. 3.1, and the solution is inverted numerically to determine $T_{\text{sub}}$ as a function of grain size for different timescales. The sublimation curves for pure silicate and carbon-rich grains ($P \approx 0$) is displayed in Fig. 3.1, together with that of porous carbonaceous grains mimicking mixtures of Si and C for which Si has sublimated. We observe that porosity tends to increase $T_{\text{sub}}$, as do smaller sublimation timescales. These are interpreted respectively as a loss of efficiency in the sublimation process for lower grain densities, and as a consequence of the exposure time. Compared with the usual constant $T_{\text{sub}}$ approximation, large and small grains have their sublimation temperatures re-evaluated by as much as plus or minus $\sim$20%.

### 3.3.2 Timescales

From what precedes, we are able to define a size- and composition-dependent estimate for the sublimation temperature of a dust grain. Fig. 3.1 reveals an important deviation from the constant sublimation temperature. However, we first need to know on which timescale the sublimation has to be considered. One can notice that an order of magnitude error on the timescale estimate will result only in a modest shift in the sublimation curve. Sublimation can be expected to alter the steady-state grain size distribution because it introduces an additional (size-dependent) destruction mechanism; nonetheless we take advantage of the fact that we
Figure 3.1: Sublimation temperatures of carbon and silicate as a function of grain size with several possible survival timescales, and assuming different porosities for carbonaceous grains or equivalently different volume fractions of silicate. The horizontal dashed lines show the constant $T_{\text{sub}}$ values often used in past studies.

are using a parametric model and we address only the question: how big must a grain be to survive a temperature $T_{\text{sub}}$ for some time $t$.

A grain lifetime is limited by its removal processes. In a debris disk, the main removal process for bound grains is generally destructive collisions, but when the optical depths and/or the dust stellocentric distance is sufficiently small, Poynting-Robertson (PR) drag can become the dominant effect (Wyatt 2005). Unbound grains are placed on hyperbolic orbits and ejected from the system before they are destroyed: their survival timescale can be equaled to a “blowout timescale”. Assuming near-circular orbits for the parent-bodies, the limit between bound and unbound grains is set by $a_{\text{blow}} = a(\beta = 1/2)$ (Fig. 3.2), where $\beta$ is the size-dependent ratio of radiation pressure to gravitational forces.

Here we want to determine the survival timescale of a grain, in the sublimation zone so depending on its size $a$. The sublimation zone is defined, for each of the materials that the grains are made of, as the interval $[d_1, d_2]$ between the minimum and maximum sublimation distances (all grain sizes considered) assuming constant sublimation temperature. $d_1$ is essentially independent of the maximum grain size $a_{\text{max}}$ because the sublimation distance does not vary with size for grains larger than a few $\mu$m (Fig. 3.4).

We calculate a preliminary grid of models for each grain composition (with no fine computation of the sublimation temperature), and we identify the grain properties and disk surface density that best fit the observations. This provides an estimate of the vertical optical depths required to calculate the collisional timescale: the mean timescale a barely bound
3.3 Dust sublimation model

Figure 3.2: $\beta$ ratio of a dust grain as a function of its size in the Fomalhaut environment. A 50-50 carbon-silicate mixture is assumed and depicted by the lower curve while the upper curve considers half porous carbon grains. The vertical and horizontal lines marks the $\beta = 0.5$ (blowout size for grains released by parent-bodies on initially circular orbits) and $\beta = 1.0$ (blowout size for any initial orbit) limits, before and after the sublimation of silicates.

A grain ($\beta \simeq 1/2$) can survive in a collision-dominated disk:

$$t_{\text{col}}^0(a_{\text{blow}}) = \frac{t_{\text{orbit}}}{2\pi \tau_{\text{geo}}^\perp}$$

(3.20)

with $t_{\text{orbit}}$ the orbital period and $\tau_{\text{geo}}^\perp$ the vertical geometrical optical depth at distance $d_1$, a distance representative of the sublimation distance of bound grains.

For larger grains, the collision timescale is scaled with size as (Thébault & Augereau 2007)

$$t_{\text{col}}(a) = t_{\text{col}}^0 \left( \frac{a}{a_{\text{blow}}} \right)^{0.3}$$

(3.21)

where $t_{\text{col}}$ depends on the sublimation zone of a given material and on the assumed surface density profile.

We define the PR drag timescale as the time needed for bound grains to spiral from the outer edge of the sublimation zone, to the inner edge of the sublimation zone (Wyatt 2005)

$$t_{\text{PR}}(a) = 400 \beta(a) \frac{(d_2 - d_1)^2}{M_*}.$$  

(3.22)

As soon as they are produced, unbound grains ($\beta > 1/2$) are placed onto hyperbolic orbits, they are ejected from the sublimation zone and eventually from the field of view. In
the sublimation zone, the hyperbolic orbit is approximated by rectilinear uniform motion; we assume the grains are produced at the inner edge of the sublimation zone $d_1$ with initial velocity $v_{\text{Kep}}(d_1)$. A grain will travel typically a distance between $\sqrt{d_2^2 - d_1^2} (\beta(a) = 1)$ and $d_2 - d_1 (\beta \gg 1/2)$. An estimate of the blowout timescale is given by the average between these two extremes (see Eq.3.31 and Appendix 3.A for a more general estimate)

$$t_{\text{blow}} = \frac{1}{2} \left[ \frac{(d_2^2 - d_1^2)^{0.5}}{v_{\text{Kep}}} + \frac{d_2 - d_1}{v_{\text{Kep}}} \right].$$ (3.23)

Eventually, the sublimation timescale is equaled to the survival timescale, namely the longest time a grain can be exposed to sublimation, according to

$$t_{\text{sub}}(a) = \begin{cases} \min(t_{\text{col}}(a), t_{\text{PR}}(a)) & \text{, for } a > a_{\text{blow}} \\ t_{\text{blow}} & \text{, for } a \leq a_{\text{blow}} \end{cases}$$ (3.24)

A representative example for Fomalhaut is shown in Fig. 3.3. The properties of the silicate population and of the carbon population described in details in Sec. 3.4 have been assumed. $t_{\text{sub}}(C)$ is always smaller than $t_{\text{sub}}(Si)$ because of the respective locations of the two grain populations. The sharp discontinuity between the survival timescale of bound and unbound grains translates into a big jump in the sublimation temperatures and then the distances at the blowout limit. We stress that Fig. 3.3 only illustrates the survival time in the sublimation zone, it should not be used to find the dominant mechanism in the entire dust disk.

### 3.3.3 Sublimation distances

The GRaTeR code handles the optical properties of various materials. Broad-band spectra and excesses attributed to warm circumstellar dust disks are well matched using materials of the silicate family or carbonaceous materials. For each of them, we use respectively the thermodynamical quantities associated to amorphous olivine ($\text{MgFeSiO}_4$) and graphite (C). They are indicated in Tab. 3.1.

After equaling the sublimation timescale to a timescale relevant for each grain (Sec. 3.3.2), we calculate the thermal equilibrium using the usual formula (Eq. 3.4). This provides the size dependent-sublimation distance presented in Fig. 3.4 for a few representative material mixtures. The overall shape of the curves is dictated by the thermal equilibrium distances. The net effect of the new model is generally an alteration of the sublimation distances of the small ($< 10\mu\text{m}$) grains, depending on the timescales used for each model. The inner edge of the sublimation is close to 0.2 AU for silicate, 0.05 AU for carbon, while their outer edge lies respectively at 0.7 - 1.2 AU, 0.18 - 0.25 AU respectively.
Figure 3.3: Examples of survival timescales against collisions, PR drag and radiation pressure blowout calculated for an exemplary grain composition ($v_C = v_S$ for silicate, $P = 50\%$ for carbonaceous material). For each effect, the upper curve corresponds to silicate and the lower curve to carbon, each in its sublimation zone and for the disk properties derived from preliminary modeling. The horizontal black lines give the orbital periods for reference. The vertical lines mark the blowout size before (left) and after (right) silicate sublimes.

**Table 3.1: Material properties.**

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Carbonaceous material</th>
<th>Silicates</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Graphite</td>
<td>Olivine</td>
</tr>
</tbody>
</table>

| Thermodynamical properties | | |
|----------------------------|----------------------------|
| 37215                      | 1.3028                    | 12.0107   |
| 128030                     | 12.471                    | 172.2331  |
| $\mu [m_p]$               | 12.0107                   | 172.2331  |
| 1.95                       | 3.5                       |

| Optical properties | aC ACAR | Astrosilicates |

**Notes.** The thermodynamical constants $A$ and $B$ are tabulated values from Kama et al. (2009) and Zavitsanos & Carlson (1973).
3.3.4 Sublimation model: summary

We presented an innovative model that is used to calculate size-dependent sublimation distances of dust grains depending on their compositions, porosities and survival timescales. The method can be summarized as follows:

1. A preliminary grid of models is adjusted to the data assuming fixed sublimation temperatures and solving the size-dependent equilibrium temperatures. This provides estimates of the dust disk location and density (Paper II).

2. Size-dependent survival timescales are estimated by searching for the most efficient effect between destructive collisions, PR drag and photo-gravitational blowout (Fig. 3.3).

3. Size-dependent sublimation temperatures are calculated from the kinetic theory of gases and thermodynamics as a function of the timescale for when a grain is exposed to sublimation (Fig. 3.1).

4. This sublimation timescale is equaled to the survival timescale for each grain yielding a size-dependent sublimation temperature that accounts for each grain specific dynamical regime. In particular, this causes a discontinuity at the blowout limit (Fig. 3.2).

5. Finally, size-dependent sublimation distances are re-evaluated by solving the equilibrium distance of each grain knowing its specific sublimation temperature (Eq. 3.4).

3.4 Application and results

3.4.1 Observational constraints and star properties

The model presented above is now confronted with the observations of Fomalhaut already presented in Table 3 of Paper II. Preliminary analysis of the near-infrared VLTI/VINCI excess, of mid-infrared KIN null excesses, complemented by (low accuracy) spectrophotometric data, suggested the coexistence of two populations of dust in the Fomalhaut environment located within the field of view of the interferometers (~ 4 AU and ~ 2 AU HWHM for FLUOR and KIN respectively). This architecture is imposed by the high level of the K-band excess with respect to N-band, and the inversion of the null excess slopes upon ~ 10µm, with a rising profile toward the longer wavelengths. Stark et al. (2009) observed a similar break around 10 µm in the Keck null depths of 51 Ophiuchi. They were only able to reproduce it with a double-population of dust grains, however with much larger spatial scales due to the distance of the star. In Paper II, the two dust populations of Fomalhaut were adjusted separately and the model suffered from an inaccurate prescription for the sublimation distances. In the present study we propose a self-consistent modeling of the exozodi and we characterize precisely the properties of the emitting grains and their location. We assume the disk position
3.4 Application and results

Figure 3.4: Sublimation distances of silicate (top panel) and carbonaceous material (bottom panel) in composite grains (C+Si or C+vacuum respectively) as a function of grain size in the Fomalhaut environment, with either a constant (dashed line) or a size-dependent (solid line) sublimation temperature. Relevant disk properties obtained from preliminary modeling are used (Sec. 3.4). Size-dependent sublimation temperatures are a function of the size-dependent survival timescales shown in Fig. 3.3 causing jumps at the material-dependent blowout limit.
angle and inclination to equal those of the cold ring \((i = 65.6^\circ, \text{PA} = 156^\circ)\). A NextGen photosphere (Hauschildt et al. 1999) is scaled to the V magnitude of Fomalhaut \((m_V = 1.2 \text{ mag})\) in order to model the total flux received by the grains. It also serves to estimate the excesses attributable to the disk in the photometric data. The interferometric observables on the other hand are independent from the chosen star spectrum.

### 3.4.2 Data and modeling strategy

We adopt a strategy in which we fit (1) the “hot dust ring” \((\lesssim 0.4 \text{ AU})\) probed by the shortest wavelengths data, using the same data subset as Paper II (most importantly the VINCI 2.18 \(\mu\text{m}\) excess), and then (2) the “warm dust belt” probed by the KIN nulls from 8 to 13 \(\mu\text{m}\) and mid- to far-infrared photometric measurements of the warm “on-star” excess (Fig. 3.5 and 3.6).

These photometric measurements are derived from unresolved observations of the inner Fomalhaut debris disk, well differentiated from the contribution of the cold belt by Spitzer/MIPS at 23.68\(\mu\text{m}\) \((F_{24} = 3.90 \pm 0.40 \text{ Jy}, \text{Stapelfeldt et al. 2004})\), Herschel/PACS at 70\(\mu\text{m}\) \((F_{70} = 0.51 \pm 0.05 \text{ Jy}, \text{Acke et al. 2012})\) and ALMA at 870\(\mu\text{m}\) (taken as a 3\(\sigma\) upper limit: \(F_{850} < 4.8 \text{ mJy}, \text{Boley et al. 2012}\)); their spatial location is constrained by the instruments point spread functions / beam to be smaller than \(\sim 20\text{AU}\). Additional Spitzer-IRS spectroscopic data is available: a small excess emission \((\lesssim 1 \text{ Jy})\) shortward of 30 \(\mu\text{m}\) is reported by Su et al. (2013). Due to large calibration uncertainties in the absolute photometry \((\sim 10\%)\) compared with the interferometric measurements, we do not attempt to fit this spectrum. The IRS spectrophotometry is represented with 3\(\sigma\) upper limits in Figure 3.5 to verify the compatibility of the model at these wavelengths.

The GRaTeR code is used to adjust the dataset in several steps. We point out that all the results presented below correspond to thermal emission by the dust and that the scattering of the stellar spectra by the grains is always negligible (although systematically calculated).

As a first step, we assess the hot population, assumed to be composed of compact carbonaceous grains, because they are very small and lie within the silicate sublimation zone (Paper II). We adopt the same grid of models as the one presented in Table 4 of Paper II, yielding \(~200,000\) models for 5 free parameters: the minimum grain size \(a_{\text{min}}\), the slope of the size distribution \(\kappa\), the surface density peak \(r_0\), outer slope \(\alpha_{\text{out}}\) and the total disk mass \(M_{\text{dust}}\) (with the maximum grain size fixed to 1 mm). The inner density slope is assumed to be steep \((\alpha_{\text{in}} = +10)\) - but this has no impact as this region is within the sublimation radii of carbons as we will see.

As a second step we focus on the disk’s warm component and take advantage of the linearity of the data that allows us to sum the contributions from the two components. Using the same parameter space, we adjust simultaneously 6 free parameters: the mass, geometry and disk properties of the warm component and the mass of the hot component (modeled in step...
3.4 Application and results

![Graph showing spectral energy distribution](image)

**Figure 3.5**: Measured spectral energy distribution, and SED of the best-fitting double-dust belt model (green: hot ring, red: warm belt, blue: total). **Top panel**: global SED including a NextGen photosphere model (solid black line), **bottom panel**: circumstellar excess emission. From left to right, thick squares denote the VLTI/VINCI 2.18μm excess, MIPS 24 μm and Herschel/PACS 70 μm photometry with 1σ error bars. Spitzer/IRS spectrum (grey crosses, not fitted) and ALMA 870 μm photometry (purple cross) are shown as 3σ upper limits.

1) to the 36 measurements composing the full dataset (VINCI, KIN, MIPS, PACS, ALMA). The warm ring is made of a 50-50 mixture of astronomical silicates and carbonaceous material characteristic of the material properties commonly inferred from debris disks and solar system asteroids studies. We do not vary this parameter as it is found to be essentially unconstrained by the observations in the previous study.

Two approaches are used. In the first approach, the outer slope of the density profile is taken as a free parameter, and the inner slope is assumed to be very steep (fixed to +10). In the second approach, the outer slope is fixed to $-1.5$ - close to the best-fit for the first approach and consistent with the profile expected for a collisional equilibrium under the effect of size-dependent radiation pressure (Thébault & Wu 2008) - and we vary the inner slope $\alpha_{in}$.

A $\chi^2$ minimization is performed as well as a Bayesian statistical analysis to measure the likelihood of the model parameters. The best-fitting parameters and Bayesian estimates discussed in this section are summarized in Tab. 3.2 and the resulting models are shown in Fig. 3.5 and 3.6. Probability curves are shown in Appendix B.

### 3.4.3 The hot population

As expected, the hot component is mostly constrained by the near-infrared data and best explained by a very narrow ring of $\gtrsim 10$ nm refractive grains, with a density profile peak...
Figure 3.6: Measured KIN excess null depths and nulls of the best-fitting double-dust belt model (green: hot ring, red: warm belt, blue: total) for the four data subsets. The grey regions denote the 1σ confidence intervals on the data.

matching the sublimation distance of carbons ($r_0 = 0.09$ AU), where only the largest grains survive. The best model ($\chi^2 = 1.4$ with 20 degrees of freedom, $d.o.f.$) is found for a minimum grain size matching the smallest values of the parameter space ($a_{\text{min}} = 10$ nm, $\kappa = -6$), although the Bayesian analysis favors $a_{\text{min}} = 20$ nm. In fact, the exact properties of this ring have little impact on the resulting SED, as long as the grain size-dependent temperatures and total mass are consistent with the K-band excess. The slopes of the density profile and of the size distribution are qualitatively very steep but their exact values are not well identified due to the strong dependence of temperature with distance and grain size. However, the data
also carries spatial information. In particular the dust must be confined within the (Gaussian) field-of-view of the VLTI and it must not let too much emission through the complex KIN transmission map in the mid-infrared. Grains larger than $\sim \lambda/2\pi \approx 0.3 \mu m$ are also inefficient emitters in the K-band, and due to the single power law used, very small grains can become dominant. For these reasons and despite the modeling degeneracy between grain size and disk location, the minimum grain size cannot exceed a few $\sim 10$ nm. For instance, the best model found with $a_{\text{min}} = 1 \mu m$ has a $\chi^2_r$ of 2.9. This result is reminiscent of what was found by Defrère et al. (2011b) who show that the exozodi of Vega must be composed of grain much smaller than 1 $\mu m$ based on multiwavelength constraints in the near-IR. We use $a_{\text{min}} = 10$ nm as a working assumption for the rest of the study although what really matters is to determine which of the grains are the emitters.

In the inner solar system, the dust probed by the NASA's deep Impact mission likely consists of $\sim 20 \mu m$-sized highly porous dust aggregates (e.g. Kobayashi et al. 2013). The hot exozodi of Fomalhaut rather consists of nanometer to submicrometer grains that we interpret as the elemental monomers produced after the break-up of larger dust particles. As demonstrated in Paper II (Fig. 6 and Tab. 5), models with higher porosity provide less good fit to the data. For instance, setting the porosity to 85 % yields a smallest $\chi^2_r$ of 1.8 for $a_{\text{min}} = 1.5 \mu m$. A large amount of sensitive measurements would be needed to go beyond this solid carbon grain model (see e.g. Lebreton et al. 2012).

The new sublimation prescription provides a more reliable estimate of the dust location with respect to the constant sublimation temperature assumption (Fig. 3.7). With $\alpha_{\text{out}} = -6$ (i.e. a very narrow ring), the sublimation distance of 0.01 $\mu m$ grains is 0.235 AU (\(\simeq d_2\)); for these grains the emission falls down to 10% of the peak flux at 0.34 AU. Grains larger than $a_{\text{min}}$, in particular those in the range 0.1-0.5 $\mu m$ that lie close to $d_1$ contribute to thermal emission in similar proportions compared with 10 nm grains. Thus, independent from our choices for the parameter space limits, the emission is by far dominated by unbound grains and a robust result is that this hot exozodi is composed of grains smaller than $\sim 0.5 \mu m$. The dust mass is dominated by the smallest grains, due to $\kappa < -4$. We would like to stress that forcing the material sublimation temperatures to higher values would not yield better fit to the data as it would only move the peak emission toward even shorter wavelengths (Tab. 3.2, Fig. 3.5). In the following, we adopt the above parameters for the hot ring and keep the total dust mass as the single free parameter for this component (Tab. 3.2).

### 3.4.4 The warm population

With the first approach (variable outer slope, fixed inner slope), the analysis indicates that the mid- to far-infrared data is best fitted by a dust belt peaking at $r_0 = 1.6$ AU and declining slowly as $r^{-1}$. The minimum size is close to the blowout limit ($d_{\text{blow}} = 8.8 \mu m$), in a rather steep distribution ($\kappa < -3.8$), yielding a reduced $\chi^2$ of 1.56 (with 30 d.o.f.). These properties are consistent with the theoretical expectations that the warm population is produced through a collisional cascade in a parent-body belt, although the steep distribution might be more
Figure 3.7: Maps showing the absolute distribution of flux as a function of distance from the star and grain size. **Top panel**: $\lambda = 2.18$ µm, **bottom panel**: $\lambda = 12.0$ µm. The orange and green lines mark the sublimation distances of silicate and carbon grains respectively.

indicative of a recent catastrophic collision than a steady-state debris disk.

Interestingly, the second approach (fixed outer slope, variable inner slope) yields similar results to those of the first one, except that a second family of solutions arises, with very small grains located at the outer edge of the range of explored peak radii. Nonetheless this solution can be excluded based on the absence of silicate features in the Spitzer/IRS spectrum that would be created by such tiny silicate grains. We add prior information in the Bayesian analysis to reject solutions with $a_{\text{min}} < a_{\text{blow}}/10$ (Appendix B.2) and find a best reduced $\chi^2$ of 1.60. Due to the different geometrical profile assumed, the peak radius is found to lie further out, at $\sim 2.5$ AU. The inner ring is not as steep as previously assumed ($\alpha_{\text{in}} = +3$), which is suggestive of an inward transport of material by Poynting-Robertson drag mitigated by destructive collisions (See for example Löhne et al. 2012).
3.5 Origin of the dust

In this section, we test several mechanisms as possible explanations for the peculiar morphology suggested by the modeling results. First, we discuss the viability of parent belts at the locations of the hot ring and the warm belt (Sec. 3.5.1). Subsequently, we review possible replenishing mechanisms for the warm belt (Sec. 3.5.2), and test whether the pile-up of dust in the sublimation zone can explain the hot ring (Sec. 3.5.3). In Sec. 3.5.4, we examine whether the observed population of unbound grains can be produced by the disruption of larger bodies due to sublimation. Finally, we investigate the role of gas in retaining the small grains in the hot ring by slowing down both their blowout and sublimation (Sec. 3.5.5). A short summary of our findings is presented in Sec. 3.5.6.

To compare theory with observation, we approximate the fractional luminosity of dust at radial distance $r$ as the fraction of the star covered by dust at that location:

$$\frac{L_D}{L_*} (r) \approx \frac{\sigma_D(r)}{4\pi r^2}. \quad (3.25)$$

Here, $\sigma_D(r)$ is the collective cross section of the dust at radial distance $r$. When the excess flux can be assumed to be entirely due to spherical grains of a single size $a$ (and hence with a
The Fomalhaut inner debris disk

Table 3.2: Properties of the Fomalhaut exozodiacal disk derived from the fit to the data.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Hot ring</th>
<th>Warm belt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach 1</td>
<td>Approach 2</td>
<td>Approach 1</td>
</tr>
<tr>
<td>Density peak</td>
<td>$r_0$</td>
<td>$r_0$</td>
</tr>
<tr>
<td></td>
<td>(AU)</td>
<td>(AU)</td>
</tr>
<tr>
<td>Inner density slope</td>
<td>$\alpha_{\text{in}}$</td>
<td>$\alpha_{\text{in}}$</td>
</tr>
<tr>
<td>Outer density slope</td>
<td>$\alpha_{\text{out}}$</td>
<td>$\alpha_{\text{out}}$</td>
</tr>
<tr>
<td>Mass up to 1 mm</td>
<td>$M_\oplus$</td>
<td>$M_\oplus$</td>
</tr>
<tr>
<td>Maximum surface density</td>
<td>$\chi_2$ (dof = 30)</td>
<td>$\chi_2$ (dof = 30)</td>
</tr>
</tbody>
</table>

Notes: Approach 1 labels the model with free outer slope, approach 2 that with free inner slope. Numbers in bold are the parameters of the smallest $\chi^2$ fits, while the intervals give the 1σ confidence intervals from the Bayesian analysis. The * exponent marks the fixed parameters.

[1] Approximate density peak position of the smallest grains (they vanish below that distance).
[3] The optical depth is directly proportional to the dust mass and therefore constitutes another definition of the mass parameter (given in V-band by convention).

**Table 3.2** Properties of the Fomalhaut exozodiacal disk derived from the fit to the data.
Figure 3.8: Radial profiles for the best-fitting double-ring model (approach 1. Red: hot ring; green: warm belt): geometrical vertical depth profiles (top panel), optical depth profiles at $\lambda = 2.18\mu m$ (middle panel) and $\lambda = 12\mu m$ (bottom panel). The dashed regions represent the sublimation zones of silicates (blue region) and carbons (yellow). The effect of size-dependent sublimation is evidenced by the ragged profile of the hot ring optical depth. A logarithmic sampling is used with 56 sizes ranging from 0.01 to 1000 $\mu m$.

mass of $m = 4\pi\rho_d a^3/3$), the fractional luminosity is related to the total dust mass according to

$$\frac{L_D}{L_*}(r) \approx \frac{3M_D}{16\pi\rho_d a r^2}.$$  

(3.26)
3.5.1 In-situ dust production through a collisional cascade?

Excess infrared emission from debris disks is normally interpreted as thermal emission from dust produced through mutual collisions between larger bodies in a planetesimal belt. Therefore, we first test whether the observed NIR excess can be explained by the production of small dust grains by asteroid belts at the locations of the hot ring and the warm belt.

In steady-state collisional evolution, a planetesimal belt at a given radial distance from the star can only contain a maximum amount of mass at any given time, because more massive belts process their material faster (Dominik & Decin 2003; Wyatt et al. 2007a). Assuming that the size distribution follows the classical Dohnanyi (1969) power law ($\kappa = -3.5$, valid if the critical specific energy for dispersal $Q^*_D$ is independent of particle size) at all sizes, and extends down to the blowout size, the mass corresponds to a maximum fractional luminosity of (Wyatt et al. 2007a)

$$\max \left[ \frac{L_D}{L_*} \right] = 7.0 \times 10^{-9} \left( \frac{r}{1 \text{ AU}} \right)^{7/3} \times \left( \frac{dr/r}{0.5} \right) \left( \frac{a_c}{30 \text{ km}} \right)^{0.5} \left( \frac{Q^*_D}{150 \text{ J kg}^{-1}} \right)^{5/6} \left( \frac{e}{0.05} \right)^{-5/3} \times \left( \frac{M_*}{1.92 M_\odot} \right)^{-5/6} \left( \frac{L_*}{16.63 L_\odot} \right)^{-5/6} \left( \frac{t_{age}}{440 \text{ Myr}} \right)^{-1}. \quad (3.27)$$

Here, we inserted fiducial values for the relative width of the planetesimal belt $dr/r$, the radius of the largest bodies $a_c$, the critical specific energy for dispersal $Q^*_D$, and the mean planetesimal eccentricity $e$.\(^4\) For the stellar mass $M_*$, the stellar luminosity $L_*$, and the age of the system $t_{age}$, we used the parameters of Fomalhaut found by Mamajek (2012). Cratering collisions, which have a specific energy lower than $Q^*_D$, lead to an increased erosion of large bodies, and therefore a faster processing of the available material, and a lower maximum fractional luminosity at any given age. However, they are not accounted for in the model of Wyatt et al. (2007a). Including cratering collisions would lower the numerical factor in Eq. 3.27 by about a factor 4 to 5 (Kobayashi & Tanaka 2010). Our choice of $Q^*_D = 150 \text{ J kg}^{-1}$ can be seen as a conservative estimate.

Evaluating Eq. 3.27 at the radial locations of the hot ring ($r \approx 0.25 \text{ AU}$) and the warm belt ($r \approx 2 \text{ AU}$), gives maximum fractional luminosities of $2.7 \times 10^{-10}$ and $3.5 \times 10^{-8}$, respectively. We note that the modeling results indicate steeper size distributions ($\kappa < -4.0$) for both components, which would yield much higher maximum fractional luminosities (using Eq. 16 of Wyatt et al. 2007a). However, the observations only probe the lower end of the size distribution, and it is unlikely that the steep power law extends all the way to parent body sizes. In contrast, the size distribution is expected to be shallower at large sizes, where the strengthening of bodies due to self-gravity becomes important. Furthermore, the theoretical $\kappa = -3.5$ is confirmed observationally for km-sized objects in the solar system’s asteroid

\(^4\) These values were found to give a good fit to debris disks around a sample of A stars (Wyatt et al. 2007b), and can be used as first order estimates for these poorly constrained parameters in the case of exozodiacal dust.
3.5 Origin of the dust

In a more thorough discussion of the stringentness of the maximum fractional luminosity, Wyatt et al. (2007a) find that by pushing the parameters, the fractional luminosity can be made to exceed the maximum given by Eq. 3.27 by a factor 100 at most. Since the fractional luminosities derived from modeling the interferometric data (Tab. 3.2) are more than two orders of magnitude higher than the maximum ones, we conclude that neither of the two components can be explained by in-situ asteroid belts that have been in collisional equilibrium for the age of the system.

3.5.2 Replenishing the warm belt

If the warm belt is in steady state (and not a transient phenomenon), some mechanism must operate to replenish the observed dust, other than an in-situ collisional cascade. A possible source of the material is the outer cold belt at about 140 AU. We now proceed to estimate the rate at which the dust needs to be replenished, and then examine whether PR drag from the cold belt is capable of providing this mass flux.

The warm belt is found to have a relatively steep size distribution ($\kappa < -4.0$), and therefore its total dust mass is dominated by the smallest grains present. In the warm belt these are particles close to the blowout size, which are destroyed by collisions on a timescale of several thousands of years (Eq. 3.21). With a dust mass of a few times $10^{-6} M_\oplus$, the mass flux through the warm belt must be of the order of $10^{-9} M_\oplus \text{yr}^{-1}$.

PR drag can only supply a limited amount of material, since grains undergo mutual collisions as they migrate inwards, and the fragments produced in these collisions can be blown out. This was demonstrated by Wyatt (2005), using a model that assumes a single particle size, fully destructive collisions, and circular orbits. Based on this model, we can estimate the collision-limited PR drag mass flux from a dust source located at $r_{\text{source}}$ inward to a radial distance $r$, which is

$$\max \left[ M_{\text{PR}}(r) \right] = \frac{L_* \sqrt{GM_* \beta Q_{\text{pr}}}}{2c^3 \left( \sqrt{r_{\text{source}}} - \sqrt{r} \right)}.$$  (3.28)

Here, $L_*$ is the stellar luminosity, $G$ is the gravitational constant, $M_*$ is the stellar mass, $Q_{\text{pr}}$ is the wavelength-averaged radiation pressure coefficient, $c$ is the speed of light. A detailed derivation of this equation is presented by van Lieshout et al. (2014a). This maximum is independent of the amount of material at the dust source. Also, its dependence on grain properties is through $Q_{\text{pr}}$ and $\beta$, for which we know $0 < Q_{\text{pr}} < 2$, and $\beta < 0.5$ for dust released from large ($\beta \approx 0$) parent bodies on circular orbits.

We evaluate Eq. 3.28 at the location of the warm belt ($r \approx 2 \text{ AU}$), using $r_{\text{source}} = 140 \text{ AU}$, the stellar parameters of Fomalhaut, $Q_{\text{pr}} = 1.0$, and $\beta = 0.5$. The $\beta$ ratio is that of the smallest bound grains, which are dragged in the most efficiently, and can therefore provide the highest mass flux. The resulting maximum mass flux is about $1.2 \times 10^{-12} M_\oplus \text{yr}^{-1}$, which is several

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5 The model of Wyatt (2005) ignores stellar wind drag, but this process is not expected to be important for Fomalhaut and other A stars, because their mass loss rates are predicted to be very low.
orders of magnitude smaller than the mass flux required to maintain the warm belt. This indicates that PR drag is not the main replenishing mechanism for the warm belt.

Another possible mechanism for transporting material from the outer cold belt to the warm belt is the inward scattering of small bodies by planets (Bonsor & Wyatt 2012; Bonsor et al. 2012). The outer belt mass is constrained to be between 4 $M_\oplus$ (Boley et al. 2012) and 110 $M_\oplus$ (Acke et al. 2012) and the star age is 440 $\pm$ 40 Myr (Mamajek 2012). If a chain of planets were to orbit between the cold and warm belts, these planets could scatter small bodies inwards, in essence resupplying the warm and hot belt with material. Bonsor et al. (2012) used N-body simulations to investigate this scattering process and determined a maximum flux of scattered particles as a function of time. This maximum occurs if a chain of tightly packed, low mass planets, were to orbit between the belts. Here, we apply these models to the case of Fomalhaut, scaling the simulations to account for the stellar mass of 2.1 $M_\odot$ and the inner edge of the outer belt at 133AU. The results are shown in Fig. 3.9. The important information given by this plot is the maximum possible rate at which material can be scattered inwards, given by the upper envelope of the scattered points. By assuming that the material is efficiently converted to small dust, this also gives us the maximum rate at which the observed small dust in the hot and warm belts could be resupplied. This process could potentially provide a mass flux of up to $5 \times 10^{-11} M_\oplus$ yr$^{-1}$ continuously for the age of the system, which is still not sufficient to compensate the collision rate ($4 \times 10^{-10} M_\oplus$ yr$^{-1}$). This value is
very much a maximum mass flux, as it was calculated using the planet configuration that is the most efficient at scattering material inwards and it assumes that the scattered objects are entirely disrupted upon entering the inner regions of the Fomalhaut planetary system.

### 3.5.3 Accumulation of sublimating dust grains?

For the hot ring, the radial distribution of dust exhibits a strong peak in surface density in the carbon sublimation zone, with much lower levels of dust further out (see Fig. 3.8). This spatial profile suggests the existence of a mechanism to confine the carbonaceous dust in the sublimation zone.

Kobayashi et al. (2009) predicted that the dual effect of PR drag and radiation pressure blowout can result in an accumulation of grains around the sublimation radius. In the following we briefly explain the pile-up mechanism in terms of the three stages identified by Kobayashi et al. (2009). (1) Initially, the grains are far away from the star, and sublimation is negligible. The grains spiral inward due to PR drag and gradually heat up as they come closer to the star. (2) As the dust temperature approaches the sublimation temperature, the grains start to lose mass due to sublimation. As a consequence of the increasing cross-section-to-mass ratio of the dust grains, radiation pressure gains in relative importance compared to gravity (i.e. the $\beta$ ratio of the dust grains becomes higher). This increases the eccentricity of the dust orbits, and therefore their orbital size. Hence, the inward radial migration is slowed down. Eventually, the decrease in semi-major axis due to PR drag is compensated by the increase due to sublimation. This happens roughly at the radial distance where the PR drag timescale equals the sublimation timescale (Kobayashi et al. 2008). (3) Finally, the size of the dust grains drops below the blowout radius, and their orbits become unbound. At this point the grains either leave the system, or they fully sublimate before they exit the sublimation zone. The net outcome of this process is an accumulation of dust in the sublimation zone, and this result is very robust against various grain properties (composition, porosity, fractal structure).

The pile-up mechanism is a result of the interplay between PR drag and sublimation, and was investigated for drag dominated disks (i.e. disks in which collisions are unimportant). If collisions are significant, they may inhibit the process of dust pile-up. Furthermore, a significant pile-up requires very low orbital eccentricities $e \lesssim 10^{-2}$ (Kobayashi et al. 2008, 2011). PR drag can circularize the orbits of dust particles, but only if the source region is distant enough. These caveats are examined in further detail by van Lieshout et al. (2014a).

The material in the pile-up needs to be replenished from further out by PR drag. Assuming that a dust source is located at the radius of the warm component ($r_{\text{source}} = 2$ AU), without specifying how this source can be maintained, we now use Eq. 3.28 to estimate how much material can be transported inward to the hot ring at $r = 0.23$ AU.$^6$ Inserting the parameters

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$^6$ In the best fit model, two populations of dust contribute equally to the hot component emission, one at $r = 0.1$ AU, and one at $r = 0.23$ AU. The latter is most optimistic choice for the PR drag scenario, resulting in the highest mass flux.
of Fomalhaut, together with $Q_{pr} = 1.0$ and $\beta = 0.5$, yields a maximum mass flux of $1.4 \times 10^{-11} \, M_\oplus \, yr^{-1}$.

If the orbital eccentricities of the dust grains are sufficiently low, a pile-up of dust will occur at roughly the radial distance where the PR drag timescale equals the sublimation timescale. The dust stays in the pile-up until it is completely sublimated, or sublimated to below the blowout size, which approximately takes a sublimation timescale. Therefore, the dust stays in the pile-up for a PR drag timescale.

A rough estimate of the maximum total mass of piled up dust can be found by multiplying the maximum mass flux rate found earlier with the PR drag timescale at the pile-up location. For $r = 0.23$ AU, the PR drag timescale is 22 yr, resulting in a maximum dust mass of $3.1 \times 10^{-10} \, M_\oplus$, which is compatible with the modeled hot component mass. However, the maximum fractional luminosity due to this amount of mass in bound grains at this location (Eq. 3.26), is only $9.2 \times 10^{-6}$, which is several orders of magnitude lower than the value of the best fit model. This estimate is independent of the amount of material at the dust source, and only depends on grain properties through $\beta$, which should not be higher than $\beta = 0.5$. The reason for the discrepancy is that the dust grains found by the model are much smaller than the $\beta = 0.5$ particles considered for the pile-up mechanism, and therefore constitute much more cross section for the same amount of mass. Since these small grains are below the blowout size, they are removed from the system on timescales much shorter than the PR drag timescale. Hence, the pile-up of dust alone cannot explain the observed excess emission of the hot component.

### 3.5.4 The release of small dust grains in the sublimation zone

The observation of dust grains with sizes far below the blowout size presents a problem. These particles have very short survival timescales, and therefore have to be replenished quickly. Their detection in the sublimation zone indicates that they could be released from unseen larger bodies that fall apart as they sublimate. The increase in the number of particles, with conservation of total mass, would lead to an increase in collective cross section, and the steep dependence of sublimation on temperature could explain the sharp peak in emission in the sublimation zone.

Larger bodies could be transported into the sublimation zone by various processes, such as P-R drag, or inward scattering by planets. Alternatively, the small particles could be released by an evaporating planet that is present in the sublimation zone for an extended period and gradually loses material. For now, we ignore what is the exact mechanism that provides the material, but rather focus on the mass source term $\dot{M}$ required to explain the observed

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7. Individual dust grains survive for longer than a sublimation timescale, since they end up on eccentric orbits that take them out of the sublimation zone. However, the time they spend in the pile-up, where they are observed, corresponds to the sublimation timescale.

8. Very small grains sometimes have $\beta$ ratios below unity, due to their low optical efficiencies. However, for the material types tested here, and Fomalhaut as the host star, $\beta$ stays well above unity, and the smallest grains are unbound (see Fig. 3.2).
fractional luminosity.

The fractional luminosity of the hot ring can be approximated by Eq. 3.26. This is possible because its size distribution is very steep, so the cross section is dominated by the smallest grains, and because its radial distribution is a sharp spike, so all grains are roughly located at the same radius. Furthermore, because of the steep size distribution ($\kappa < -4.0$), the total dust mass in the hot ring is dominated by the smallest grains. Hence, the modeling results provide a rough constraint on the product of the mass source term $\dot{M}$ and the survival timescale of the small grains $t_{\text{surv}}$, according to

$$\frac{L_D}{L_*} = \frac{3\dot{M}t_{\text{surv}}}{16\pi \rho_d a r^2}.$$  

(3.29)

A closer inspection of the modeling results reveals that the flux of the hot ring is mainly due to two populations of dust, which have similar contributions (see Fig. 3.7). The reason for this complication is that the radial distribution is truncated at the size dependent sublimation radius. One population consists of very small grains ($a = 0.01 \mu m$) located at $r = 0.23$ AU, the other of slightly larger grains ($a = 0.2 \mu m$) at $r = 0.10$ AU. These slightly larger grains are still well below the blowout size. In the following we will test both sets of parameters when evaluating the survival timescale. For the material density we assume that of carbon with a porosity of 5% ($\rho_d = 1.85 \text{ g cm}^{-3}$).

We now make an estimation of the survival timescale of the relevant dust grains, in order to find the required mass production or influx to explain the observed fractional luminosity. The mechanisms that are critical to the survival of small grains close to a star are sublimation and blowout. Sublimation destroys the grains on a timescale of

$$t_{\text{subl}} = \frac{a}{|\dot{a}|},$$  

(3.30)

where $\dot{a}$ is given by Eq. 3.17. This is the time it takes for a grain to sublimate, assuming the sublimation rate remains constant. $t_{\text{subl}}$ is highly dependent on $r$ (through dust temperature), and therefore uncertain. For the blowout timescale, we take

$$t_{\text{dyn}} = \sqrt{\frac{5r^3}{2GM_* (\beta - 1)}},$$  

(3.31)

which is the time it takes a particle to move outward from its release point to twice that radial distance, valid for $\beta \gg 1$ (a derivation is presented in App. 3.A.1).

Figures 3.10a and 3.11a show the constraints that the observations put on the product of $\dot{M}$ and $t_{\text{surv}}$ for the $a = 0.01 \mu m$ grains at $r = 0.23$ AU and the $a = 0.2 \mu m$ grains at $r = 0.10$ AU, respectively. Also shown are the typical survival timescales of these dust grains, and the maximum mass influx due to PR drag from a source region at 2 AU. The figures reveal that a mass source term of at least $\sim 10^{-7} \text{ M}_\oplus \text{ yr}^{-1}$ is required to explain the observations, if the grains survive for $t_{\text{dyn}}$. Comparing this with the maximum mass influx due to PR drag indicates that this mechanism cannot provide enough material.
Figure 3.10: A summary of the constraints on the hot ring, assuming it consists of 0.01 \( \mu \)m carbon grains located at 0.23 AU. Panel (a): The diagonal line shows the relation between the mass flux and the dust grain survival timescale, as constrained by the observed fractional luminosity (black, with error margins in grey, Eq. 3.29). The horizontal lines indicate the typical timescales (in a gas free environment) for destruction by sublimation (red, dash-dotted, Eq. 3.30), and removal by blowout (blue, dashed, Eq. 3.31). The vertical line with the arrow indicates the maximum mass flux due to P-R drag from a very dense source region located at 2 AU (green, Eq. 3.28). Panel (b): The dependence on gas density of the sublimation timescale (red, dash-dotted, Eq. 3.30), and the blowout timescale (blue, dashed, the sum in quadrature of Eq. 3.31 and Eq. 3.32). The top axis gives the total gas mass corresponding to the midplane gas densities on the bottom axis, assuming the gas is located in a vertically isothermal ring of width \( \Delta r = r \) (Eq. 3.34). The horizontal, black, dotted line marks the minimum survival time required if the observed material is to be provided by PR drag. It extends across both panels to show the gas density and total gas mass this would imply.

Other processes that release small dust particles in the sublimation zone may yield higher mass source terms. For instance, Rappaport et al. (2012) report a mass loss rate from the possible evaporating planet KIC 12557548 b of \( \sim 10^{-9} \) M\(_{\oplus}\) yr\(^{-1}\), sustainable for \( \sim 0.2 \) Gyr. This rate would still be insufficient to explain Fomalhaut’s hot ring, and the dust morphology inferred for KIC 12557548 b is very different from a ring. However, the actual mass loss rate of an evaporating planet is highly dependent on variables such as its mass, size, temperature, and composition.

Alternatively, the estimates of the dust survival timescale could be too low. If the small dust grains would somehow be retained in the sublimation zone, their survival timescale could be larger. One possibility is that the assumption that the disk is completely gas free is not valid in the sublimation zone. In the next subsection, we explore the influence of gas on
3.5 Origin of the dust

3.5.5 The influence of gas on dust survival timescales

Because dust sublimation converts solid dust to gas, some gas is expected to be present in the sublimation zone. The presence of gas increases the blowout timescale, because particles on their way out are slowed down by gas drag. Assuming that the subsonic Epstein drag law can be used for the gas drag force on the small dust grains, we find that the blowout timescale at high gas densities is given by

\[ t_{\text{dyn}}(\Delta r, \rho_g) = \frac{\rho_g v_{\text{th}} \Delta r r^2}{G M_* (\beta - 1) \rho_d \Delta s}. \]  

(3.32)

Here, \( \rho_g \) is the mass density of the gas, \( v_{\text{th}} \) is the mean thermal speed of the gas, and \( \Delta r \) is the distance to be travelled by the blowout grain (this equation is derived in App. 3.A.2). This timescale is shown for \( \Delta r = r \) in Figs. 3.10b and 3.11b, using the gas densities on the lower axes.

By assuming that the gas is confined to a ring around the star with a radial width of \( \Delta r \), and that it is vertically isothermal, we can express \( t_{\text{dyn}} \) in terms of the total gas mass, and eliminate the dependence on \( \Delta r \) and \( v_{\text{th}} \). The gas surface density of a vertically isothermal...
disk with midplane density $\rho_g$ is given by

$$\Sigma_g = \frac{\pi \sqrt{\gamma}}{2} \rho_g v_{\text{th}} \sqrt{\frac{r^3}{GM_*}},$$

(3.33)

where $\gamma$ is the adiabatic index, for which we assume $\gamma = 1.5$. Then, the total gas mass is given by

$$M_g = 2\pi r \times \Delta r \times \Sigma_g$$

(3.34)

$$= \pi^2 \sqrt{\gamma \rho_g v_{\text{th}} \Delta r} \sqrt{\frac{r^5}{GM_*}},$$

(3.35)

and the blowout timescale becomes

$$t_{\text{dyn}}(M_g) = \frac{M_g}{\pi^2 \sqrt{\gamma GM_* r (\beta - 1) \rho_d s}}.$$  

(3.36)

This is shown in Figs. 3.10b and 3.11b with the gas masses on the upper axes.

In the optimistic case that the hot ring is due to $a = 0.01 \mu$m grains at $r = 0.23$ AU, the hypothesis that the dust is supplied by PR drag from the warm belt requires a survival timescale of about 30 yr (the dotted black line in Fig. 3.10). This timescale is reached at a gas density of approximately $4 \times 10^{-12}$ g cm$^{-3}$, which corresponds to a total gas mass of about $5 \times 10^{-3} M_\oplus$. Assuming that all this gas is provided by sublimating dust at the maximum PR drag rate, this mechanism needs to have operated for approximately $M_g / \max[M_{\text{PR}}] \approx 5 \times 10^{-3} M_\oplus / 1.4 \times 10^{-11} M_\oplus \text{yr}^{-1} \approx 0.4 \text{ Gyr}$, which equals the age of the system, to explain the total gas mass.

The gas will also affect the sublimation of dust grains, since the presence of gas raises the sublimation temperature (see Sec. 3.3). Figs. 3.10b and 3.11b also show the dependence of the sublimation timescale on gas density. This assumes that the ambient gas only consists of carbon, so the gas densities on the lower axes correspond to the partial densities used to compute the sublimation rate in Sec. 3.3. For high gas densities, the dust particles will not sublimate, but grow. This would stabilize the dust grains, possibly explaining their very high temperatures.

### 3.5.6 Summary of theoretical findings

The distribution of dust in the inner few AU of the Fomalhaut system that was found by modeling the observations, is difficult to reconcile with a steady-state model. Both the hot and warm components contain more dust than can be explained by in-situ production via a collisional cascade that has operated for the age of the system. If these two components are in steady state (as opposed to being transient phenomena), other processes must be operating to maintain the observed dust populations. We considered several mechanisms as potential
Table 3.3: Mass fluxes of several potential supply or production mechanisms, and of the main destruction mechanisms (destructive collisions and radiative transfer blowout).

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Mass flux (M$_\oplus$ yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Warm belt</strong></td>
<td></td>
</tr>
<tr>
<td>PR drag from 140 AU</td>
<td>$&lt; 1.2 \times 10^{-12}$</td>
</tr>
<tr>
<td>Planetesimal scattering</td>
<td>$\leq 5 \times 10^{-11}$</td>
</tr>
<tr>
<td>Collisions</td>
<td>$\sim -4 \times 10^{-10}$</td>
</tr>
<tr>
<td><strong>Hot ring</strong></td>
<td></td>
</tr>
<tr>
<td>PR drag from 2 AU</td>
<td>$&lt; 1.4 \times 10^{-11}$</td>
</tr>
<tr>
<td>Evaporating planet$^{[1]}$</td>
<td>$\sim 10^{-9}$</td>
</tr>
<tr>
<td>Blowout</td>
<td>$\sim -8 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

**Notes.** $^{[1]}$ This is the value found by Rappaport et al. (2012) for KIC 12557548 b. It is given here for comparison.

solutions of the dust. Table 3.3 gives an overview of the mass fluxes that can be attained by these mechanisms.

The warm belt is dominated by barely bound grains that are primarily destroyed by mutual collisions. It needs a mass flux of the order of $10^{-9}$ M$_\oplus$ yr$^{-1}$ in order to be sustained. This rate cannot be maintained through PR drag of material from the outer cold belt at about 140 AU, but the inward scattering of small bodies by a chain of planets can marginally provide the required mass flux.

The hot ring seems to require an even higher replenishment rate ($\sim 10^{-7}$ M$_\oplus$ yr$^{-1}$). It consists of small particles that are removed from the system by blowout. A lower mass flux is possible if these grains are somehow retained near their production site, lengthening their survival timescale. The pile-up of dust due to the interplay of PR drag and sublimation is insufficient to explain the observations. PR drag of dust grains from the warm belt could provide the required mass flux, if a large amount of gas is present in the sublimation zone, which slows down the blowout of unbound grains. The viability of such a substantial gas ring and its consistency with existing observations remain to be tested. Both oxygen and carbon will remain unaffected by radiation pressure around Fomalhaut. In the case of the β Pictoris debris disk, Fernández et al. (2006) show that several species can potentially act as self-braking agents on the gas disk, and that more than 0.03 M$_\oplus$ of gas could be retained consistent with observed upper limits on the column densities.

### 3.6 Discussion

In the last section, we showed that the hot exozodi of Fomalhaut could be the result of an accumulation of small unbound grains at the carbon sublimation distance brought there by the PR drag effect. For this mechanism to work, a trapping mechanism such as braking by a gaseous component needs to be invoked. Alternative sources of continuum emission, such
as free-free emission from a stellar wind, mass-loss events or hot gas, have been discussed e.g. in Paper II, Absil et al. (2008) and Defrère et al. (2012) and can be considered unsatisfying explanations: emission by very hot dust is the most convincing explanation to date. In the solar system, nanometer-sized particles are detected with the STEREO and Ulysses spacecrafts (Meyer-Vernet et al. 2009; Krüger et al. 2010, respectively), with an increase in the particle flux in the inner solar system. These very small grains, which are affected by the Lorentz force (Czechowski & Mann 2010), could thus be expected in exozodiacal disks. In fact, magnetic trapping of nanograins could be a valuable alternative to the gas-braking mechanisms we have investigated in this study as discussed by Su et al. (2013). We note that it cannot be excluded that these nanograins might produce non-thermal emission, such as a PAH continuum.

The bright hot component can be seen as the “tip of the iceberg” in the sense that it may be a bright counterpart to the warm belt. Our model of the warm belt confirms previous attempts to constrain its properties based on unresolved observations, although our detailed treatment of grain optics and the addition of spatial constraints point towards a closer in location than previously suspected. In particular using a blackbody model, Su et al. (2013) estimated the belt location to be around 11 AU. While preserving the consistency with their dataset, the small FOV of the nulling interferometer impose the warm dust peak distance to be in the [1.5, 2.5] AU range after an appropriate subtraction of the hot dust contribution.

Furthermore, future detection of a polarization signal could provide a confirmation of the model, and additional constraints on the grain properties and the disk geometry (in particular its inclination, assumed to match that of the cold belt in this study). We use the MCFOST radiative transfer code (Pinte et al. 2006, 2009) with identical model parameters and assumptions as those discussed above9 in order to predict polarimetric signals for the exozodi. We find that the linear polarization integrated over the disk range from $3 \times 10^{-7}$ to $5 \times 10^{-7}$ for the warm belt, and reaches $2 \times 10^{-5}$ to $3 \times 10^{-6}$ for the hot ring in bands U to I. These values are compatible with the upper limits of $9 \times 10^{-3}$ to $3 \times 10^{-3}$ given by Chavero et al. (2006). For the hot ring, such signals would potentially be detectable by future sensitive polarimeters.

The warm belt has a distribution of temperatures that ranges from $\sim 320$ to $470$ K. The suspected parent-body belt location is thus consistent with the prediction that such belts form preferentially before the snowline, $R_{\text{snow}} = 2.7 \left(L/L_{\odot}\right)^{1/2}$ AU = 11 AU, because giant planet accretion is triggered beyond that limit. At such distance, the parent-body population should have eroded in a short timescale compared to the age of the star. Given the properties of the cold dust belt, Poynting-Robertson drag is not a valuable mechanism to transport sufficient amount of material from $\sim 140$ AU down to the warm belt. Even in a very favorable planetary system configuration, scattering of small bodies (comets, asteroids, planetesimals) from the cold belt by a chain of planets could not sustain the required mass production rate in the warm disk for the age of the star.

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9MCFOST uses a 3D geometry: we assume linear flaring, and we test scale heights of 0.001, 0.01 or 0.1 (unitless) at the reference distance of 1 AU, with little impact on the results. $r_{\text{max}}$ is fixed to 1 AU and 20 AU for the hot ring and the warm belt respectively. Here the grain temperatures are independent of their sizes.
Another explanation to the unexpectedly large mass in the warm component could be that the dust originates in stochastic and/or isolated catastrophic events, such as planetesimal collisions or break-up, or major dynamical perturbations. In the solar system, the Late Heavy Bombardment (LHB) was responsible for the depletion of the Kuiper belt, the release of large numbers of icy objects into its inner regions and probably a durable increase of the inner Zodiacal cloud infrared luminosity (Nesvorný et al. 2010; Booth et al. 2009). The orbital parameters of Fomalhaut b have recently been reevaluated based on an fourth epoch detection with the HST. The planet orbit is found to be very eccentric (≥ 0.8) such that it likely approaches the innermost parts of the system at periastron, where additional planetary perturbers might be present (Kalas et al. 2013; Beust et al. 2014). Thus an LHB-like event may be occurring around Fomalhaut as a result of a planet-planet scattering event causing delayed stirring in both the cold and the warm belt. In summary, a valuable scenario to understand the global debris disk is that a high collisional activity has been triggered by the presence of perturbing planets, reminiscent of the solar system history.

Finally, it is attractive to place our study in the context of the long-term objective of finding and characterizing an Earth-like planet in the habitable zone of a star. Scaling the Mars and Venus criteria for Fomalhaut (Selsis et al. 2007), the habitable zone ranges between 2.5 and 5.5 AU. Under favorable 100% cloud-cover conditions, it would extend from 1.4 to 8.1 AU. At these distances, the level of warm dust emission around Fomalhaut is high and represent therefore a threat for future spectroscopic and direct imaging missions (e.g., Defrère et al. 2010; Roberge et al. 2012). In turn, the existence of a massive asteroid belt may be an indication that there is no planet in these region as it would have cleared its neighborhood around its orbit. A noticeable feature that we have not discussed yet is that the gap between the two exozodi components could be sculpted by the gravitational influence of a hidden planet at around 1 AU. Constraints from radial velocities, astrometric measurements, and high-contrast imaging have been summarized in Paper I and are currently compatible with an hypothetical small mass companion in these regions.

3.7 Conclusion

In a series of three papers, we have performed an interferometric study of the Fomalhaut inner debris disk. Paper I presented the detection of a circumstellar excess in K-band, attributed to very hot dust, confined well inside the 3AU-HWHM FOV of the VINCI instrument. Despite the limited spatial constraints, the brightness temperature required calls for extremely hot, refractory grains lying very close to the star, at the sublimation limit. Conversely, KIN null depth measurements presented in paper II are indicative of a warmer dust component at a few AU (2AU-HWHM FOV) that produces a rising excess upward of 10μm. In the present study, we have presented the detailed results of self-consistent modeling of these two components by means of a parametric radiative transfer code and accurate treatment of debris disk physics. To account for the expected size-dependent sublimation temperature of dust, we introduced a
new prescription for the treatment of grain sublimation accounting for their specific dynamics and lifecycle. This enabled us to assess realistically the spatial and size distribution of the emitting grains. We find that the Fomalhaut exozodiacal disk consists of two dust populations, one “classical”, though massive, disk of warm (\sim 400\text{K}) dust peaking at \sim 2\text{AU} and declining slowly with distance, responsible for most of the mid-infrared emission, and a hotter (\sim 2000 \text{K}) and brighter counterpart dominated by small (0.01 - 0.5 \mu \text{m}), unbound dust particles at the limit of sublimation. The stellar radius is approximately $9 \times 10^{-3} \text{AU}$ and the hot grains are actually located at typically 10 to 35 stellar radii. The degeneracy inherent to SED fitting is partially broken by the spatial information contained in the interferometric data. We find that the model also fits the photometric mid/far-infrared measurements from Spitzer/MIPS and Herschel/PACS, and is consistent with the flux level measured in the Spitzer/IRS mid-infrared spectrum. If the warm dust, or an additional colder (but unresolved) component, were present further out in the system - as suggested by the suspected on-star excess from ALMA - it should produce moderate emission in the mid/far-infrared to preserve the compatibility between the KIN and Herschel / Spitzer data.

We analytically explored the various processes that can affect a dust grain: photo-gravitational and Poynting-Robertson drag forces, collisions, sublimation and disruption of big aggregates. We propose a framework for interpreting self-consistently the simultaneous prevalence of both hot and warm dust in the inner regions of Fomalhaut, similar to that also reported for samples of nearby main sequence stars by near- and mid- infrared exozodi surveys. Firstly we find that neither of the two inner belts can be explained by a steady-state collisional cascade in a parent-body reservoir. Ignoring the production mechanism for the warm dust, we estimate that PR drag, from this component down to the sublimation radius, could transport enough mass into the hot component, provided that it can accumulate there. We showed that small carbon monomers released by the disruption of larger aggregates that originate from the warm component can explain the observed flux level in the near-infrared, because this process considerably enhances the effective cross section of the dust population. Finally braking by a gaseous component could preserve these unbound grains from radiative transfer blowout for a sufficient time providing enough gas mass is available. If sublimation is the main source for this gas, it must have accumulated for a timescale comparable to the age of the star.

In summary, the intriguing hot dust phenomenon reported by various interferometric surveys could be understood in the light of the cumulation of multiple effects that eventually yield an accumulation of very small grains at a fraction of an AU. These hot rings are likely the counterparts of warm debris disks orbiting at a few AU that have their dust production triggered by intense collisional activity. In the near future, this scenario will need to be tested against statistical samples of objects, including later type stars.

\textit{Acknowledgements.} We would like to acknowledge Paul Kalas, James Graham, Kate Su and Alexis Brandeker, for contributing to interesting discussions on various aspects of this study, as well as the anonymous referee for the valuable advises he provided. The research leading
3.A The blowout timescale

Particles with high $\beta$ ratios are removed from the system by radiation pressure. Here, we derive the typical timescale for this process, for the case of $\beta \gg 1$. In this limit, the transverse movement of the particles is small compared to the radial movement, and only the radial acceleration of the particle needs to be considered.

3.A.1 The gas free case

We need to consider the forces of gravity and direct radiation pressure (the PR drag force is negligibly small, and we do not consider gas drag at this stage). These forces are given by

$$F_{\text{rad}} + F_{\text{grav}} = \frac{(\beta - 1)GM_*m}{r^2}. \quad (3.37)$$

For small radial displacements $\Delta r$, the acceleration $\ddot{r} = (F_{\text{rad}} + F_{\text{grav}})/m$ can be assumed to be independent of $r$, and the displacement as a function of time is given by $\Delta r = \frac{1}{2}\ddot{r}t^2$. The resulting timescale is

$$t_{\text{dyn, } \Delta r \to 0}(\Delta r) = \sqrt{\frac{2\Delta r^2_{\text{release}}}{GM_*(\beta - 1)}}. \quad (3.38)$$

At large distances from the release point, the acceleration tends to zero, and the velocity of the particle approaches a constant: $\dot{r}(r \to \infty) = \sqrt{\frac{2GM_*\beta}{r_{\text{release}}}}$ (Lecavelier Des Etangs et al. 1998). Hence, for large displacements, the removal happens on a timescale of

$$t_{\text{dyn, } \Delta r \to \infty}(\Delta r) = \frac{\Delta r}{\dot{r}(r \to \infty)} = \sqrt{\frac{(\Delta r)^2_{\text{release}}}{2GM_*(\beta - 1)}}. \quad (3.39)$$

Adding Eqs. 3.38 and 3.39 in quadrature leads to the removal timescale

$$t_{\text{dyn}}(\Delta r) = \sqrt{\frac{\Delta r_{\text{release}}}{GM_*(\beta - 1)}} \left(2r_{\text{release}} + \frac{\Delta r}{2}\right). \quad (3.40)$$

Comparing this equation with a numerical evaluation of the equation of motion shows that its relative errors are less than 5% for the values of $\beta$ and $r_{\text{release}}$ considered here.

to these results has received funding from the European Community’s Seventh Framework Programme under Grant Agreement 226604. We also thank the French National Research Agency (ANR) for financial support through contract ANR-2010 BLAN-0505-01 (EXO-ZODI), the Programme National de Planétologie (PNP) and the CNES for supporting part of this research.
We define $t_{\text{dyn}}$ as the time it takes for a particle to fly from its release point ($r_{\text{release}}$) to a point twice that radial distance from the star ($2r_{\text{release}}$). This is motivated by the fact that the small grains are seen in a very narrow radial range, so only the time they spend close to the release point is relevant. Setting $\Delta r = r_{\text{release}} = r$ gives the blowout timescale given by Eq. 3.31.

### 3.A.2 The high gas density case

For high gas densities, the gas drag force cannot be ignored. Since the particles considered here are small (compared to the mean free path of the gas molecules), and their velocities are low (compared to the sound speed of the gas), the gas drag force is given by the subsonic Epstein drag law. It is given by

$$F_{\text{drag}} = -\frac{4\pi a^2 \rho_g v_{\text{th}} \Delta v}{3},$$

where $\rho_g$ is the mass density of the gas, $\Delta v$ is the relative speed between the dust grain and the gas, and $v_{\text{th}}$ is the mean thermal speed of the gas. The latter is given by $v_{\text{th}} = \sqrt{\frac{8k_B T_g}{\pi \mu_g m_u}}$, where $\mu_g$ is the molecular weight of gas molecules, $m_u$ is the atomic mass unit, $k_B$ is the Boltzmann constant, and $T_g$ is the temperature of the gas. To calculate $v_{\text{th}}$, we assume that the gas temperature equals the dust temperature ($T_g = T_d$), and that the gas consists of the same molecules as the dust grains ($\mu_g = \mu_d$).

For high gas densities, the particles quickly reach the terminal velocity $v_{\text{term}}$, which is found by solving $F_{\text{rad}} + F_{\text{grav}} + F_{\text{drag}} = 0$ for $\Delta v$. This gives

$$v_{\text{term}} = \frac{GM_* (\beta - 1) \rho_d s}{\rho_g v_{\text{th}} r^2}.$$

In the high gas density case, the average velocity over the radial range $\Delta r$ can be approximated by the terminal velocity. The blowout timescale for high gas densities is then found from $t_{\text{dyn}, \rho_g \to \infty}(\rho_g, \Delta r) = \Delta r / v_{\text{term}}$, which leads to Eq. 3.32.
3.B Bayesian probability curves

Figure 3.12: Bayesian probability curves obtained when fitting models to the warm component with approach 1 (inner density slope fixed to 10, free outer density slope $\alpha$).
Figure 3.13: Bayesian probability curves obtained when fitting models to the warm component with approach 2 (outer density slope fixed to 1.5, free inner density slope $\alpha$). The analysis uses some prior information regarding the grain sizes $P(\alpha < a_{\text{blow}}/10 = 0)$. 
Dusty tails of evaporating exoplanets
I. Constraints on the dust composition

R. van Lieshout, M. Min, and C. Dominik

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Abstract

Context. Recently, two exoplanet candidates have been discovered, KIC 12557548b and KOI-2700b, whose transit profiles show evidence of a comet-like tail of dust trailing the planet, thought to be fed by the evaporation of the planet’s surface.

Aims. We aim to put constraints on the composition of the dust ejected by these objects from the shape of their transit light curves.

Methods. We derive a semi-analytical expression for the attenuation of the dust cross-section in the tail, incorporating the sublimation of dust grains as well as their drift away from the planet. This expression shows that the length of the tail is highly sensitive to the sublimation properties of the dust material. We compute tail lengths for several possible dust compositions, and compare these to observational estimates of the tail lengths of KIC 12557548b and KOI-2700b, inferred from their light curves.

Results. The observed tail lengths are consistent with dust grains composed of corundum (Al₂O₃) or iron-rich silicate minerals (e.g., fayalite, Fe₂SiO₄). Pure iron and carbonaceous compositions are not favoured. In addition, we estimate dust mass loss rates of $1.7 \pm 0.5 \, M_\oplus \, \text{Gyr}^{-1}$ for KIC 12557548b, and $> 0.007 \, M_\oplus \, \text{Gyr}^{-1}$ (1σ lower limit) for KOI-2700b.
4.1 Introduction

The recently discovered exoplanet candidates KIC 12557548b (hereafter KIC 1255b) and KOI-2700b show asymmetric transit profiles, that can be explained by the occultation of stars by comet-like tails of dust emanating from evaporating exoplanets (Rappaport et al. 2012, 2014). The plausibility of this scenario has been strengthened by quantitative modelling of the transit light curve (Brogi et al. 2012; Budaj 2013; van Werkhoven et al. 2014), and of the Parker-type thermal wind that can eject dust grains from the planetary atmosphere (Perez-Becker & Chiang 2013). Table 4.1 lists the basic properties of the two systems.

If the evaporating-planet scenario is correct, KIC 1255b and KOI-2700b may provide a rare chance to probe the interiors of small exoplanets. Since the dust tails are thought to originate in the atmospheres of the planets, which are fed by the evaporating planetary surfaces, knowledge of the dust composition would provide information about the composition of the planets. This, in turn, would shed light on the origin and evolutionary history of the planets, and would therefore comprise a valuable constraint for theories of planet formation and evolution.

In this paper, we demonstrate that measurements of the length of a dust tail can be used to put constraints on the composition of the dust grains in the tail (see also Kimura et al. 2002). In Sect. 4.2, we show how the tail length is related to the rate at which dust grains become smaller as a result of sublimation and the rate at which they drift away from the planet. In Sect. 4.3, we apply this theory to the two candidate evaporating planets discovered thus far, testing how well several possible dust species explain the observations. Finally, in Sect. 4.4 we discuss our findings, and in Sect. 4.5 we list our conclusions.

4.2 Cross-section decay in a dusty tail

In this section, we derive an expression for the decay of extinction cross-section per unit angle $W$ with angular separation from the planet $\Delta \theta$. This equation will describe how the depth of the transit depends on the mass loss rate of the planet, and how the length of the tail depends on the material properties of the dust. In our derivation we make use of the following assumptions, whose validity is discussed in the derivation and in Sect. 4.4.1:

1. The dust tail can be treated as if it were in steady state.
2. The tail is radially optically thin throughout.
3. The cross-section in the tail is dominated by dust grains with the same initial size.
4. The grains can be treated as simple spheres.
5. The optical efficiencies of the grains (i.e., $Q_{\text{abs}}$, $Q_{\text{scat}}$, etc.) do not change significantly as the grains become smaller.
4.2 Cross-section decay in a dusty tail

Table 4.1: Host star and system parameters of the two evaporating exoplanet candidates

<table>
<thead>
<tr>
<th></th>
<th>KIC 1255b</th>
<th>KOI-2700b</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff,}}$ [K]</td>
<td>$4550^{+140}_{-131}$</td>
<td>$4296^{+131}_{-146}$</td>
<td>H14</td>
</tr>
<tr>
<td>$M_{\star}$ [M$_\odot$]</td>
<td>$0.666^{+0.067}_{-0.059}$</td>
<td>$0.546 \pm 0.044$</td>
<td>H14</td>
</tr>
<tr>
<td>$R_{\star}$ [R$_\odot$]</td>
<td>$0.660^{+0.060}_{-0.059}$</td>
<td>$0.540^{+0.048}_{-0.051}$</td>
<td>H14</td>
</tr>
<tr>
<td>$L_{\star}$ [L$_\odot$]</td>
<td>$0.168^{+0.037}_{-0.036}$</td>
<td>$0.089^{+0.019}_{-0.021}$</td>
<td>H14</td>
</tr>
<tr>
<td>$P_p$ [days]</td>
<td>$0.6535538(1)$</td>
<td>$0.910022(5)$</td>
<td>V14, R14</td>
</tr>
<tr>
<td>$a_p$ [AU]</td>
<td>$0.0129(4)$</td>
<td>$0.0150(4)$</td>
<td></td>
</tr>
</tbody>
</table>

Notes. From top to bottom: stellar effective temperature, stellar mass, stellar radius, stellar luminosity (derived from $T_{\text{eff,}}$ and $R_{\star}$), orbital period of the candidate planet, and corresponding semi-major axis (as given by Kepler’s third law). Numbers in brackets indicate the uncertainty on the last digit.

References. H14 Huber et al. (2014); R14 Rappaport et al. (2014); V14 van Werkhoven et al. (2014).

6. The dust grains survive for at least one orbit.
7. The orbit-averaged sublimation rate of a dust grain does not change substantially as a grain becomes smaller.
8. Sublimation of dust grains can be treated as in vacuum. Gas released by the planet does not offset the sublimation through recondensation.
9. The planet’s orbit is circular.
10. The size of the planet is negligible compared to the length of the dust tail (i.e., grains are released from a single point).
11. Dust temperatures are determined by absorption and reradiation of stellar radiation.
12. The dynamics of the dust grains are dominated by drift due to radiation pressure. The initial relative velocities with which the particles are released from the planet are negligible.

The basic geometry of the system is shown in Fig. 4.1. We express the collective extinction cross-section of the dust as a fraction of the area of the stellar disk. Its angular density $W$ therefore has units [rad$^{-1}$]. Under the assumptions listed above, $W(\Delta \theta)$ can be written as

$$W(\Delta \theta) = \frac{dn}{d\Delta \theta} \frac{\sigma_{\text{ext}}(\Delta \theta)}{\pi R_{\star}^2},$$  \hspace{1cm} (4.1)

where $dn/d\Delta \theta$ is the number of particles per unit angle, $\sigma_{\text{ext}}$ is the extinction cross-section of a single dust grain, and $R_{\star}$ is the stellar radius. The angular number density can be expanded
Dusty tails of evaporating exoplanets. I. Semi-analytical constraints

Figure 4.1: Diagrams of the path followed by a dust grain released from an evaporating planet (ignoring sublimation of the dust grain) for 3 orbital periods of the planet, in the inertial frame and in the frame corotating with the planet. The positions of the planet and the dust particle are indicated at a time 1.6 orbital periods of the planet after release.

\[
\frac{dn}{d\Delta \theta} = \frac{dn}{dt} \left( \frac{dt}{d\Delta \theta} \right) = \frac{\dot{M}_d}{m_0} \left( \frac{dt}{d\Delta \theta} \right).
\]

(4.2)

Here, \( t \) denotes time, \( \frac{dn}{dt} \) is the rate at which the planet releases particles, the brackets \( \langle \ldots \rangle \) indicate averaging over the orbit of the dust grain, \( \dot{M}_d \) is the dust mass loss rate of the planet (i.e., excluding the mass lost in gas), and \( m_0 \) is the mass of an individual dust grain when it leaves the planet. The extinction cross-section of a single dust grain as a function of angular separation \( \sigma_{\text{ext}}(\Delta \theta) \) can be inferred from its derivative, which can be expanded into

\[
\frac{d\sigma_{\text{ext}}}{d\Delta \theta} = \frac{d\sigma_{\text{ext}}}{ds} \left( \frac{ds}{dt} \right) \left( \frac{dt}{d\Delta \theta} \right).
\]

(4.3)

Here, \( s \) is the grain radius.

In the following subsections, we derive expressions for the size dependence of extinction cross-section \( \frac{d\sigma_{\text{ext}}}{ds} \) (Sect. 4.2.1), the azimuthal drift rate \( \langle d\Delta \theta/dt \rangle \) (Sect. 4.2.2), and the grain radius change rate \( \langle ds/dt \rangle \) (Sect. 4.2.3). These are then combined using Eqs. 4.1–4.3 to yield the equation for the decay of extinction cross-section (Sect. 4.2.4).

4.2.1 Extinction cross-section

The extinction cross-section of a spherical dust grain with radius \( s \) can be expressed as

\[
\sigma_{\text{ext}}(s) = \pi \tilde{Q}_{\text{ext}}(s)s^2.
\]

(4.4)
4.2 Cross-section decay in a dusty tail

Table 4.2: Bulk densities and sources for optical properties of the dust species considered in this study

<table>
<thead>
<tr>
<th>Dust species</th>
<th>$\rho_d$ [g cm$^{-3}$]</th>
<th>Refs. for opt. prop.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron (Fe)</td>
<td>7.87</td>
<td>O88</td>
</tr>
<tr>
<td>Silicon monoxide (SiO)</td>
<td>2.13</td>
<td>P85, W13</td>
</tr>
<tr>
<td>Cryst. fayalite (Fe$_2$SiO$_4$)</td>
<td>4.39</td>
<td>Z11, F01</td>
</tr>
<tr>
<td>Cryst.$^a$ enstatite (MgSiO$_3$)</td>
<td>3.20</td>
<td>D95, J98</td>
</tr>
<tr>
<td>Cryst. forsterite (Mg$_2$SiO$_4$)</td>
<td>3.27</td>
<td>Z11, F01</td>
</tr>
<tr>
<td>Quartz (SiO$_2$)</td>
<td>2.60</td>
<td>Z13</td>
</tr>
<tr>
<td>Corundum (Al$_2$O$_3$)</td>
<td>4.00</td>
<td>K95</td>
</tr>
<tr>
<td>Silicon carbide (SiC)</td>
<td>3.22</td>
<td>L93</td>
</tr>
<tr>
<td>Graphite (C)</td>
<td>2.16</td>
<td>D84</td>
</tr>
</tbody>
</table>

Notes.  
$^a$ The optical properties at wavelengths below 8 $\mu$m use amorphous enstatite.

References.  
D84 Draine & Lee (1984); D95 Dorschner et al. (1995); F01 Fabian et al. (2001); J98 Jaeger et al. (1998); K95 Koike et al. (1995); L93 Laor & Draine (1993); O88 Ordal et al. (1988); P85 Palik (1985); W13 Wetzel et al. (2013); Z11 Zeidler et al. (2011); Z13 Zeidler et al. (2013).

Here, $\bar{Q}_{\text{ext}}$ is the extinction efficiency averaged over the stellar spectrum and the spectral response function of the instrument. We calculate monochromatic extinction efficiencies from (material dependent) refractive index data using Mie (1908) theory. The sources of refractive index data are listed in Table 4.2. For the stellar spectra we take Kurucz (1993) models.$^1$ The Kepler response function is described by Koch et al. (2010). Figure 4.2 shows the resulting extinction efficiencies.

Figure 4.2 shows that the extinction efficiency does not change substantially with grain size ($d\bar{Q}_{\text{ext}}/ds \approx 0$) for $s \gtrsim 0.1$–0.5 $\mu$m, depending on material type. In this regime, we can make the approximation

$$\frac{d\sigma_{\text{ext}}}{ds} = 2\pi\bar{Q}_{\text{ext}}s.$$  (4.5)

4.2.2 Drift due to radiation pressure

The dynamics of small dust grains is significantly affected by radiation pressure from the star. Since the radiation pressure force scales the same way with distance from the star as gravity, it is parametrised by $\beta$, the ratio between the norms of the direct radiation pressure force and the gravitational force (i.e., $\beta = |F_{\text{rad}}|/|F_{\text{grav}}|$). For spherical dust grains, this parameter is given

$^1$ For both stars, we use model atmospheres with an effective temperature of $T_{\text{eff,} \star} = 4500$ K and a surface gravity of log $g = 4.5$, compatible with the stellar parameters given by Huber et al. (2014).
Dusty tails of evaporating exoplanets. I. Semi-analytical constraints

![Figure 4.2: Wavelength-averaged extinction efficiency $\bar{Q}_{\text{ext}}$ as a function of grain radius $s$ for the dust species considered in this study (cf. Fig. 13 of Croll et al. 2014). Since KIC 12557548 and KOI-2700 are of similar stellar type, these values apply to both stars.](image)

by (e.g., Burns et al. 1979)

$$\beta = \frac{3}{16\pi c G} \frac{L_* \bar{Q}_{\text{pr}}(s)}{M_* \rho_d s}. \quad (4.6)$$

Here, $c$ is the speed of light, $G$ is the gravitational constant, $L_*$ is the stellar luminosity, $M_*$ is the stellar mass, $\bar{Q}_{\text{pr}}$ is the radiation pressure efficiency averaged over the stellar spectrum, and $\rho_d$ is the bulk density of the dust. Figure 4.3 shows the $\beta$ ratios for dust grains of different compositions, using the stellar parameters listed in Table 4.1, bulk densities from Table 4.2, and $\bar{Q}_{\text{pr}}$ values derived from the Mie calculations described in Sect. 4.2.1.

After being released from a planet that follows a circular Keplerian orbit, radiation pressure-affected particles will continue on new Keplerian orbits with eccentricities $e_d$, semi-major axes $a_d$, and periods $P_d$ given by (e.g., Rappaport et al. 2014)

$$e_d = \frac{\beta}{1 - \beta}; \quad a_d = a_p \frac{1 - \beta}{1 - 2\beta}; \quad P_d = P_p \frac{1 - \beta}{(1 - 2\beta)^{3/2}}. \quad (4.7)$$

Here, $a_p$ and $P_p$ denote the semi-major axis and orbital period of the planet, respectively. We note that particles with $\beta \geq 0.5$ leave the system on unbound orbits.

Because of its eccentric orbit, a dust grain will drift away azimuthally from the planet and oscillate in radial distance, giving rise to a rosette-like path in the corotating frame (see Fig. 4.1). Averaged over the particle’s orbit (but ignoring sublimation), the azimuthal drift rate equals the synodic orbital frequency, which is given by (Rappaport et al. 2014)

$$\omega_{\text{syn}} = \frac{2\pi}{P_p} = \frac{2\pi}{P_d} \frac{1 - \beta - (1 - 2\beta)^{3/2}}{1 - \beta} \approx \frac{4\pi\beta}{P_p}. \quad (4.8)$$
4.2 Cross-section decay in a dusty tail

The mathematical approximation in the last step gives an error of less than about 5% for all $\beta < 0.5$, and less than 0.5% for all $\beta < 0.1$. Compared to the actual (non-orbit-averaged) drift, this constant drift rate gives errors in $\Delta \theta$ of less than 15% after the first orbit for $\beta < 0.1$ (see also Fig. 4.7).

The azimuthal drift of a grain is described by $\Delta \theta(t) = \omega_{\text{syn}} t$. Since $\omega_{\text{syn}}$ depends on $\beta$, a particle’s drift rate will change as it becomes smaller because of sublimation. Using the assumption that the radiation pressure efficiency does not change substantially with decreasing grain size ($d \tilde{Q}_\text{pr}/ds \approx 0$, which roughly holds for $s \gtrsim 0.1–0.5 \mu m$, depending on material type), we find

$$\langle d\Delta \theta/dt \rangle = \omega_{\text{syn}} + \frac{d\omega_{\text{syn}}}{d\beta} \frac{ds}{dt}$$

$$\approx \frac{4\pi \beta}{P_p} \left(1 - \frac{t}{s} \frac{ds}{dt}\right) = \frac{4\pi \beta}{P_p} s_0$$

where $s_0$ denotes the initial grain radius. The final step uses the assumption that the orbit-averaged sublimation rate remains constant, which implies that the evolution of a grain’s radius is described by $s(t) = s_0 + \langle ds/dt \rangle t$.

4.2.3 Dust sublimation

While a dust particle drifts away from the planet, it will gradually sublimate because of the equilibrium temperature it reaches when illuminated by the stellar radiation. Since we
Dusty tails of evaporating exoplanets. I. Semi-analytical constraints

consider spherical dust grain, whose mass and surface area are given by \( m = \frac{4}{3}\pi s^3 \rho_d \) and \( A = 4\pi s^2 \), respectively, the orbit-averaged rate at which the grain radius changes can be rewritten as

\[
\left\langle \frac{ds}{dt} \right\rangle = \frac{ds}{dm} \left\langle \frac{dm}{dt} \right\rangle = -\frac{ds}{dm} A \left\langle J \right\rangle = -\frac{\left\langle J \right\rangle}{\rho_d}.
\]

(4.11)

Here, \( J \) denotes the mass loss flux from the surface of the dust grain (units: [g cm\(^{-2}\) s\(^{-1}\)]; positive for mass loss).

According to the kinetic theory of gases, a solid surface with temperature \( T \) in vacuum loses mass at a rate of (Langmuir 1913)

\[
J(T) = \alpha_{\nu}(T) \sqrt{\frac{\mu m_u}{2\pi k_B T}}.
\]

(4.12)

Here, \( \alpha \) is the evaporation coefficient that parameterises kinetic inhibition of the sublimation process (which we assume to be independent of temperature), \( p_{\nu} \) is the material-specific phase-equilibrium vapour pressure, \( \mu \) is the molecular weight of the molecules that sublime, \( m_u \) is the atomic mass unit, and \( k_B \) is the Boltzmann constant. The temperature dependence of the equilibrium vapour pressure can be approximated by

\[
p_{\nu} = \exp(-A/T + B) \text{ dyn cm}^{-2},
\]

(4.13)

where \( A \) and \( B \) are material-dependent sublimation parameters that can be determined experimentally. Table 4.3 lists estimates of the sublimation parameters for the materials we consider.

A particle on an eccentric orbit has different temperatures \( T_d \) at different parts of its orbit. When computing the orbit-averaged mass loss flux, \( J(T_d) \) has to be weighed by the time spent in each part of the orbit. This gives

\[
\left\langle J \right\rangle = \frac{1}{2\pi} \int_0^{2\pi} J[T_d(\theta_d)] \frac{(1 - e_d^2)^{3/2}}{(1 + e_d \cos \theta_d)^2} d\theta_d,
\]

(4.14)

where \( \theta_d \) is the true anomaly of the dust grain, and \( T_d \) depends on \( \theta_d \) through distance \( r = a_d(1 - e_d^2)/(1 + e_d \cos \theta_d) \).

The temperature \( T_d(s, r) \) of a dust grain with size \( s \) at distance \( r \) from the (centre of the) star can be found by solving the energy balance

\[
\frac{\Omega_d(r)}{\pi} \int Q_{\text{abs}}(s, \lambda) F_\star(\lambda) d\lambda = 4 \int Q_{\text{abs}}(s, \lambda) B_\lambda(\lambda, T_d) d\lambda.
\]

(4.15)

\(^2\) More precisely, \( \mu \) should reflect the average molecular weight of the molecules recondensing from the gas phase in an equilibrium between sublimation and condensation. For simplicity, we use the molecular weights of the dust compound. Discrepancies may result in errors of the order of unity in the final value of \( J \), which are negligible compared to the uncertainties on \( \alpha \) and \( p_{\nu} \).
Table 4.3: Sublimation characteristics of the dust species considered in this study

<table>
<thead>
<tr>
<th>Dust species</th>
<th>$\mu$</th>
<th>$\alpha$</th>
<th>$A$</th>
<th>$B$</th>
<th>Temp. range</th>
<th>Refs.</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron (Fe)</td>
<td>55.845</td>
<td>1.0</td>
<td>48354</td>
<td>1573 – 1973</td>
<td>F04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon monoxide (SiO)</td>
<td>44.085</td>
<td>0.04</td>
<td>49520</td>
<td>1275 – 1525</td>
<td>G13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cryst. fayalite (Fe$_2$SiO$_4$)</td>
<td>203.774</td>
<td>0.1$^a$</td>
<td>60377</td>
<td>1373 – 1433</td>
<td>N94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cryst. enstatite (MgSiO$_3$)</td>
<td>100.389</td>
<td>0.1$^a$</td>
<td>68908</td>
<td>1573 – 1923</td>
<td>M88, K91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cryst. forsterite (Mg$_2$SiO$_4$)</td>
<td>140.694</td>
<td>0.1$^b$</td>
<td>65308</td>
<td>1673 – 2133</td>
<td>N94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quartz (SiO$_2$)</td>
<td>60.084</td>
<td>1.0</td>
<td>69444</td>
<td>1833 – 1958</td>
<td>H90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corundum (Al$_2$O$_3$)</td>
<td>101.961</td>
<td>0.1$^c$</td>
<td>77365</td>
<td>1833 – 1958</td>
<td>L08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon carbide (SiC)</td>
<td>40.10</td>
<td>0.1$^a$</td>
<td>78462</td>
<td>1500 – 2000</td>
<td>L93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graphite (C)</td>
<td>12.011</td>
<td>0.1$^a$</td>
<td>93646</td>
<td>2400 – 3000</td>
<td>Z73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes.  
(a) For materials for which no measurements of the evaporation coefficient are available we arbitrarily adopt $\alpha = 0.1$.  
(b) From Gail (2010).  
(c) From Schaefer & Fegley (2004).  
(d) Where no uncertainty on sublimation parameters was reported, we arbitrarily set the standard deviation to 5%.  
(e) Range of temperatures for which $A$ and $B$ were determined.  
(f) These sublimation parameters were measured using a different polymorph of SiO$_2$, namely high cristobalite. Furthermore, they assume the sublimation proceeds following the reaction SiO$_2$(solid) $\rightarrow$ SiO$_2$(gas), for which the evaporation coefficient $\alpha$ is close to unity (Hashimoto 1990).  
(g) Using the equilibrium vapour pressure of SiC$_2$.  
(h) At the temperatures relevant to this study, graphite sublimates mostly as C$_3$-clusters, and the quoted parameters correspond to this component (Zavitsanos & Carlson 1973).

References.  
F04 Ferguson et al. (2004); G13 Gail et al. (2013); H90 Hashimoto (1990); K91 Kushiro & Mysen (1991); L93 Lilov (1993); L08 Lührmann (2008); M88 Mysen & Kushiro (1988); N94 Nagahara et al. (1994); Z73 Zavitsanos & Carlson (1973).
Here, $\lambda$ denotes wavelength, $Q_{\text{abs}}$ is the monochromatic absorption efficiency of the dust grain, $F_*$ is the stellar spectrum, $B_\lambda$ denotes the Planck function, and $\Omega(r)$ is the solid angle subtended by the star as seen from a distance $r$, given by $\Omega(r) = 2\pi \left[ 1 - \sqrt{1 - (R_*/r)^2} \right]$. Figure 4.4 shows the grain temperatures of dust particles made of several different materials at the orbital distance of KIC 1255b and KOI-2700b. We use $Q_{\text{abs}}$ values derived from the Mie calculations described in Sect. 4.2.1. For $F_*(\lambda)$, we use Kurucz (1993) models (see footnote 1) scaled with the stellar luminosity. At large grain sizes, temperatures of most of the investigated materials approach the black-body temperature

$$T_{\text{bb}}(r) = \left[ \frac{\Omega(r)}{4\pi} \right]^{1/4} T_{\text{eff}, *}. \quad (4.16)$$

At the distance of the planet, this gives $T_{\text{bb}}(a_p) = 1577^{+91}_{-89}$ K for KIC 1255b and $T_{\text{bb}}(a_p) = 1244^{+69}_{-75}$ K for KOI-2700b (indicated by the horizontal bands in Fig. 4.4).

### 4.2.4 The decay equation

We can now combine the above equations into an expression for the decay of extinction cross-section. By combining Eqs. 4.3–4.6, 4.10, and 4.11, we find

$$\frac{d\sigma_{\text{ext}}}{d\Delta\theta} \approx -\frac{8cG M_* P_p}{3 L_* Q_{\text{pr}}(s)} \frac{\langle J \rangle}{\sigma_{\text{ext}, 0}} \sqrt{\frac{\sigma_{\text{ext}}^3}{\sigma_{\text{ext}, 0}}}, \quad (4.17)$$

where $\sigma_{\text{ext}, 0}$ is a particle’s extinction cross-section when it is released from the planet. Integration then yields

$$\sigma_{\text{ext}}(\Delta\theta) \approx \sigma_{\text{ext}, 0} \left( 1 + \frac{\Delta\theta}{4\Delta\theta_{\text{tail}}} \right)^{-2}, \quad (4.18)$$
4.3 Constraints from observed light curves

Table 4.4: Constraints on the dust tail cross-section distribution parameters derived from transit light curve fitting

<table>
<thead>
<tr>
<th></th>
<th>KIC 1255b</th>
<th>KOI-2700b</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_0 ) [rad(^{-1})]</td>
<td>0.0227(^{+0.0020})(^{-0.0013})</td>
<td>&gt; 0.0004 (1(\sigma))</td>
</tr>
<tr>
<td>( \Delta \theta_{\text{tail}} ) [rad]</td>
<td>0.172 ( \pm ) 0.006</td>
<td>0.42 ( \pm ) 0.17</td>
</tr>
</tbody>
</table>

References. R14 Rappaport et al. (2014); V14 van Werkhoven et al. (2014, their 1-D model fit of the average transit profile).

where \( \Delta \theta_{\text{tail}} \) is the characteristic angle of the tail’s decay (defined below). Inserting this into Eq. 4.1, together with Eqs. 4.2, 4.6 and 4.10, gives the final expression for the decay of extinction cross-section per unit angle with angular separation from the planet

\[
W(\Delta \theta) \approx W_0 \left(1 + \frac{\Delta \theta}{4 \Delta \theta_{\text{tail}}} \right)^{-4}
\]  
(4.19a)

\[
W_0 = \frac{c G M_\star P_p M_d \bar{Q}_{\text{ext}}(s)}{\pi L_\star R_\star^2 \bar{Q}_{\text{pr}}(s)}
\]  
(4.19b)

\[
\Delta \theta_{\text{tail}} = \frac{3}{16c G M_\star P_p} \frac{\bar{Q}_{\text{pr}}(s)}{\langle J \rangle}.
\]  
(4.19c)

4.3 Constraints from observed light curves

With the theoretical background in place, we now investigate what constraints can be derived from the observed light curves of KIC 1255b and KOI-2700b. To do this properly, one would have to use a transit model that computes the light curve resulting from the occultation of a star by a dust tail whose extinction cross-section distribution is given by Eq. 4.19a. Such models exist (Brogi et al. 2012; Budaj 2013; van Werkhoven et al. 2014; Rappaport et al. 2014), but they generally adopt an ad hoc exponential decay for \( W(\Delta \theta) \). While the decay predicted by Eq. 4.19a is shallower than exponential, the profiles are similar enough to use the normalisation constants and \( e \)-folding angles derived from light curve fits as estimates for \( W_0 \) and \( \Delta \theta_{\text{tail}} \), respectively.\(^3\)

The literature values we adopt for the cross-section distribution parameters are listed in Table 4.4. KOI-2700b only has a lower limit on \( W_0 \). This is because its ingress is not resolved well, which allows tails with higher cross-section densities at higher transit impact parameters. We determined the lower limit on \( W_0 \) from the transit depth of \( \delta = 360 \pm 29 \) ppm

\(^3\) The two equations are identical to first order at \( \Delta \theta = 0 \), and, for a large exponent, Eq. 4.19a approaches exponential decay (cf. Rappaport et al. 2014, and see their Fig. 12):

\[
\lim_{n \to \infty} \left(1 + \frac{\Delta \theta}{n \Delta \theta_{\text{tail}}} \right)^{-n} = \exp \left( \frac{\Delta \theta}{\Delta \theta_{\text{tail}}} \right)
\]
(Rappaport et al. 2014). In the limit that the tail is short compared to the chord on the stellar disk crossed by the planet, the transit depth corresponds to the total amount of extinction cross-section in the tail, which can be found by integrating Eq. 4.19. Hence, we find

\[ \delta \leq \int_0^\infty W(\Delta \theta) d\Delta \theta \approx \frac{4}{3} W_0 \Delta \theta_{\text{tail}} = \frac{1}{4\pi} \frac{\dot{M}_d}{R_\star^2} \frac{\bar{Q}_{\text{ext}}(s)}{\langle J \rangle}. \]  

(4.20)

This ignores the limb darkening of the star as well as the forward scattering of starlight into the line of sight. It also assumes that the planet does not contribute significantly to the flux deficit.

### 4.3.1 Mass loss rates of the planets

The observational constraints on the normalisation constant \( W_0 \) can be used to estimate the dust mass loss rate of the evaporating planet (i.e., excluding mass lost in gas). Rewriting Eq. 4.19b gives

\[ \dot{M}_d \approx 0.45 \left( \frac{L_\star}{1 \text{ L}_\odot} \right) \left( \frac{R_\star}{1 \text{ R}_\odot} \right)^2 \left( \frac{M_\star}{1 \text{ M}_\odot} \right)^{-1} \left( \frac{P_p}{1 \text{ day}} \right)^{-1} \]

\[ \times \left( \frac{W_0}{0.001 \text{ rad}^{-1}} \right) \left( \frac{\bar{Q}_{\text{pr}}}{1} \right) \left( \frac{\bar{Q}_{\text{ext}}}{2} \right)^{-1} \text{ M}_\oplus \text{ Gyr}^{-1}. \]

(4.21)

For large grains, we can approximate \( \bar{Q}_{\text{pr}} \approx 1 \) and \( \bar{Q}_{\text{ext}} \approx 2 \). For KIC 1255b, this yields \( \dot{M}_d \approx 1.7 \pm 0.5 \text{ M}_\oplus \text{ Gyr}^{-1} \), which is somewhat higher than earlier (grain-property-dependent) estimates by Rappaport et al. (2012), Perez-Becker & Chiang (2013), and Kawahara et al. (2013). For KOI-2700b, we find a 1σ lower limit of \( \dot{M}_d \gtrsim 0.007 \text{ M}_\oplus \text{ Gyr}^{-1} \), consistent with the estimate of Rappaport et al. (2014).

### 4.3.2 Dust composition

The observed tail length can be used together with system parameters and \( \bar{Q}_{\text{pr}} \approx 1 \) (valid for large grains) in Eq. 4.19c to estimate the required orbit-averaged mass loss flux of the grains

\[ \langle J \rangle \approx 2.1 \times 10^{-9} \left( \frac{L_\star}{1 \text{ L}_\odot} \right) \left( \frac{M_\star}{1 \text{ M}_\odot} \right)^{-1} \left( \frac{P_p}{1 \text{ day}} \right)^{-1} \]

\[ \times \left( \frac{\Delta \theta_{\text{tail}}}{1 \text{ rad}} \right)^{-1} \left( \frac{\bar{Q}_{\text{pr}}}{1} \right) \text{ g cm}^{-2} \text{ s}^{-1}. \]

(4.22)

This gives \( \langle J \rangle \approx (4.7 \pm 1.1) \times 10^{-9} \text{ g cm}^{-2} \text{ s}^{-1} \) for KIC 1255b and \( \langle J \rangle \approx (9.0^{+4.3}_{-4.4}) \times 10^{-10} \text{ g cm}^{-2} \text{ s}^{-1} \) for KOI-2700b.

\(^4\) I.e., \( \Delta \theta_{\text{tail}} \ll \hat{\theta}_c \), where \( \hat{\theta}_c \) is the angle subtended by the chord (see Fig. 4 of Brogi et al. 2012).
4.3 Constraints from observed light curves

Figure 4.5: Mass loss flux as a function of temperature (Eq. 4.12) for different materials, whose properties are listed in Table 4.3. Typical uncertainties (in temperature) are indicated by the error bars next to the legend. The horizontal lines mark the sublimation rates required to explain the observed tail lengths (Eq. 4.22), with light grey bands indicating their uncertainties. The black-body temperatures (Eq. 4.16) at the distance of the planets and their uncertainties are indicated by vertical line segments and dark grey patches.

In Fig. 4.5, we compare these observational constraints on the sublimation rate with laboratory $J(T)$ curves for different materials. Typical uncertainties on the temperature were estimated at $J = 10^{-8}$ g cm$^{-2}$ s$^{-1}$ using a Monte Carlo technique (i.e., by varying $A$ and $B$ according to their respective uncertainties, and then numerically solving Eq. 4.12 for $T$). We also mark the black-body temperature (Eq. 4.16) at the distance of the planet for both systems. The difference in temperature between the two systems (greater than what can be explained by the trend of any of the material curves) indicates that the dust in the two systems may have a different composition, with KIC 1255b requiring a more refractory species than KOI-2700b. Furthermore, some materials reach the required sublimation rates at temperatures much lower or higher than the typical (black-body) temperatures of the systems, suggesting that the dust is unlikely to be composed of these materials. However, dust temperatures can depart considerably from black-body temperatures (see Fig. 4.4). In addition, particles on eccentric orbits experience lower temperatures when they are farther away from the star.

To investigate the effects of size-dependent grain temperatures, we use Eq. 4.19c to calculate tail lengths as a function of grain size, employing orbit-averaged sublimation rates from Eq. 4.14, realistic dust temperatures from Eq. 4.15, and size-dependent $\tilde{Q}_{pr}$ values derived from the Mie calculations described in Sect. 4.2.1. The results are shown in Fig. 4.6, together with the observed tail lengths for comparison. Typical uncertainties were estimated at $s = 1000$ µm using Monte Carlo simulations. For some materials, particles of certain sizes sublimate entirely before completing one orbit ($s_0 / \langle ds/dr \rangle < P_d$), meaning that assumption 6 of our derivation is violated. In these cases, the tail length as predicted by Eq. 4.19c
Dusty tails of evaporating exoplanets. I. Semi-analytical constraints

Figure 4.6: Characteristic angle of the tail’s decay as a function of grain size (Eq. 4.19c) for KIC 1255b and KOI-2700b, with different colours for different materials. Dashed lines indicate that particles sublimate entirely before completing one orbit. Gaps (neither a solid nor a dashed line) appear where particles are unbound ($\beta \geq 0.5$). Typical uncertainties are shown by the error bars at the bottom of each panel. The horizontal black lines indicate the values of $\Delta \theta_{\text{tail}}$ derived from the observed light curves.

is shown in Fig. 4.6 with dashed lines (while strictly Eq. 4.19c is no longer valid; also see Sect. 4.4.1). At small grain sizes ($s \lesssim 0.1 \mu m$), assumption 5 breaks down, because $\bar{Q}_{\text{pr}}$ then varies strongly with grain size. At the largest grain sizes considered here ($s \gtrsim 1000 \mu m$), grain temperatures approach the black-body temperature, and hence the results in Fig. 4.6 converge to those of Fig. 4.5 (but we note that such large dust grains are not very plausible; see Sect. 4.4.2).

Figure 4.6 displays an extremely large dynamic range in predicted tail lengths, caused by the exponential dependence of sublimation rate on grain temperature (see Eqs. 4.12 and 4.13). For most of this range, dust tails are probably not detectable by transit photometry. Small tail lengths ($\Delta \theta_{\text{tail}} \lesssim 0.01$) effectively leave only the parent planet, which makes a regular, symmetric light curve. Large values ($\Delta \theta_{\text{tail}} \gtrsim 100$) occur if dust grains hardly sublimate at all, giving rise to a nearly uniform ring of dust around the star. Such a ring would scatter and absorb the same amount of light at all phases, and would therefore not produce a signal in the normalised light curve.

The tail length predictions of Fig. 4.6 allow us to put constraints on the composition of the dust in the tails of the two evaporating exoplanet candidates. For KIC 1255b, we find the following: (1) Iron, fayalite, and silicon monoxide give tail lengths that are smaller than the observed value, because of their low sublimation temperatures. (2) The silicate minerals quartz, forsterite, and enstatite can produce the observed tail length, but only at very large grain sizes ($s \gtrsim 100 \mu m$). Smaller grains have lower temperatures (owing to their transparency in the optical), and therefore sublimate much slower. (3) Corundum grains with sizes of around $s \sim 10 \mu m$ are consistent with the observations. (4) The carbonaceous materials graphite and silicon carbide generally give tail lengths that are much longer than the observed value, because of their refractory nature. An exception are very small ($s \sim 0.01 \mu m$) graphite grains, which marginally fit the observed tail length.
The constraints for KOI-2700b, which is cooler, are somewhat different: (1) Iron, which seems to be a good candidate in Fig. 4.5, yields tails that are shorter than observed, because of its higher-than-black-body temperature. (2) Fayalite grains of many different sizes are consistent with the observed tail length. (3) Silicon monoxide gives the observed tail length for very large grain sizes ($s \sim 1000 \, \mu m$). (4) The silicate minerals quartz, forsterite, and enstatite, as well as the carbonaceous materials graphite and silicon carbide all yield tails that are much longer than indicated by the observations. (5) Corundum grains with sizes of $s \leq 1 \, \mu m$ give the observed tail length.

The above results are discussed in more detail in Sect. 4.4.3, where they are examined in the light of independent constraints on the typical size of the dust grains.

### 4.4 Discussion

#### 4.4.1 Validity of assumptions

Our semi-analytical model makes use of a number of assumptions, listed at the beginning of Sect. 4.2. In the above derivation and analysis, we already give arguments for some of these assumptions, or discuss under what conditions they are valid. Many of the assumptions, however, warrant some further discussion, which we provide here.

**Steady state** (assumption 1): It is clear that the dust tail of KIC 1255b is not in steady state, because it experiences drastic changes in transit depth from orbit to orbit (Rappaport et al. 2012). This variability is, in fact, an important argument for the evaporating-planet scenario. For KOI-2700b, the *Kepler* data do not have the signal-to-noise ratio required to detect orbit-to-orbit changes in transit depth, but a secular trend in transit depth was found (Rappaport et al. 2014). The results we find under the assumption of a steady-state dust tail, using the average observed light curve, reflect the properties of the system averaged over time.

**Optical depth** (assumption 2): Rappaport et al. (2012) argue that the star-to-planet part of the dust cloud must be optically thin or marginally so (i.e., $\tau \lesssim 1$) in order for the planet to be heated sufficiently by stellar radiation to sublimate. Indeed, Perez-Becker & Chiang (2013) find that the mass loss rate of the planet goes down for high dust abundances, and reaches a maximum at optical depths of about $\tau \approx 0.1$ (see their Fig. 4). Furthermore, shielding of dust grains by each other from the stellar radiation is reduced by the large angular diameter of the star from the vantage point of the dust cloud ($27^\circ$ for KIC 1255b; $19^\circ$ for KOI-2700b). Regarding the optical depth from the star to the observer, it is important to take into account that the dust cloud is inclined with respect to the line of sight (for non-zero impact parameters). This lowers the optical depth because it ensures that the radial extent of the dust cloud (which can be substantial because of the different paths followed by grains of different sizes) contributes to its vertical extent with respect to the line of sight.
Single grain size (assumption 3): Without knowledge of the actual grain size distribution, our analysis assuming a single grain size can still be used to exclude many dust compositions. If a given dust species yields tail lengths that, for all plausible grain sizes, are either always longer or always shorter than the observed value (i.e., its curve in Fig. 4.6 lies either fully above or fully below the horizontal black line), it is not possible to create the observed tail length with a combination of different-sized grains of this composition.

Constant optical efficiencies (assumption 5): Figure 4.2 shows for which grain sizes the extinction efficiency remains constant. The independence of transit depth on wavelength (Croll et al. 2014) indicates that this assumption is at least correct for the extinction efficiency in the case of KIC 1255b, although the possible effects of the optical depth of the dust cloud should be investigated. For the emission efficiency (relevant to the energy balance of the dust grains) the assumption only becomes accurate at much larger grain sizes (see discussion of assumption 7 below).

Survival timescale (assumption 6): We assume that particles survive against sublimation for at least one orbit. Arguments based on the variability timescale presented by Rappaport et al. (2012) and Perez-Becker & Chiang (2013) indicate that this is marginally the case for KIC 1255b. For KOI-2700b, dust grains with $\beta \lesssim 0.03$ need to survive for longer than one dust orbit to explain the length of the tail (ignoring sublimation; see also Sect. 4.5 of Rappaport et al. 2014). This assumption is needed to justify averaging over the orbit of the dust particle. For low-eccentricity orbits, however, this does not introduce a large error.

Constant sublimation rate (assumption 7): Our assumption that the orbit-averaged sublimation rate does not change as particles become smaller means that we make a zeroth-order approximation in dust temperature. This is only valid at very large grain sizes, for which black-body temperatures are a good approximation. Avoiding this simplification would require a numerical approach to modelling the dust tail. Assumption 7 also means that we do not take into account the changes in orbital eccentricity that are caused by the changes in $\beta$ of a shrinking dust grain. This second point is a good approximation as long as orbital eccentricities are low, such that the sublimation rate does not vary substantially from its periastron value.

Dust temperatures (assumption 11): When computing dust temperatures from the energy balance Eq. 4.15, we assume that the contribution of latent heat due to sublimation is negligible. This is valid for the materials and temperatures we consider (Lamy 1974; Rappaport et al. 2014). Collisional heating by stellar wind particles can also be ignored (Rappaport et al. 2014).

Radiation pressure dominated dynamics (assumption 12): For the dynamics of the dust grains, we only take into consideration the gravity and radiation pressure of the star.
Rappaport et al. (2014) find that the ram pressure force due to the stellar wind is one or two orders of magnitude lower than the radiation pressure force, and can therefore be ignored. Assumption 12 also requires that the initial relative velocity \( \Delta v_0 \) with which the grain is launched away from the planet is negligible compared to the radiation-pressure-induced drift. This holds if \( \Delta v_0 \ll 2\beta v_p \), where \( v_p = \sqrt{GM_*/a_p} \) is the planet’s Keplerian velocity. Using Kepler’s third law, this can be rewritten as

\[
\beta \gg 0.002 \left( \frac{M_*}{1 \text{ M}_\odot} \right)^{-1/3} \left( \frac{P_p}{1 \text{ day}} \right)^{1/3} \left( \frac{\Delta v_0}{1 \text{ km s}^{-1}} \right).
\]  

(4.23)

The magnitude of the initial relative velocity \( \Delta v_0 \) is very uncertain. It may be comparable to the escape speed of the planet, but this depends on the precise coupling of the dust grains to the gas flow. We do not expect \( \Delta v_0 \) to be more than a few km s\(^{-1}\), and therefore assumption 12 should be valid for grains with radii up to about \( s \lesssim 10 \mu m \). Importantly, if this condition were violated, and dust grains are launched isotropically, some particles would end up drifting ahead of the planet. The transit light curve resulting from such a configuration would display a gradual ingress. In the observed light curve, only the egress is gradual, consistent with grain dynamics that are dominated by radiation pressure.

### 4.4.2 Constraints on the grain size

Section 4.3.2 revealed that our constraints on the dust composition are dependent on the size of the dust grains, primarily because the grain size influences the grain temperature. By using independent constraints on the grain size, it is therefore possible to make better inferences about the dust composition. Here, we list the available constraints on the grain size (most of which only concern KIC 1255b):

1. Brogi et al. (2012) find that the scattering properties of the dust trailing KIC 1255b, as imprinted on the light curve, are best explained by small particles (0.04 \( \mu m < s < 0.19 \mu m \)).

2. In a similar analysis, Budaj (2013) finds that the pre-ingress brightening of KIC 1255b is best explain by grains of \( s \sim 0.1–1 \mu m \), while smaller grains (\( s \sim 0.01–0.1 \mu m \)) are better at explaining its egress.

3. From the independence of the transit depth of KIC 1255b on wavelength, Croll et al. (2014) derive a 3\( \sigma \) lower limit on the grain size of \( s \gtrsim 0.5 \mu m \) (\( s \gtrsim 0.2 \mu m \) for iron grains).

4. In the phase-folded short cadence light curve of KIC 1255b, Croll et al. (2014) tentatively detect a small decrement of flux in the egress, about 0.15 phase units after the midpoint of the transit (corresponding to an angular separation from the planet of \( \Delta \theta_{\text{decr}} \approx 0.15 \times 2\pi \approx 0.94 \text{ rad} \)). If this feature is real, it may be related to the first
periastron passage of the dust grains after launch. The periastron passage gives an enhancement in the dust density because the relative angular velocity between dust and planet vanishes when dust particles pass their periastron. This is illustrated by Fig. 4.7, which shows the drift of dust particles with respect to the planet (ignoring sublimation), calculated by solving Kepler’s equation (see also Fig. 4.1, as well as Sect. 4.1 and Fig. 11 of Rappaport et al. 2014). The figure demonstrates that the angular offset of the dust density enhancement is very sensitive to $\beta$, and hence to the size of the dust grains. This also implies that, if the flux decrement feature is real and it is caused by the periastron passage of dust grains, the grain size distribution must be very narrow. Ignoring sublimation, the relation between the $\beta$ ratio of the dust grains and the position of the density enhancement is

$$\Delta \theta_{\text{decr}} = \omega_{\text{syn}} P_d = 2\pi \frac{1 - \beta - (1 - 2\beta)^{3/2}}{(1 - 2\beta)^{3/2}}.$$

The observed angular offset of the possible flux decrement of $\Delta \theta_{\text{decr}} \approx 0.94$ rad corresponds to $\beta \approx 0.064$. This $\beta$ ratio is reached by particles of various sizes, depending on material type (see Fig. 4.3), but leads to a tentative upper limit of roughly $s \lesssim 2$ µm, since larger grains always have lower $\beta$ ratios.

5. For very small values of $\beta$, the motion of dust grains is probably dominated by the initial velocity with which they are launched away from the planet, rather than radiation-pressure-induced drift (see Eq. 4.23). Large grains may therefore result in a more symmetric distribution of dust around the planet, contrary to what is observed. For this reason, we deem dust grains with sizes of $s \gtrsim 100$ µm unlikely.
6. Perez-Becker & Chiang (2013) assume $s = 1 \, \mu m$ for their atmospheric outflow model, and find that grains up to this size couple sufficiently to the gas to be lifted out of the planetary atmosphere by a thermal wind. Although the resulting upper limit on grain size is model-dependent (it varies with planet mass, for example), this is an additional argument against very large grain sizes.

Combined, the above pieces of evidence point to grain sizes of about $s \sim 0.1$–$1 \, \mu m$ for KIC 1255b, although some of the clues are contradictory. For both objects, there are arguments to exclude very large dust grains ($s \gtrsim 100 \, \mu m$). Future research should examine whether a uniform grain size is a good approximation. It is also important to determine the optical depth of the dust clouds, and investigate what effects it may have on the shape of the pre-ingress brightening and on the wavelength dependence of the transit depth.

### 4.4.3 The composition of the dust and the planets

Applying the above grain size constraints to the results from Sect. 4.3.2 allows us to exclude some of the solutions seen in Fig. 4.6. Specifically, the silicate minerals that reproduce the observed tail length of KIC 1255b at very large grain sizes now seem unlikely. The same holds for SiO in the case of KOI-2700b.

One of the remaining candidates for the dust material is corundum ($\text{Al}_2\text{O}_3$). It yields the observed tail lengths of both KIC 1255b and KOI-2700b at plausible grain sizes. Because of the relatively low cosmic abundance of aluminium, it may seem unlikely that a minor species such as corundum accounts for all the dust in the tails of the evaporating planets. However, as already noted by Rappaport et al. (2012), high $\text{Al}_2\text{O}_3$ abundances can be created by a distillation of the planetary surface. This mechanism was proposed by Léger et al. (2011) in a study of the lava ocean which is thought to dominate the dayside surface of the hot super-Earth CoRoT-7b. Starting with a silicate composition, more volatile elements such as Si, Fe, and Mg evaporate preferentially from the lava ocean, and, if these components leave the system permanently, the ocean over time consists of increasingly refractory species. After about 1.5 Gyr of steady evaporation, the ocean reaches a stable composition of 13% CaO and 87% $\text{Al}_2\text{O}_3$. A similar distillation scenario for KIC 1255b and KOI-2700b may explain the prevalence of corundum grains.

Previous work on the evaporating exoplanet candidates proposed pyroxene ([Mg,Fe]$\text{SiO}_3$) as the main constituent of the dust in the tails, and (in the case of KIC 1255b) rejected olivine ([Mg,Fe]$\text{Si}_2\text{O}_4$) as too volatile (Rappaport et al. 2012, 2014). These suggestions are based on scaling the sublimation time scales found by Kimura et al. (2002) for cometary dust grains in the vicinity of the Sun to the exoplanetary systems. Here, we show that dust grains made of enstatite (the Mg-rich end-member of pyroxene) survive for too long to explain the observed tail lengths.\(^5\) Regarding olivine, the Mg-rich end-member forsterite behaves similar to enstatite, while the Fe-rich end-member fayalite gives much shorter tails. For KOI-2700b, the

---

\(^5\) We only considered crystalline minerals, because amorphous forms are expected to transition to crystalline on timescales much shorter than the sublimation timescale (see Sect. 4.2.3 of Kimura et al. 2002).
observed tail length is consistent with dust grains composed of fayalite, while for KIC 1255b fayalite is too volatile. Since these two end-members of olivine give tail lengths on opposite sides of the observed value for KIC 1255b, it is conceivable that an intermediate form (iron-rich, but with some magnesium) yields the right length.

The list of dust materials we tested is far from exhaustive. In addition, we only considered dust grains of a single, pure composition. This type of investigation only allows one to test whether a dust species is consistent with the observations or not, and there may be more dust species, different from the ones we found, that can also explain the data. Perhaps the most stringent constraint resulting from this work is that pure iron and carbonaceous dust compositions are not favoured.

Broadly speaking, the composition of the dust ejected by evaporating planets reflects that of the parent planet. Hence, pure iron and carbonaceous compositions are not favoured for the planets KIC 1255b and KOI-2700b. However, the relation between dust and planet composition may be complicated by processes such as preferential condensation of certain species and fractionation of a lava ocean (Léger et al. 2011). Therefore, to gain more insight into the planetary composition, a thorough investigation of the link between planet and dust composition is required. Roughly, this would entail the following modelling steps: (1) Given a planetary composition, temperature, and gravity, calculate the composition of the atmosphere resulting from evaporation (e.g., Schaefer & Fegley 2009; Miguel et al. 2011; Schaefer et al. 2012). (2) Determine the dynamical structure of the atmospheric outflow (see Perez-Becker & Chiang 2013). (3) Calculate which dust species would condensate in this outflow (as has been done for stellar outflows; e.g., Gail 2010). By doing this for different potential planetary compositions, and as a function of temperature and gravity, it may be possible to use the dust composition as a probe for the composition of the planet.

4.5 Conclusions

This paper describes the decay of cross-section in dusty tails trailing evaporating planets, such as KIC 1255b and KOI-2700b. The analytical expression we derive (Eq. 4.19) can be used to model transit light curves. Specifically, it provides a physical interpretation for two properties of a dust tail. The density of the tail (found from the transit depth) is related to the mass loss rate of the planet. The tail length (which can be derived from the duration of the transit) is determined by the sublimation rate of the dust in the tail. This sublimation rate depends sensitively on the optical and thermodynamical properties of the material that the dust grains in the tail are made of. Therefore, given accurate laboratory measurements of these properties, the tail length can be used to constrain the composition of the dust.

The constraints we find for the dust composition of the two evaporating exoplanet candidates are summarised in Table 4.5. Our analysis has lead to the following conclusions about

---

6 This assumes that the outflow is loaded with dust grains as a result of condensation from the gas phase. Another possible origin of the dust is explosive vulcanism (see Sect. 4.2 of Rappaport et al. 2012, and references therein).
Table 4.5: Summary of the dust composition constraints

<table>
<thead>
<tr>
<th>Dust species</th>
<th>KIC 1255b</th>
<th>KOI-2700b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron (Fe)</td>
<td>too volatile</td>
<td>dust temperature too high</td>
</tr>
<tr>
<td>Silicon monoxide (SiO)</td>
<td>too volatile</td>
<td>required grain size implausibly large</td>
</tr>
<tr>
<td>Cryst. fayalite (Fe$_2$SiO$_4$)</td>
<td>too volatile</td>
<td>consistent with observed tail length</td>
</tr>
<tr>
<td>Cryst. enstatite (MgSiO$_3$)</td>
<td>required grain size implausibly large</td>
<td>too refractory</td>
</tr>
<tr>
<td>Cryst. forsterite (Mg$_2$SiO$_4$)</td>
<td>required grain size implausibly large</td>
<td>too refractory</td>
</tr>
<tr>
<td>Quartz (SiO$_2$)</td>
<td>required grain size implausibly large</td>
<td>too refractory</td>
</tr>
<tr>
<td>Corundum (Al$_2$O$_3$)</td>
<td>consistent with observed tail length</td>
<td>consistent with observed tail length</td>
</tr>
<tr>
<td>Silicon carbide (SiC)</td>
<td>too refractory</td>
<td>too refractory</td>
</tr>
<tr>
<td>Graphite (C)</td>
<td>too refractory</td>
<td>too refractory</td>
</tr>
</tbody>
</table>
these systems:

1. The mass loss rate in dust of KIC 1255b is approximately \( \dot{M}_d \approx 1.7 \pm 0.5 \, M_\oplus \, \text{Gyr}^{-1} \). For KOI-2700b, we find a 1\( \sigma \) lower limit of \( \dot{M}_d \gtrsim 0.007 \, M_\oplus \, \text{Gyr}^{-1} \).

2. Dust grains composed of corundum can explain the tail lengths of both candidates.

3. The tail length of KOI-2700b is also consistent with a fayalite composition. A composition of iron-rich silicate minerals may also work for KIC 1255b.

4. Dust grains made of pure iron, graphite, or silicon carbide are not favoured for both objects.

Acknowledgements. We thank H.-P. Gail for sharing refractive index data of SiO. We also appreciate the constructive comments of an anonymous referee.
Dusty tails of evaporating exoplanets
II. Physical modelling of the KIC 12557548b light curve


*Astronomy & Astrophysics*, to be submitted

**Abstract**

*Context.* Evaporating rocky exoplanets, such as KIC 12557548b, eject large amounts of dust grains, which can trail the planet in a comet-like tail. When such objects occult their host star, the resulting transit signal contains information about the dust in the tail.

*Aims.* We aim to use the detailed shape of the *Kepler* light curve of KIC 12557548b to constrain the size and composition of the dust grains that make up the tail, as well as the mass loss rate of the planet.

*Methods.* Using a self-consistent numerical model of the dust dynamics and sublimation, we calculate the shape of the tail by following dust grains from their ejection from the planet to their destruction due to sublimation. From this dust cloud shape, we generate synthetic light curves (incorporating the effects of extinction and angle-dependent scattering), which are then compared with the phase-folded *Kepler* light curve. We explore the free parameter space thoroughly using a Markov chain Monte Carlo method.

*Results.* Our physics-based model is capable of reproducing the observed light curve in detail. Good fits are found for initial grain sizes of $1.2^{+4.4}_{-0.9}$ μm and dust mass loss rates of $2.5^{+13.1}_{-1.9}$ M$_{⊕}$ Gyr$^{-1}$. We find that only certain combinations of material parameters yield the
correct tail length. These constraints are consistent with dust made of corundum (Al$_2$O$_3$), but do not agree with a range of carbonaceous, silicate, or iron compositions.

**Conclusions.** Using a detailed, physically motivated model, it is possible to constrain the composition of the dust in the tails of evaporating rocky exoplanets. This provides a unique opportunity to probe to interior composition of the smallest known exoplanets.

### 5.1 Introduction

Determining the chemical composition of exoplanets is an important step in advancing our understanding of the Earth’s galactic neighbourhood and provides valuable benchmarks for theories of planet formation and evolution. For small (i.e., Earth-sized and smaller) exoplanets, most efforts so far have been directed at determining a planet’s mean density from independent measurements of its size and mass, which gives an indication of the bulk composition (e.g., Howard et al. 2013). This method, however, is restricted by the observational lower limits for which planetary radii and masses can reliably be determined (although recent progress is pushing the limits ever further down; Jontof-Hutter et al. 2015). Another, more fundamental, problem is that exoplanets with different (combinations of) chemical compositions can have the same mean density, making it impossible to distinguish between these compositions using just the planet’s size and mass (Seager et al. 2007). In particular, whether small carbon-based exoplanets exist is an open question that cannot be resolved with bulk density measurements alone (see Fig. 9 of Seager et al. 2007; Madhusudhan et al. 2012).

Another method of investigating the chemical composition of exoplanetary material is the study of white-dwarf atmospheres that are polluted by the accretion of tidally disrupted asteroids or minor planets (see Jura & Young 2014 for a review). This method allows the bulk composition of the accreted bodies to be measured with unprecedented precision. However, it can only be applied to white-dwarf systems, and the exact relation between the measured compositions and those of the exoplanets in the original, main-sequence-stage planetary systems is not yet clear.

The recent discovery of transiting evaporating rocky exoplanets (Rappaport et al. 2012) has opened up a possible new channel for determining the chemical compositions of small exoplanets that is complimentary to the methods mentioned above. Through the evaporation of their surface, these objects present material from their interior to the outside, where it can be examined as it blocks and scatters star light. We recently showed how the composition of the outflowing material can be determined from the shape of the object’s transit light curve using semi-analytical expressions (van Lieshout et al. 2014b, hereafter Paper I). In the present paper, we revisit this problem using a numerical model, which allows us to let go of several of the simplifying assumptions made in Paper I and to use more directly all the information contained in the light curve.
5.1 Introduction

Figure 5.1: The phase-folded long-cadence Kepler light curve of KIC 1255b (bottom), together with schematic views of the system at different orbital phases (top), illustrating how an asymmetric dust cloud can explain the peculiar transit profile. Arrows indicate which sketch corresponds to which orbital phase. For details on the observational data, see Sect. 5.2.3 of this work and Sect. 2 of van Werkhoven et al. (2014). The error bars on flux include the spread caused by the variability in transit depth, making the in-transit error bars greater than the out-of-transit ones (which are mostly smaller than the size of the symbols). In the sketches, the star, the orbit of the planet, and the length of the dust tail are all drawn to scale; the tick marks on the axes are spaced one stellar radius apart. The thickness of the dust cloud and its colour gradient (which illustrates the gradually decreasing dust density) are chosen for illustrative purposes.

5.1.1 Evaporating rocky exoplanets

To date, there are three known (candidates of) transiting evaporating rocky exoplanets: KIC 12557548b (hereafter KIC 1255b; Rappaport et al. 2012), KOI-2700b (Rappaport et al. 2014), and EPIC 201637175b (Sanchis-Ojeda et al. 2015), all three discovered using the Kepler telescope (Borucki et al. 2010). They all orbit K- and M-type main-sequence stars in orbital periods of less than a day and their light curves are marked by asymmetric transit profiles and variable transit depths. Both light-curve properties can be explained by a scenario in which the extinction of star light is caused by an asymmetric cloud of dust grains, whose collective cross-section changes from transit to transit. This scenario was first put forward by Rappaport et al. (2012) for the prototype KIC 1255b; we briefly summarise it here.

The dust grains that make up the cloud originate in a small evaporating planet. Once they have left the planet, radiation pressure from the host star pushes them into a comet-like tail trailing the planet. With increasing distance from the planet, the dust grains speed up with respect to the planet. They also gradually sublimate due to the intense stellar irradiation, decreasing their size. Both effects cause the angular density of extinction cross-section to decrease further into the tail. The resulting asymmetric shape of the dust cloud can explain

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1 A search for more such objects amongst short-period Kepler exoplanet candidates did not find any additional ones (Garai et al. 2014).
the sharp ingress and gradual egress of the observed transit light curve (see Fig. 5.1).\(^2\) In addition, forward scattering of star light by dust grains results in a brightening just before the transit, when the bulk of the dust cloud is not in front of (the brightest part of) the stellar disk, but close enough to yield small scattering angles. The asymmetry of the dust cloud means that this effect is much stronger around ingress than egress.

The dust cloud scenario has been validated by the colour dependence of the transit depth (Bochinski et al. 2015; Sanchis-Ojeda et al. 2015), while many false positive scenarios for this type of event have been ruled out based on radial velocity measurements, high angular resolution imaging, and photometry (Croll et al. 2014). Morphological modelling of the KIC 1255b light curve has allowed some properties of its dust cloud to be determined (Brogi et al. 2012; Budaj 2013; van Werkhoven et al. 2014). In particular, both the wavelength dependence of the transit depth and the morphological dust cloud models indicate that the dust grains have radii in the range 0.1 to 1.0 \(\mu\)m.

To explain the variation in transit depth, the dust cloud scenario invokes erratic variations in the planet’s dust production rate. By making some assumptions about the dust grains, it is also possible to infer the average dust mass loss rate of the evaporating planet (i.e., excluding the mass lost in gas) from the light curve. For KIC 1255b and EPIC 201637175b, the dust mass loss rates are estimated to be of the order of \(1 M_\oplus\) Gyr\(^{-1}\) (Rappaport et al. 2012; Perez-Becker & Chiang 2013; Kawahara et al. 2013; Paper I; Sanchis-Ojeda et al. 2015), while for KOI-2700b it may be two orders of magnitude lower (Rappaport et al. 2014; Paper I).

The planet’s mass loss is thought to be fuelled by the total bolometric flux from the host star (Rappaport et al. 2012).\(^3\) Stellar radiation heats the planetary surface to a temperature exceeding 2000 K, which causes the solid surface to vaporise, creating a metal-rich atmosphere (as has been modelled in detail for super-Earths; Schaefer & Fegley 2009; Miguel et al. 2011; Schaefer et al. 2012). This atmosphere is hot and expands into the open space around the planet, driving a “Parker-type” thermal wind (Rappaport et al. 2012). As the gas expands and cools, its refractory constituents can condense into dust grains.\(^4\) Small dust grains are entrained in the gas flow until the gas thins out, from which point the dust dynamics are controlled by stellar gravity and radiation pressure.

Perez-Becker & Chiang (2013) modelled the planetary outflow in detail, finding that the mass loss rate is a strong function of the mass of the evaporating body. According to their model, the mass loss rate of KIC 1255b indicates that the planet cannot be more massive than about 0.02 \(M_\oplus\). The planetary radius corresponding to this mass is consistent with the upper limits on the size of the planet, derived from the non-detection of transits in some parts of the

\(^2\) The light curve of EPIC 201637175b has a markedly different shape, which can be explained by a streamer of dust grains leading (rather than trailing) the planet. In this object, whose host star is less luminous, the initial launch velocities of the dust grains (instead of radiation pressure) may dominate the dynamics (Sanchis-Ojeda et al. 2015).\(^3\) X-ray-and-ultraviolet-driven evaporation was suggested as an alternative, based on a relation between transit depth and stellar rotational phase (Kawahara et al. 2013). A more straightforward explanation for this relation, however, is the occultation of starspots by the transiting dust cloud (Croll et al. 2015).\(^4\) Another mechanism that could be responsible for loading the planet’s atmosphere with dust is explosive vulcanism (Rappaport et al. 2012).
light curve (Brogi et al. 2012) and secondary eclipses in the entire light curve (van Werkhoven et al. 2014), and, if correct, would make KIC 1255b one of the smallest exoplanets known.

5.1.2 Dusty tail composition

Regardless of how exactly the evaporating planet produces and ejects dust, the composition of the dust in the tail will reflect that of the planet. The precise relation between the two compositions may be complicated by selection effects such as preferential condensation of certain dust species in the atmosphere (e.g., Sect. 3.2.2 of Schaefer et al. 2012) and possibly the fractional vaporisation of a magma ocean (Sect. 6.1 of Léger et al. 2011). Nevertheless, identifying the composition of the dust can lead to insights into the composition of the surface of the planet and possibly its interior (if prior evaporation has already removed the original surface, exposing deeper layers). Such insights are invaluable for theories of planet formation and evolution.

Building upon the work of Kimura et al. (2002) and Rappaport et al. (2012, 2014), we recently demonstrated how the length of a dusty tail trailing an evaporating exoplanet can be used to constrain the composition of the dust in this tail (Paper I). In a nutshell, the length of the tail is determined by the interplay of radiation-pressure-induced azimuthal drift of dust grains and the decrease in size of these grains due to sublimation. Because the sublimation rate of the dust is strongly dependent on its compositions, the tail length is a proxy for grain composition. By comparing tail-length predictions for potential dust species with the observed tail length (derived from the duration of the transit egress), it is possible to put constraints on the composition of the dust in the tail.

Paper I presents a semi-analytical description of dusty tails, in which the shape of the tail is described using just two parameters: the tail’s characteristic length and its initial density. The values of these two parameters are taken from the morphological tail models, which derive them from light curve fitting (Brogi et al. 2012; Budaj 2013; van Werkhoven et al. 2014; Rappaport et al. 2014). However, describing the tail morphology in just two parameters ignores many details of the tail’s shape, which may be used to constrain the dust composition from the detailed shape of the transit light curve. Furthermore, the derivation of a semi-analytical description of the dust tail in Paper I requires many assumptions, which may undermine the applicability of the resulting equations.

To take the next step in modelling the dusty tails of evaporating planets, it is desirable to employ a physics-based (in contrast to morphological) model of the tail that self-consistently takes into account the interplay of grain-size-dependent radiation-pressure dynamics and temperature-dependent grain-size evolution. In this paper, we develop such a model. In brief, the model consists of a particle-dynamics-and-sublimation simulation, followed by a transit-profile generation using a light-scattering code. Similar modelling work has been done previously to predict the light curves due to possible extrasolar comets in the β Pictoris system (Lecavelier Des Etangs et al. 1999; Lecavelier Des Etangs 1999), with the major differences that these comets have orbital periods of years rather than hours and sublimation...
Table 5.1: Host star and system parameters of KIC 1255b

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stellar effective temperature</td>
<td>$T_{\text{eff},\star}$</td>
<td>K</td>
<td>$4550^{+140}_{-131}$</td>
</tr>
<tr>
<td>Stellar mass</td>
<td>$M_\star$</td>
<td>$M_\odot$</td>
<td>$0.666^{+0.067}_{-0.059}$</td>
</tr>
<tr>
<td>Stellar radius</td>
<td>$R_\star$</td>
<td>$R_\odot$</td>
<td>$0.660^{+0.060}_{-0.059}$</td>
</tr>
<tr>
<td>Stellar luminosity</td>
<td>$L_\star$</td>
<td>$L_\odot$</td>
<td>$0.168^{+0.037}_{-0.036}$</td>
</tr>
<tr>
<td>Planet’s orbital period</td>
<td>$P_p$</td>
<td>days</td>
<td>$0.6535538(1)$</td>
</tr>
<tr>
<td>Planet’s semi-major axis</td>
<td>$a_p$</td>
<td>AU</td>
<td>$0.0129(4)$</td>
</tr>
</tbody>
</table>

Notes. The stellar parameters are taken from Huber et al. (2014); the planet’s orbital period is from van Werkhoven et al. (2014). Numbers in brackets indicate the uncertainty on the last digit.

does not have to be taken into account. For the dusty tails of evaporating planets, Rappaport et al. (2012, their Sect. 4.6) and Sanchis-Ojeda et al. (2015, their Sect. 6.2) did particle-dynamics simulations, but using a constant lifetime of the dust grains against sublimation and without generating light curves. In order to derive constraints on the dust composition from broadband transit profiles, it is essential to treat dust sublimation in a self-consistent, time-dependent way.

We apply our model to the prototypical evaporating rocky exoplanet KIC 1255b. Of the three candidates, this object has the best quality data. In principle, the model can be applied to the other two candidate evaporating rocky exoplanets after some additional work. Specifically, for KOI-2700b it would be necessary to obtain a better constraint on the dust survival time from possible correlations between subsequent transits or lack thereof. Modelling EPIC 201637175b would require the initial launch velocity of the grains to be explored in more detail.

The basic parameters of the KIC 1255b system are listed in Table 5.1. For the stellar parameters, there are different estimates in the literature, casting doubt on whether the star has evolved off the main sequence or not. In Appendix 5.A, we investigate the different claims and conclude that the star is most likely still on the main sequence.

The rest of this paper is organised as follows. Section 5.2 provides a detailed description of the dust cloud model and of how it is compared to the observations. Section 5.3 gives the resulting constraints on the free parameters of the model and shows what they imply for the dust composition. In Sect. 5.4, we discuss our findings in the light of previous work and examine one of our modelling assumptions. Finally, we summarise our work and draw conclusions in Sect. 5.5.
Table 5.2: Modelling assumptions

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Assumption</th>
<th>Used</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steady state</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Optically thin dust cloud</td>
<td>✓</td>
<td>(a)</td>
</tr>
<tr>
<td>3</td>
<td>Single initial grain size</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Spherical dust grains</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Constant optical efficiency factors</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Long dust survival time</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Constant dust sublimation rate</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Dust sublimation as in vacuum</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Circular planet orbit</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Small planet</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Radiation controlled dust temperatures</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Radiation pressure dominated dynamics</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Notes. An overview of the modelling assumptions introduced in Paper I. The check marks indicate whether these assumptions are still used in the present work.

(a) This assumption is investigated in more detail in Sect. 5.4.2.

(b) In Paper I, the dust survival time was assumed to be long (compared to the orbital period of the dust grains). Here, we assume instead that it is short (no longer than about one orbital period of the planet; see Sect. 5.2.3 for further details).

5.2 Methods

In our modelling efforts, we adopt the following approach. First, we calculate the shape of the dust tail by following dust grains from their release from the planet to their complete sublimation (Sect. 5.2.1). We then generate the light curve that would result from this dust tail (Sect. 5.2.2). Finally, the synthetic light curve is compared to the phase-folded *Kepler* data, yielding a goodness of fit (Sect. 5.2.3). These three steps are carried out for different values of input parameters (which include the material properties of the dust) in a Markov chain Monte Carlo (MCMC) framework to obtain constraints on those parameters (Sect. 5.2.4).

Our model makes use of a number of assumptions, which were introduced in Paper I (see Table 5.2). They are motivated in the rest of this section when they are encountered and in Sect. 4.1 of Paper I. The numerical approach of this work allows us to let go of several of the assumptions made in Paper I: The optical efficiency factors of the dust (e.g., $Q_{\text{abs}}$, $Q_{\text{pr}}$) are allowed to change as dust grains become smaller (assumption 5). We do not use orbit-averaged quantities, and hence the survival time of the dust grains is not required to be long compared to the orbital period of the dust grains (assumption 6). Sublimation rates of dust grains are calculated in a time-dependent manner, rather than assumed to remain equal to their initial value (assumption 7).
In contrast to Paper I, we do not primarily test several possible dust species for which detailed laboratory measurements of their properties are available, but rather describe the optical and thermodynamical properties of the dust material with a set of free parameters. The constraints on these parameters obtained from the MCMC fitting can then be compared with laboratory measurements of candidate dust species.

5.2.1 Tail morphology
To determine the shape of the dust tail, we compute the trajectory of a single dust grain. Assuming that all dust grains released from the planet are identical (i.e., they have the same composition and initial size), this trajectory gives the shape of a stream of dust particles, with individual time steps corresponding to separate particles launched from the planet at different times. Although the single-initial-grain-size assumption makes our model somewhat less realistic, it lowers the computation time of the model significantly, allowing us to calculate many different model realisations in a statistical fashion. The initial size of the dust grains that we eventually find should be interpreted as a typical initial size. In addition, we note that the origin of the dust grains through condensation in a planetary outflow may favour a narrow size distribution, since large grains cannot be lifted out of the atmosphere (Perez-Becker & Chiang 2013) and the saturated-atmosphere crossing time could result in a minimum size.

The path of an individual dust grain is determined by solving the equations of motion and sublimation. These equations are coupled because (1) the grain-size evolution influences the dust dynamics through size-dependent radiation pressure, and (2) the dynamics influence the sublimation rate through distance-dependent grain temperatures.

Dust dynamics
After being released, the dust particles drift away from the planet as a result of the direct radiation pressure force of the star. Aside from the stellar gravity, we assume this to be the only relevant force for the dynamics of the dust grains. Therefore, we ignore the gravitational influence of the planet (insignificant at spatial scales of the dust tail because the planet’s mass is small), Poynting–Robertson drag (only relevant over many orbits), stellar wind pressure (see Appendix A of Rappaport et al. 2014), and gas drag from the planetary outflow (assumed to diminish rapidly). Because the radiation-pressure-induced drift is slow with respect to the local Keplerian velocity, the dynamics are best solved in a rotating reference frame (i.e., centred on the star and co-rotating with the planet). Hence, the motion of a dust grain is described by

\[
\frac{d^2 r}{dt^2} = -\frac{G M_* (1 - \beta)}{r^3} r - 2 \omega \times \frac{dr}{dt} - \omega \times (\omega \times r) .
\]  (5.1)

Here, the vector \( r \) is the position of the particle (and hence its magnitude \( r \) is the distance to the centre of the star), \( t \) denotes time, \( G \) is the gravitational constant, \( \beta \) is the ratio between the
norms of the direct radiation pressure force and the gravitational force, and \( \omega \) is the rotation vector of the reference frame (with magnitude \( \omega_p = 2\pi/P_p \)). The Coriolis and centrifugal acceleration terms represent fictitious forces due to the rotating reference frame.

For spherical dust grains, the \( \beta \) ratio is given by (e.g., Burns et al. 1979)

\[
\beta = \frac{3}{16\pi c G M_*} \frac{L_* \tilde{Q}_{pr}(s)}{\rho_d s}.
\]  

(5.2)

Here, \( c \) is the speed of light, \( L_* \) is the stellar luminosity, \( \tilde{Q}_{pr} \) is the radiation pressure efficiency averaged over the stellar spectrum (see Sect. 5.2.1), \( \rho_d \) is the bulk density of the dust, and \( s \) is the grain radius.

**Dust sublimation**

For a spherical dust grain in a gas-free environment, the rate at which the grain radius \( s \) changes is given by (Langmuir 1913; see also Eq. (11) of Paper I)

\[
\frac{ds}{dt} = -\frac{J(T_d)}{\rho_d} = -\frac{\alpha p_v(T_d)}{\rho_d} \sqrt{\frac{\mu m_u}{2\pi k_B T_d}}.
\]  

(5.3)

Here, \( J \) is the mass loss rate per unit surface (units: \([g \, cm^{-2} \, s^{-1}]\); positive for mass loss), \( \alpha \) is the evaporation coefficient (which parameterises kinetic inhibition of the sublimation process), \( p_v \) is the partial vapour pressure at phase equilibrium, \( \mu \) is the molecular weight of dust molecules, \( m_u \) is the atomic mass unit, \( k_B \) is the Boltzmann constant, and \( T_d \) is the temperature of the dust. The equilibrium vapour pressure is material-specific and depends strongly on temperature \( T \). This material and temperature dependence is captured by the parameters \( A \) and \( B \) in the Clausius–Clapeyron relation

\[
p_v(T) = \exp(-A/T + B) \, \text{dyn cm}^{-2}.
\]  

(5.4)

We assume \( \alpha \), \( A \), and \( B \) to be independent of temperature and ignore any temperature dependence of \( p_v \) beyond that given by Eq. 5.4.

The sublimation rate depends only linearly on the parameters \( \mu, \alpha \), and \( \exp(B) \). Therefore, these can be combined into the new parameter\(^5\)

\[
B' \equiv B + \ln(\alpha \sqrt{\mu}),
\]  

(5.5)

reducing the number of material-specific free parameters of our model, without changing the functional form of the sublimation equation 5.3. With this substitution, the sublimation properties of dust materials are fully described by the parameters \( A \) and \( B' \). Hence, these are both free parameters in our model.

---

\(^5\) Generally, the difference between \( B \) and \( B' \) is relatively small (because \( \alpha \) and \( \mu \) affect \( B' \) with opposite sign). For the dust species listed in Table 3 of Paper I, \( 0.01 \leq \alpha \leq 1 \), \( 10 \leq \mu \leq 200 \), and \( |B - B'|/B \lesssim 7\% \), which is comparable to the typical uncertainties on \( B \).
Dust grain temperatures $T_d(s, r)$ are calculated from the power balance between incoming stellar radiation and outgoing thermal radiation. This ignores the latent heat of sublimation and the collisional heating by stellar wind particles, which are both insignificant (Lamy 1974; Rappaport et al. 2014). The power balance reads

$$\Omega(r) \int Q_{\text{abs}}(s, \lambda) F_{\star}(\lambda) \, d\lambda = 4\pi \int Q_{\text{abs}}(s, \lambda) B_{\lambda}(\lambda, T_d) \, d\lambda,$$

where $\lambda$ denotes wavelength, $Q_{\text{abs}}$ is the monochromatic absorption efficiency of the dust grain, $F_{\star}$ is the stellar spectrum, $B_\lambda$ denotes the Planck function, and $\Omega(r)$ is the solid angle subtended by the star as seen from the dust particle. The distance dependence is given by $\Omega(r) = 2\pi \left[ 1 - \sqrt{1 - \left( R_\star / r \right)^2} \right]$. For the stellar spectrum of KIC 12557548 we take a Kurucz (1993) model with $T_{\text{eff,}\star} = 4500$ K and a surface gravity of $\log g = 4.5$ (see Fig. 5.10), scaled with the stellar luminosity. Equation 5.6 is solved numerically for $T_d$ as a function of grain size and distance from the star.

**Optical properties of the dust**

Several parts of our model (the calculation of $\beta$ and $T_d$, as well as the synthetic-light-curve generation) require values for the dimensionless efficiency factors $Q_{\text{abs}}$, $Q_{\text{ext}}$, $Q_{\text{sca}}$, and $Q_{\text{pr}}$ with which dust particles interact with the stellar radiation field as a function of grain size and wavelength. We assume the dust grains to be solid spheres, which allows us to calculate these quantities using Mie (1908) theory. While Mie theory is often not applicable to interplanetary dust grains, which are aggregates, the freshly condensed grains concerned here are expected to be compact, and therefore we expect Mie theory to be a good approximation. As input, the Mie calculations require (material dependent) complex refractive indices as a function of wavelength. The complex refractive index consists of a real part $n(\lambda)$, which we refer to as the refractive index, and an imaginary part $k(\lambda)$, known as the extinction coefficient.

To allow our model to retrieve the dust composition in a relatively unbiased manner, we wish to describe the complex refractive index in a parametric way, rather than testing a limited number of dust species with full wavelength-dependent $n(\lambda)$ and $k(\lambda)$ data (especially considering that these data often come without good estimates of their uncertainties). The parameterisation should be simple enough to be numerically feasible, but still able to capture the optical properties of a wide range of materials. In particular, the dependence of dust temperature on grain size should be described well, since the sublimation of dust grains is very sensitive to it. We tested several different prescriptions for the wavelength dependence of the complex refractive index and finally settled on a very simple recipe, in which $n(\lambda)$ is assumed to be constant over all wavelengths and $k(\lambda)$ is split into two constants for different wavelength regimes:

$$n(\lambda) = n, \quad k(\lambda) = \begin{cases} k_1 & \text{for } \lambda < \lambda_{\text{split}} \\ k_2 & \text{for } \lambda \geq \lambda_{\text{split}} \end{cases}$$

(5.7)
5.2 Methods

In our model, \( n \) and \( k_1 \) and free parameters, while \( k_2 \) and \( \lambda_{\text{split}} \) are fixed at \( k_2 = 1 \) and \( \lambda_{\text{split}} = 8 \) \( \mu \)m. The values we chose for \( \lambda_{\text{split}} \) and \( k_2 \) reflect the fact that many dust species have strong features in their \( k(\lambda) \) profile beyond 8 \( \mu \)m, which can facilitate their cooling if their heating efficiency is sufficiently low (see Appendix 5.B).

The main advantage of our simple complex-refractive-index recipe is the limited number of free parameters it uses.\(^6\) Despite its simplicity, the recipe is capable of reproducing much of the diversity seen in grain temperatures seen for real dust species. This is demonstrated in Appendix 5.B, where we compare the grain-size dependent dust temperatures predicted by our simple recipe to those found using the full wavelength-dependent complex refractive index data of real dust species.

Numerical method

To determine the path and size-evolution of a dust grain, we integrate Eqs. 5.1 and 5.3 using a classical fourth order Runge–Kutta scheme. The dust properties that couple these equations, \( \beta(s) \) and \( T_d(s, r) \), are precalculated and tabulated. The initial size of the dust grain \( s_0 \) is a free parameter of our model; the other initial conditions (position and velocity) are set to simulate a particle released at the position of the planet with zero velocity with respect to the planet. This means that we neglect the velocity with which particles are launched away from the planet by the gas outflow, assuming that radiation pressure dominates the dynamics of the dust grains. For KIC 1255b, this seems to be a good approximation (see Sect. 4.1 of Paper I and Sect. 6.2 of Sanchis-Ojeda et al. 2015). We also ignore the non-zero but poorly constrained size of the planet, which is negligible compared to the spatial extent of the tail (i.e., particles are released from a single point, which corresponds to the centre of the planet). A dynamical step-size control ensures that (1) the position of the particle (in the rotating frame) never changes by more than 1% of the stellar radius (mostly to ensure an adequate spatial sampling from which a light curve can be generated) and (2) its size does not decrease by more than a factor 2. The integrator stops when a particle has reached a size of \( s \leq 0.001 \) \( \mu \)m. At this point the particle is considered fully sublimated as it does not contribute significantly anymore to the extinction or scattering of star light.

The output of this dynamics-and-sublimation routine is a list of particle positions and sizes at each time step. Assuming that the planet ejects a continuous stream of particles with identical composition and initial size, this list corresponds to the coordinates of particles that have left the planet at different points in time. The model solutions presented in this work typically consist of several hundred output points. Figure 5.2 shows an illustrative example of a dust stream predicted by our model, including the evolution of the grain size. It is compared

\(^6\) More complex prescriptions for the optical properties, with more free parameters (e.g., with \( k_2 \) and/or \( \lambda_{\text{split}} \) kept free, or with \( k(\lambda) \) split up into more wavelength regimes), pose computational difficulties to the MCMC analysis, which for increasing dimensionality needs more iterations to reach convergence and obtain reliable parameter constrains. Conversely, an even more simple recipe, using a constant \( k \) for all wavelengths, fails to capture the cooling through mid-infrared features, giving relatively high dust temperatures even for very low extinction efficiencies (\( k \sim 10^{-5} \)).
Figure 5.2: Example dust tail morphology of KIC 1255b found by our numerical dynamics and sublimation calculation (black lines). The ◦-symbols shown at several points along the stream are placed at equal intervals of travel time. Their size is proportional to the local grain size. An ×-symbol marks the point where the dust grains are fully sublimated. For comparison, a dust path ignoring sublimation is shown in grey. Several orbits of the dust grain are shown, which results in a rosette-like shape in the corotating frame (see, also, Fig. 1 of Paper I). The dotted line indicates the orbit of the planet. **Top:** The dust tail and host star from the observer’s point of view at the time of mid-transit. **Middle:** View looking down onto the planet’s orbital plane. **Bottom left:** Time needed by a dust grain to travel from the planet to a point in the tail as a function of the angle behind the planet. The horizontal dashed line indicates the orbital period of the planet $P_p$. **Bottom right:** Grain size as a function of angle behind the planet.
5.2 Methods

to a dust path that does not take into account the size evolution due to sublimation.

The dust dynamics are solved in the orbital plane of the planet (middle panel in Fig. 5.2).
In order to find the coordinates with respect to the star as seen from the observer (top panel in Fig. 5.2), the output coordinates are rotated by an angle $i$, the inclination of the planet’s orbit with respect to the sky plane. This is given by $i = \arcsin(bR_\star/a_p)$, where $b$ is the transit impact parameter (a free parameter of our model).

5.2.2 Light-curve generation

The previous step gives the spatial and size distribution of dust in the form of grain sizes and three-dimensional position coordinates at each time step, which correspond to coordinates and sizes of dust particles released from the planet at different times. From these data, we now have to compute the normalised flux into the direction of the observer as a function of phase. Rather than determining spatial densities of dust grains, we compute the effect on the light curve of each individual particle in the dust stream. After computing the effects on the light curve of a single dust grain, these are scaled using the dust mass loss rate of the planet $\dot{M}_d$, a free parameter of the model.

At each step in phase, we determine which particles are in front of the star and which are behind the star as seen from the observer. Any particle that is in front of the star reduces the flux because of absorption and scattering. This is achieved by subtracting an amount of flux proportional to the grain’s extinction cross-section and the local intensity of the stellar disk. To describe the stellar intensity profile, we employ the four-term limb-darkening law of Claret & Bloemen (2011) with parameters $a_1 = 0.71, a_2 = -0.83, a_3 = 1.52, a_4 = -0.56$, appropriate for a star with $T_{\text{eff},\star} = 4500$ K and $\log g = 4.5$, observed in the Kepler band.

Any particles that are not behind the star increase the flux through scattering. The amount of flux to add is proportional to the grain’s scattering cross-section, their scattering phase function at the scattering angle, and the intensity of the stellar disk at the point from where the light emanates. For the scattering, it is important to take into account the non-zero size of the star. This is especially true for grains with sharply forward peaked phase functions. Rather than convolving the phase function with the angular size of the star as seen from the dust cloud, we integrate over the limb-darkened stellar disk using Monte Carlo integration. In this process, we take into account the reduced part of the star that is visible as seen from the dust particle, due to its proximity to the star.

Two final operations are done on the synthetic light curve. First, the light curve is convolved with a trapezoidal kernel to capture the effect of the 29.4 minute exposure time of the long cadence Kepler data. This smoothes the light curve and has a significant effect, since the exposure time is longer than the time scale on which the light curve varies. Second, we normalise the light curve to the out-of-transit flux, which is the same operation that has been done on the Kepler data. In principle, this may affect models that show significant forward scattering outside the in-transit phase window. In practice, however, we find that this procedure makes little difference. Figure 5.3 shows an example of a light curve generated by our
model. Note that while the figure only shows the part of the light curve around the transit, we calculate fluxes at all orbital phases.

We do not take into account the effect on the light curve of the solid planet. Given the upper limits on the size of the planet, this effect is negligible. The planet’s size (or that of any optically thick, circular part of the dust cloud) could in principle be added to the model as an additional free parameter.

5.2.3 Goodness-of-fit evaluation

We now describe the observational data and how the goodness of fit of model light curves is calculated. In principle, determining the relative likelihood of a particular set of parameter values is done by a simple $\chi^2$ comparison of the synthetic light curve with the observed one. However, we use several additional criteria to assess the viability of the model solution before the $\chi^2$ statistic is computed. If a given model solution does not satisfy any of these criteria, the likelihood of this solution is set to zero.

Observations

We compare the model solutions with the *Kepler* light curve of KIC 1255b. Specifically, we use the long-cadence data of quarters 1 through 15, as reduced by van Werkhoven et al. (2014, see their Sect. 2), which we phase folded and binned into 128 phase bins. The length of one bin corresponds to 0.25 of the *Kepler* long-cadence integration time. Figure 5.1 shows the resulting light curve.
Uncertainties were computed from the spread of individual Kepler data points within each phase bin. This results in phase-dependent error bars. Because of the orbit-to-orbit variability in the transit depth, the in-transit part of the light curve has greater uncertainties than the out-of-transit part. The variability-caused variance is roughly proportional to the local (i.e., local in phase) transit depth. In a second approach, we estimated the average error on the binned data from the variance in normalised flux amongst the out-of-transit ($\phi \not\in [-0.15, 0.2]$) bins.\(^7\) This turned out to be a factor of about 1.4 greater than the median of the uncertainties found from the spread within each bin. The latter were therefore multiplied by this factor to get the final uncertainties shown in Fig. 5.1 and used for our fitting.

**Dust-survival-time constraints**

The full Kepler light curve contains information about the system that is not preserved in the phase-folding process. Specifically, the variability of the transit depths is sensitive to the survival time of the dust grains. Correlations (or the lack thereof) between the depths of subsequent transits or the depth of a transit core and the egress depth of the following transit can be used to determine how long after their release from the planet dust grains influence the transit depth. In their thorough analysis of the long-cadence Kepler data, van Werkhoven et al. (2014) found no evidence of any such correlations. Their absence indicates that the survival time of the dust grains is not much longer than the planet’s orbital period $P_p$.$^8$

To use this information, we set a maximum survival time for the dust grains, $t_{\text{surv}} < t_{\text{cut}}$. This is numerically convenient, because it severely restricts the maximum computation time for a given model. The maximum survival time is set to $t_{\text{cut}} = 1.2P_p$ rather than $1P_p$ (orbital period of the planet) to give some leeway to solutions in which dust grains survive for slightly longer than $1P_p$, but with the last part contributing very little to the extinction, such that no significant correlations are produced between subsequent transits.$^9$ We find that the posterior probability distribution of survival time peaks around $1P_p$, confirming that the cut-off time of $1.2P_p$ is reasonable.

**Light-curve criteria**

Our model is very sensitive to some of the input parameters. In particular, the dust sublimation rate depends strongly on temperature and, as a result, small changes in the values of

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\(^7\) This assumes that the out-of-transit part of the light curve is flat, which was demonstrated by van Werkhoven et al. (2014, see their Sect. 3.3). It gives a good estimate of the actual uncertainties on the data, less affected by noise that is correlated on timescales longer than the length of an individual bin (i.e., red noise).

\(^8\) Estimates of the survival time based on the length of the tail indicate that it is close to $1P_p$. The assumption in Paper I that the dust survival is long (compared to the orbital period of the dust grains), therefore holds marginally for KIC 1255b.

\(^9\) A more thorough way of using the absence of correlation between subsequent transits would be to simulate a varying mass loss rate and inspect the resulting correlation between transits, or to compute the light curve for material older than $1P_p$ and compare this to switch-off events in the observed light curve. However, these methods come with prohibitively large computational costs and are therefore beyond the scope of the present work.
the material properties can result in large changes in the light curve. To prevent the fitting algorithm from wasting computational resources on parts of the parameter space that produce light curves that are clearly unrealistic, we only allow solutions that display the general features of the light curve, i.e., the asymmetric shape with a gradual egress and a pre-ingress brightening. Technically, this is achieved by the following three cumulative criteria:

1. Only model solutions that are within $20\sigma$ of the observational data at all phases are allowed. This excludes synthetic light curves without a main transit feature, or with (additional) strong transit features at other phases than the observed one.

2. At phases $\varphi \in [-0.05, -0.04]$, we require a minimum normalized flux of $1+10^{-4}$. This excludes models that do not exhibit any pre-ingress brightening.

3. At phases $\varphi \in [0.08, 0.10]$, we allow a maximum normalized flux of $1-10^{-4}$. This excludes models without a significant tail.

We acknowledge that this procedure may in principle lead to an underestimation of the uncertainties we derive on the free parameters.

### 5.2.4 Fitting strategy

Our dust tail model consists of three steps: (1) determining the shape of the dust cloud, (2) computing a synthetic light curve, and (3) comparing it to the observations. In order to put constraints on the dust composition, these steps need to be repeated many times for different values of the free parameters. We now describe the parameter space that needs to be explored and the method we use to do so.

#### Free parameters

Our model contains nine free parameters, five of which describe the dust material ($\rho_d$, $n$, $k_1$, $A$, and $B'$). They are listed in Table 5.3, along with their scaling and the bounds of the range considered for each parameter. We assume flat or log flat prior probability distributions for all free parameters within their allowed ranges. In principle, the stellar parameters could be added as free parameters to account for their uncertainty, but given the already large number of free parameters, we keep them fixed at the fiducial values listed in Table 5.1.

For $\Delta\varphi_0$, $b$, and $M_d$, we allow the entire physically possible range. For the other parameters, the bounds reflect physically reasonable limits and sometimes numerical limitations. The initial grain size and bulk dust density are required to be higher than 0.01 $\mu$m and 0.01 g cm$^{-3}$, respectively, primarily for numerical reasons. Very-low-density grains have high $\beta$ ratios (see Eq. 5.2) and are put on unbound trajectories that require long computation times, but do not produce good fits. The upper bound on $\rho_d$ is set to 10 g cm$^{-3}$. This is high enough to consider dust grains made of pure iron, while only relatively rare metals such as lead and gold have even higher densities. For the complex-refractory-index and sublimation parameters ($n$, $k_1$, $A$, $B'$),
5.2 Methods

Table 5.3: Free parameters of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Scale</th>
<th>Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transit midpoint phase offset</td>
<td>Δφ₀</td>
<td>linear</td>
<td></td>
<td>[-0.5, 0.5]</td>
</tr>
<tr>
<td>Transit impact parameter</td>
<td>b</td>
<td>linear</td>
<td>[0, 1]</td>
<td></td>
</tr>
<tr>
<td>Dust mass loss rate of planet</td>
<td>M_d</td>
<td>M⊕ Gyr⁻¹</td>
<td>logarithmic</td>
<td>[0, +∞)</td>
</tr>
<tr>
<td>Initial grain size</td>
<td>s₀</td>
<td>µm</td>
<td>logarithmic</td>
<td>[0.01, +∞)</td>
</tr>
<tr>
<td>Bulk density</td>
<td>ρ_d</td>
<td>g cm⁻³</td>
<td>logarithmic</td>
<td>[0.01, 10]</td>
</tr>
<tr>
<td>Refractive index</td>
<td>n</td>
<td>linear</td>
<td>[1, 4]</td>
<td></td>
</tr>
<tr>
<td>Extinction coefficient</td>
<td>k₁</td>
<td></td>
<td>logarithmic</td>
<td>[10⁻⁵, 10]</td>
</tr>
<tr>
<td>Sublimation parameter 1</td>
<td>A</td>
<td>10⁴ K</td>
<td>linear</td>
<td>[4, 10]</td>
</tr>
<tr>
<td>Sublimation parameter 2</td>
<td>B’</td>
<td></td>
<td>linear</td>
<td>[20, 50]</td>
</tr>
</tbody>
</table>

, and B’), the bounds are set to bracket the values found by laboratory measurements for a wide range of possible dust species.

Markov chain Monte Carlo method

The parameter space we have to consider is very large. On average, a single model evaluation takes about 1 s of computation time on a desktop workstation, making it unfeasible to search for maxima in the likelihood using a grid approach. Therefore, we employ an MCMC method to explore the parameter space. Specifically, we use the affine-invariant ensemble-sampler algorithm of Goodman & Weare (2010) as implemented in the Python package emcee (Foreman-Mackey et al. 2013). This algorithm is designed to efficiently sample probability distributions with strong correlations between parameters. It uses a ensemble of “walkers” that map out the probability density landscape by moving through parameter space. Their proposed steps are based on the positions of the other walkers in the ensemble and the acceptance of a step depends on the probability ratio of the proposed and current positions in parameter space.

We use 100 walkers, initialised at positions in parameter space that were found to give good fits in earlier trial runs. The proposal-scaling factor (the parameter a in Goodman & Weare 2010) is set at its recommended standard value of 2. We find that the acceptance rate of proposed steps is rather low (about 6 to 7%), indicating bad mixing of the chain. Increasing the number of walkers only marginally improves the acceptance rate, while the associated computational costs rapidly become prohibitively high. Hence, we compensated the low acceptance rate with a large number of steps. After burn-in, we let the walkers make 9 × 10⁴ steps each, resulting in a total of 9 × 10⁶ model evaluations. We assessed the convergence of the chain using the autocorrelation length, which is found to be less than 5000 steps for all free parameters. This means that the length of the chain is longer than 18 autocorrelation times for all parameters (Foreman-Mackey et al. 2013 recommend a chain length of 10 autocorrelation
5 Dusty tails of evaporating exoplanets. II. Numerical modelling

![Graph: Normalised flux vs phase and residuals](image)

**Figure 5.4**: Top: Comparison between the best-fit model (black line) and the *Kepler* data (black circles with error bars). To show the range of variation within the Markov chain, 50 randomly selected samples from the chain are underplotted as grey lines. **Bottom**: Residuals of the best-fit model, normalised using the phase-dependent error bars on the data.

...times) and we can assume the chain to be converged. The number of unique model solutions in the flattened chain is about $6 \times 10^5$. We find that the nine-dimensional posterior probability density function can be mapped out in sufficient detail with this number of samples.

### 5.3 Results

We find that our physics-based model is capable of reproducing the observed *Kepler* light curve in detail (the best-fitting solution, shown in Fig. 5.4, has a reduced $\chi^2$ value of about 1.3). In particular, the observed “kink” in the egress (around $\varphi \approx 0.06$) is a natural result of the path and the size evolution of the dust grains and does not require an occulting object consisting of multiple components. The low initial speed of the dust grains with respect to the planet means that the angular density of extinction cross-section is very high at the head of the dust cloud, while a large section of the tail has a more constant density (see also Fig. 5.2).

Also shown in Fig. 5.4 are 50 model realisations that were picked randomly from the chain, to visualise the spread of the model solutions in data space. We find that the chain contains many poorly fitting solutions, with relatively shallow transits, prominent long egress tails, and other significant deviations from the observed profile. In data space, these solutions occupy a large part of the region allowed by the additional fitting requirements listed in Sect. 5.2.3. This indicates that the exploration of parameter space was not very efficient, relying heavily on the additional fitting requirements, with the $\chi^2$ statistic being of secondary importance.
### Table 5.4: Results of the MCMC analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constraint (2σ)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δϕ₀</td>
<td>-0.010⁺⁻⁰.013</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>&lt; 0.80 (2σ)</td>
<td></td>
</tr>
<tr>
<td>M_d</td>
<td>2.5⁺⁻¹⁺₁³ M⊕ Gyr⁻¹</td>
<td></td>
</tr>
<tr>
<td>s₀</td>
<td>1.2⁺⁻⁰.4 µm</td>
<td>(a)</td>
</tr>
<tr>
<td>ρ_d</td>
<td>&gt; 0.9 g cm⁻³ (2σ)</td>
<td>(a)</td>
</tr>
<tr>
<td>n</td>
<td>unconstrained</td>
<td></td>
</tr>
<tr>
<td>k₁</td>
<td>&gt; 7.9 × 10⁻³ (2σ)</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>see note</td>
<td>(b)</td>
</tr>
<tr>
<td>B'</td>
<td>see note</td>
<td>(b)</td>
</tr>
</tbody>
</table>

**Notes.** The values reported here are medians of the marginalised probability density distribution. The uncertainties, as well as the upper and lower limits, reflect the 2.3⁰⁻⁰.7²⁰th percentiles.

- **(a)** Combining $s_0$ and $ρ_d$ gives a constraint on the initial radiation-pressure-to-gravity ratio of $β_0 = 0.029⁺⁻⁰.004 (2σ)$.
- **(b)** Individually, the sublimation parameters $A$ and $B'$ are unconstrained, but they are highly correlated and can be combined to give the constraint $\log_{10}(J_{1700K}/(g \text{ cm}^{-2} \text{ s}^{-1})) = -7.3⁺⁻².6 (2σ)$.

Importance. As a result, the distribution in parameter space of model realisations in the Markov chain may be a poor representation of the actual probability density distribution, rather indicating the limits of the region of parameter space allowed by the observations. Indeed, we find that good-fitting model solutions are spread throughout a large part of parameter space explored by the MCMC algorithm and do not cluster around the peak of the distribution of model realisations. For these reasons, we adopt a conservative approach in interpreting the MCMC results and report 2σ uncertainties in the remainder of this section, which are conservative enough to encompass most of the good-fitting models.

#### 5.3.1 Model-parameter constraints and correlations

The constrains on the model’s free parameters resulting from the MCMC analysis are summarised in Table 5.4. Many of the individual free parameters are not well constrained. By inspecting the result in more detail, however, we can extract useful constraints. Figure 5.5 gives a more extensive overview of the MCMC results, showing one and two-dimensional projections of the nine-dimensional posterior probability density function. These indicate (1D) how symmetric or skewed the constraints on individual parameters are and (2D) how pairs of free parameters are correlated. Some three-dimensional projections of the probability density function were also inspected, but these are not shown here. We now discuss the correlations that occur between the free parameters of the model, as well as some specifics of the constraints on individual parameters.
While the planet’s dust mass loss rate $\dot{M}_d$ and the initial size of the dust grains $s_0$ are both individually relatively well constrained, there is also a clear correlation between them. Larger grains are associated with higher mass loss rates. The mass loss rate is mostly constrained by the depth of the transit. Because larger grains have a lower cross-section-to-mass ratio, more mass is needed to acquire the same transit depth with larger dust grains. Also, larger grains have scattering phase functions that are more sharply peaked in the forward direction and as a result of this the scattered light component of their light curves is greater than that of small...
grains. Since the scattered light component can partly counteract the extinction signal, larger grain sizes require more particles, and hence more mass, to yield the observed transit depth.

A clear anticorrelation is seen between the initial grain size $s_0$ and the bulk density of the dust $\rho_d$. This can be explained by requirements on the $\beta$ ratio, which is inversely proportional to both. Very small grains need be dense in order to remain bound to the star (rather than expelled from the system by radiation pressure). For large grains, the density needs to be sufficiently low that radiation pressure can make them drift far enough into the tail within the allowed time to reproduce the observed light curve. We can quantify this constraint by computing $\beta_0$ (i.e., the $\beta$ ratio at the moment the grain is released) using Eq. 5.2 with $s = s_0$ and assuming $\bar{Q}_{pr} = 1$ (which holds within at most a factor 2). Lines of constant $\beta_0$ go parallel to the $s_0-\rho_d$ correlation, and consequently the distribution of $\beta_0$ is relatively narrow (see Table 5.4). It peaks below the maximum $\beta$ ratio possible around KIC 1255b (see Fig. 3 of Paper I), but above the value for which initial launch velocities of the grains become important (see Eq. (23) of Paper I and Fig. 15 of Sanchis-Ojeda et al. 2015).

The detailed shape of the pre-ingress brightening is sensitive to the size of the dust grains, because this determines the shape of the scattering phase function. In general, we find that the best results regarding the pre-ingress brightening (and the best fits in general) are achieved with initial grain sizes of 0.2 to 0.3 $\mu$m, while larger grains provide too much forward scattering and smaller sizes (if allowed) give light curves that are too flat. Because of the correlations between model parameter described above, allowed model solutions with these initial grain sizes correspond to dust mass loss rates of the order of $1 \, M_\oplus \, \text{Gyr}^{-1}$ and dust densities close to the edge of the parameter space we consider (10 g cm$^{-3}$). These values do not, however, correspond to the peak in the probability density distribution found by the MCMC analysis.

The optical properties of the dust material influence the constraints mostly through their effects on the dust temperatures. The extinction coefficient $k_1$ has a sharp lower limit at about $k_1 \sim 10^{-2}$. This is an effect of how the $T_d(s)$ profile changes with $k_1$. For low values of $k_1$, the dust temperature only goes down with decreasing grain size (see Fig. 5.11). For dust with such a profile, the sublimation rate quickly goes down as the grain becomes smaller and as a result the grains do not sublimate. This makes it impossible to create a tail of finite length with a low-$k_1$ material. The refractive index $n$ has a weaker effect on dust temperatures and hence it is not constrained, although there is a small preference for lower values.

Finally, there is a clear correlation between the sublimation parameters $A$ and $B'$. This is to be expected, since they both appear in the exponent in Eq. 5.4 and the relative range in dust temperature is small. To aid our further analysis, we therefore define a new parameter

$$J_{1700K} \equiv J(T_d = 1700 \, \text{K}),$$

i.e., the mass loss flux from a dust grain at a temperature of 1700 K, with $J$ defined in Eq. 5.3. The value of 1700 K was chosen to minimise the width of the $J_{1700K}$ distribution (i.e., $J_{1700K}$ varies perpendicular to the correlation between $A$ and $B'$). The temperature

---

This parameter combines $A$ and $B'$ into a single parameter. However, it should not replace them as a free parameter in the model, since then the temperature dependence of the sublimation rate would be fixed.
can be thought of as the typical temperature of dust grains in well-fitting model realisations. However, the distribution we find for $J_{1700\,K}$ is still relatively broad (see Table 5.4), reflecting both the range in sublimation rates that give good-fitting light curves and the range in temperature reachable by combining different optical properties and grain sizes.

Individually, $A$ and $B'$ are unconstrained. The lack of solutions with high values of $A$ is an effect of the bound on $B'$. There is a small preference for low temperature sensitivity (corresponding to the bottom left corner of the $A$-$B'$ diagram).

### 5.3.2 Comparison with laboratory-measured dust properties

The main purpose of this work is to understand if and how the composition of the dust in the tail of KIC 1255b can be constrained by modelling the tail and resulting light curve. To see how the outcome of the MCMC analysis constrains the composition of the dust grains, we compare the posterior probability density function to the five composition parameters of a few selected dust species. Table 5.5 lists the values of these parameters for the materials we want to test. We only include crystalline forms because amorphous dust will rapidly anneal (Kimura et al. 2002). Of course, this table is a very incomplete list of possible dust species, in particular because it only includes pure materials. Because of the parametric description we use for the dust material, however, the results of the MCMC analysis provide constraints on the material properties that are independent of the availability of laboratory measurements and can be used to test any material for which good laboratory measurements become available.

Of the five free parameters of our model that describe the dust properties, two cannot be used to exclude species from Table 5.5. The refractive index $n$ is not constrained by the data at all and the lower-limit requirement for the bulk density $\rho_d$ is met by all dust species under consideration. The other three composition parameters, $k_1$, $A$, and $B'$, do give meaningful constraints. Figure 5.6 shows the marginalised probability density distribution for these parameters, together with the values of the materials from Table 5.5. The two panels are essentially two different projections of the same three-dimensional confidence region, with $J_{1700\,K}$ being used to define an axis perpendicular to the $A$-$B'$ correlation.

Comparison of the model results with the parameters of the dust species reveals that many of the tested materials cannot reproduce the observed transit profile. Only corundum (i.e., crystalline aluminium oxide) gives a satisfactory fit. We now briefly discuss each of the materials individually.

- **Iron** gives a reasonable fit in the $J_{1700\,K}$-$k_1$ projection. However, its sublimation parameters, which are established to higher accuracy than any of the other materials we consider, indicate that it is too volatile. Taking into account that its temperatures are slightly higher than predicted by its $k_1$ value (see Appendix 5.B), which would increase

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11 An important limiting factor in compiling a list of possible dust species for this research is the lack of reliable and consistent laboratory measurements of sublimation properties. In particular, knowledge of the accommodation coefficient $\alpha$ is lacking.
Table 5.5: Parameters of the dust species considered in this study

<table>
<thead>
<tr>
<th>Dust species</th>
<th>Density $\rho_d$ [g cm$^{-3}$]</th>
<th>Complex refractive index$^a$</th>
<th>Sublimation parameters$^f$</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$</td>
<td>$\log_{10}(k_1)$</td>
<td>$\mathcal{A}$ [10$^4$ K]</td>
<td>$B'$</td>
</tr>
<tr>
<td>Iron (Fe)</td>
<td>7.87</td>
<td>3.5$^b$</td>
<td>4.84 ± 0.12</td>
<td>31.3 ± 0.7</td>
</tr>
<tr>
<td>Silicon monoxide (SiO)</td>
<td>2.13</td>
<td>1.9</td>
<td>4.95 ± 0.14</td>
<td>31.2 ± 1.0</td>
</tr>
<tr>
<td>Crystalline fayalite (Fe$_2$SiO$_4$)</td>
<td>4.39</td>
<td>1.8 $-2.25 \pm 0.25$</td>
<td>6.04 ± 0.11</td>
<td>38.1 ± 0.7$^g$</td>
</tr>
<tr>
<td>Crystalline enstatite (MgSiO$_3$)</td>
<td>3.20</td>
<td>1.6 $-4.25 \pm 0.25$</td>
<td>6.89 ± 0.88</td>
<td>37.8 ± 5.0$^e$</td>
</tr>
<tr>
<td>Crystalline forsterite (Mg$_2$SiO$_4$)</td>
<td>3.27</td>
<td>1.6 $-3.75 \pm 0.25$</td>
<td>6.53 ± 0.40</td>
<td>34.3 ± 2.5</td>
</tr>
<tr>
<td>Quartz (SiO$_2$)</td>
<td>2.60</td>
<td>1.6$^e$ $-3.25 \pm 0.25$</td>
<td>6.94 ± 0.34</td>
<td>35.1 ± 1.8</td>
</tr>
<tr>
<td>Corundum (Al$_2$O$_3$)</td>
<td>4.00</td>
<td>1.6 $-1.75 \pm 0.25$</td>
<td>7.74 ± 0.39$^h$</td>
<td>39.3 ± 2.0$^h$</td>
</tr>
<tr>
<td>Silicon carbide (SiC)</td>
<td>3.22</td>
<td>2.5 $-3.50 \pm 0.25$</td>
<td>7.85 ± 0.39$^h$</td>
<td>37.4 ± 1.9$^e$</td>
</tr>
<tr>
<td>Graphite (C)</td>
<td>2.16</td>
<td>2.2 $-1.00 \pm 0.50$</td>
<td>9.36 ± 0.05</td>
<td>36.2 ± 1.8$^{e,h}$</td>
</tr>
</tbody>
</table>

Notes. ($a$) The values listed for $n$ are valid for the wavelength range 0.4 to 4 µm, which covers the Kepler bandpass as well as the peak of the stellar spectrum. For $k_1$, we list the values that give the best match between the dust-temperature-vs.-grain-size profiles computed using the full wavelength-dependent complex refractive indices of the various materials and using our simple two-parameter prescription (see Appendix 5.B). ($b$) For iron, $n(\lambda)$ steadily rises from 2.2 to 4.8 in the wavelength regime considered. ($c$) For iron and corundum, the best-matching $k_1$ values give temperatures that are slightly too low (see Appendix 5.B). ($d$) Our simple, two-parameter complex-refractive-index recipe cannot reproduce the dust-temperature-vs.-grain-size profile of SiO (see Appendix 5.B). ($e$) The $n(\lambda)$ and $k(\lambda)$ data we use for quartz only cover wavelengths of 3 µm and higher. To compute $n$ and $k_1$, these were extrapolated down. ($f$) Further details on the sublimation parameters, including evaporation coefficients $\alpha$ and molecular weights $\mu$, can be found in Table 3 of Paper I. ($g$) For these materials, no measurements of the evaporation coefficient $\alpha$ are available. In the computation of $B'$, we arbitrarily adopt $\alpha = 0.1$. ($h$) For sublimation parameters without a reported uncertainty, we set the standard deviation on the original $\mathcal{A}$ and/or $B$ to 5%, which is comparable to the level of uncertainty of sublimation parameters that do include an error bar.

References. D84 Draine & Lee (1984); D95 Dorschner et al. (1995); F01 Fabian et al. (2001); F04 Ferguson et al. (2004); G10 Gail (2010); G13 Gail et al. (2013); H90 Hashimoto (1990); J98 Jaeger et al. (1998); K91 Kushiro & Mysen (1991); K95 Koike et al. (1995); La93 Laor & Draine (1993); Li93 Lilov (1993); L08 Lihrmann (2008); M88 Mysen & Kushiro (1988); N94 Nagahara et al. (1994); O88 Ordal et al. (1988); P85 Palik (1985); S04 Schaefer & Fegley (2004); W13 Wetzel et al. (2013); Z73 Zavitsanos & Carlson (1973); Z11 Zeidler et al. (2011); Z13 Zeidler et al. (2013).
Figure 5.6: Comparison between our model results and the laboratory-measured properties of some real dust
species, as listed in Table 5.5, for the model parameters that provide meaningful constraints on the dust composi-
tion. Left: Constraints on the parameters that describe the temperature dependence of dust sublimation. The dashed
diagonal axis shows how $J_{1700 \text{K}}$ (the sublimation rate of dust at 1700 K) varies with $A$ and $B'$. Right: Combined
constraints on the extinction coefficient and the 1700 K sublimation rate. Note that the sublimation rate decreases to
the right.

the distance between the iron symbol and the edge of the allowed region of parameter
space, we deem grains of pure iron unlikely.

- Silicon monoxide is not shown in the right panel of Fig. 5.6, because its $T_d(s)$ profile
cannot be reproduced using our complex-refractory-index recipe for any value of $k_1$
(see Appendix 5.B). Since it reaches temperatures much higher than any that can be
produced by the recipe, and its sublimation parameters are on the “volatile edge” of the
allowed region, this material can be excluded as too hot and volatile.

- For fayalite (the iron-rich end-member of olivine), both the sublimation rate and $k_1$
individually give a marginal fit, but the combined constraints disfavour this material.

- The silicates enstatite (the magnesium-rich end-member of pyroxene), forsterite (the
magnesium-rich end-member of olivine), and quartz all have sublimation parameters
that are consistent with the model constraints, but their $k_1$ values are too low (i.e., in
pure form they are too transparent at visible and near-infrared wavelengths).

- Of the tested materials, corundum is the only one with properties that lie completely
within the allowed region of parameter space. It should be noted that, as for iron,
our complex-refractory-index recipe gives temperatures for corundum that are slightly
lower than the $T_d(s)$ profile computed using the full wavelength-dependent complex
refractory indices (see Appendix 5.B). This discrepancy means that corundum subli-
mates somewhat faster than its position in $J_{1700 \text{K}}-k_1$ space suggests. However, since
corundum is positioned farther from the “volatile edge” of the allowed region of param-
eter space, the argument used to disfavour silicon monoxide and iron does not apply to corundum.

- For silicon carbide, the sublimation rate is marginally compatible with the constraints, but the value of $k_1$ is too low.

- Pure graphite grains can clearly be excluded, since they are too refractory.

5.4 Discussion

5.4.1 Comparison with previous findings

We reach similar conclusions about the dust composition of KIC 1255b as in Paper I. In particular, both analyses find that corundum is the only material out of the nine tested species that can yield the right tail length. Figure 6 of Paper I indicates that the observed tail length is only reached at grain sizes of about $s \sim 10 \, \mu$m, while in this work we find that such grain sizes are too large. This discrepancy can be accounted for by the lower dust temperatures predicted for corundum by our complex-refractive-index recipe (see Appendix 5.B). Within the error bars (which include the uncertainties in both the sublimation and the stellar parameters) grain sizes of about $s \sim 1 \, \mu$m that we find here are still allowed.

5.4.2 An optically thick coma?

Throughout this work, as well as in Paper I, we have assumed that the dust tail is optically thin in the radial direction. Because we also assume that the dust grains are released from a single point in space, this optical-depth assumption naturally breaks down in our model at the head of the dust cloud. In reality, however, dust grains are released over a range of starting positions. To check the validity of the optical-depth assumption, we compute the height with respect to the planet’s orbital plane that the dust tail needs to extend in order to have a radial optical depth of unity. We denote half of this height with $h_{\tau=1}$ and in Fig. 5.7 this quantity is shown for a typical model realisation. It is compared with the height that the cloud can be expected to have based on the maximum possible size of the source region (i.e., upper limits on the size of the planet), and the vertical spreading caused by non-zero launch speeds in the vertical direction.

Figure 5.7 shows that the first few $R_\oplus$ of the dust cloud may be optically thick. Beyond about 10 $R_\oplus$, we expect the dust tail to become optically thin. In summary, the optical-depth assumption is valid for most of the dusty tail, but possibly not for the first few $R_\oplus$. This inaccuracy, however, may be mitigated by the large solid angle of the star at the distance of the dust cloud. That is, dust grains that are obscured by others in the radial direction can still be reached by radiation emanating from close to the stellar limb.

The region up to 10 $R_\oplus$ is responsible for about half of the transit depth as computed under the assumption that it is optically thin. Therefore, optical-depth effects could have a
Figure 5.7: Diagram to illustrate for what part of the tail the assumption of an optically thin dust cloud is valid. The solid line shows $h_{\tau=1}$, the dust cloud half-height for which the radial optical depth equals unity. The dashed lines give the heights above the planet’s orbital plane reached by dust grains that are launched vertically from the planet with different initial vertical speeds $v_{z,0}$. Distances on the bottom axis are measured along the planet’s orbit. All curves are drawn for a typical model realisation. The shaded regions in the lower left indicate upper limits on the planet’s radius based on (1) the absence of transits in part of the *Kepler* data (light grey; $R_p < 1.15 R_\oplus$; Brogi et al. 2012) and (2) the planet-mass-dependent mass loss rate (dark grey; $R_p \lesssim 0.3 R_\oplus$, model-dependent; Perez-Becker & Chiang 2013).

significant impact on the light curve. However, it is important to note that the optical depth in the radial direction is not the same as the optical depth from the star to the observer. The inclination of the planet’s orbital plane with respect to the line of sight means that the optical depth towards the observer will be less than the radial optical depth. This is a strong effect, since dust motion in the planet’s orbital plane are substantial, and, for an inclined orbit, they contribute to the vertical extend of the dust cloud with respect to the line of sight. Nevertheless, the high radial optical depth close to the planet warrants further investigation since it will affect the radiation pressure on the dust grains, as well as their temperatures and hence their sublimation.

Finally, the dashed curves in Fig. 5.7 show minima just beyond $100 R_\oplus$, which occur because dust particles on inclined orbits cross the orbital plane of the planet halfway through their own orbit. In principle, this could give another possible explanation for the small decrement in flux tentatively detected in the egress of the *Kepler* short cadence light curve of KIC 1255b (Croll et al. 2014). In this scenario, the increase in optical depth at the knot halfway along the orbit of the dust grains would cause a temporary brightening in the egress, and hence an apparent dimming afterwards. However, to reach an optical depth higher than unity, the source area would have to be significantly smaller than the required $h_{\tau=1}$ at the distance of the minimum, which is less than $0.1 R_\oplus$. Also, small differences in initial grain size (and therefore in $\beta$), as well as in initial launch direction and speed, can easily wash out
this optical-depth effect.

5.5 Conclusions

We have developed a numerical model to simulate the dusty tails of evaporating planets and their transit light curves, with the primarily goal of putting constraints on the composition of the dust in such tails, and applied it to the *Kepler* light curve of the prototypical evaporating planet KIC 1255b. Although the precise best-fit values and uncertainties we find for the model parameters may depend on modelling details (e.g., simulating only a single initial grain size), our analysis shows that by using a physically motivated model it is possible to put meaningful constraints on the composition of the dust in the tail of an evaporating planet based on the shape of its broadband transit light curve. Since the dust composition is related to that of the planet, such constraints can provide helpful input for theories of planet formation and evolution.

Regarding KIC 1255b, we draw the following conclusions.

1. *Dust composition.* We find that only certain combinations of material properties (specifically, sublimation parameters and extinction coefficient) can reproduce the observed transit profile. The dust grains need to have the right sublimation rate and temperature to yield the observed tail length but avoid the correlations between subsequent transits that arise when grains survive longer than an orbital period of the planet. These constraints allows us to rule out or disfavour many of the pure materials we tested for the dust composition (see Fig. 5.6): iron, silicon monoxide, fayalite, enstatite, forsterite, quartz, silicon carbide, and graphite. The only material we found to match the constraints is corundum (i.e., crystalline aluminium oxide). Grains made of combinations of the tested materials, however, cannot be ruled out.

2. *Grain sizes.* We simulate the dust cloud assuming the dust grains all have the same initial size, but let this size evolve due to sublimation. The typical initial grain sizes we find are $1.2^{+4.4}_{-0.9}$ µm. The shape of the pre-ingress brightening favours smaller initial grain sizes of 0.2 to 0.3 µm.

3. *Mass loss rate.* We find that the planet loses $2.5^{+13.1}_{-1.9}$ $M_\oplus$ Gyr$^{-1}$ in dust alone. The exact value depends on the size and bulk density of the dust grains and the best-fitting solutions are found at the lower end of the allowed range ($\sim 1$ $M_\oplus$ Gyr$^{-1}$).

4. *Tail morphology.* It is not necessary to invoke an object consisting of multiple components (e.g., a coma and a tail) to explain the detailed shape of the transit light curve, including the “kink” seen in the egress. This feature emerges naturally from the distribution of the dust extinction cross-section in the tail (see Fig. 5.2). We also find evidence that the head of the dust cloud may be optically thick in the radial direction (see Fig. 5.7).
Acknowledgements. The research leading to the presented results has received funding from the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no 338251 (StellarAges).

5.A Evolutionary status of KIC 12557548

The literature contains diverging estimates of the stellar parameters of KIC 12557548, casting uncertainty on its evolutionary status (see Table 5.6). The star is generally found to be a mid-K dwarf, but Kawahara et al. (2013) report stellar parameters that point to a subgiant status. If the host star is indeed evolved off the main sequence, this has important consequences for the story of the evaporating planet. It could mean that the evaporation of the planet was triggered by the evolution of the star: i.e., that the planet was stable at its present orbital distance throughout the main-sequence lifetime of the star, and has only recently started losing (substantial amounts of) mass due to an increase in stellar irradiation. Therefore, it is important to have a good understanding of the evolutionary status of KIC 12557548. In addition, the constraints derived from our dust tail modelling depend on the properties we assume for the host star. This appendix describes the checks we did to distinguish between the two scenarios (main-sequence star vs. subgiant).

We first performed an astroseismic analysis, which entails searching for evidence of solar-like oscillations in the light curve of KIC 12557548. The frequencies of these oscillations scale with stellar properties, and the method has the potential to pin down a star’s evolutionary status with high accuracy (Chaplin & Miglio 2013). Specifically, we inspected the power-spectrum of the decorrelated and detrended long-cadence Kepler light curve with the transit signal (phases $\varphi \in [-0.15, 0.2]$) masked out. Unfortunately, no significant solar-like oscillations could be detected. In principle, their non-detection can be used to obtain a lower limit on the star’s surface gravity (Campante et al. 2014). Given the target’s faintness, however, we expect the Kepler data to be of insufficient quality to yield a meaningful constraint.

Another way to constrain the properties of KIC 12557548 is to use the transit signal, which contains information about the star’s mean density. Using Kepler’s third law, it is possible to express a star’s mean density $\rho_\star$ in terms of the orbital period $P_p$ and scaled semi-major axis $a_p/R_\star$ of an orbiting planet:

$$\rho_\star = \frac{\rho_p}{G P_p^2} \left( \frac{a_p}{R_\star} \right)^3. \quad (5.9)$$

For a spherical transiting planet, $a_p/R_\star$ can be computed analytically from the shape of the light curve (Seager & Mallén-Ornelas 2003). It is mainly sensitive to the ratio of transit duration to orbital period, modulated by the impact parameter $b$ (a small star and a low $b$ can give the same transit duration as a large star and a high $b$). In the case of KIC 1255b, it is still possible to derive constraints on $a_p/R_\star$ from the light curve, but, because the occulting
## Table 5.6: Estimates of the stellar parameters of KIC 12557548

<table>
<thead>
<tr>
<th>Reference</th>
<th>$T_{\text{eff.}}$ [K]</th>
<th>log $g$ [cgs]</th>
<th>[Fe/H]</th>
<th>Evolutionary status</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kepler Input Catalogue$^a$</td>
<td>4400 ± 200</td>
<td>4.6 ± 0.5</td>
<td>−0.2 ± 0.5</td>
<td>main-sequence star</td>
<td>photometry</td>
</tr>
<tr>
<td>Rappaport et al. (2012)</td>
<td>4300 ± 250</td>
<td></td>
<td></td>
<td>main-sequence star</td>
<td>low-resolution spectroscopy</td>
</tr>
<tr>
<td>Kawahara et al. (2013)</td>
<td>4950 ± 70</td>
<td>3.9 ± 0.2</td>
<td>0.09 ± 0.09</td>
<td>subgiant</td>
<td>high-resolution spectroscopy</td>
</tr>
<tr>
<td>Huber et al. (2014)$^b$</td>
<td>4550 +140_{−131}</td>
<td>4.622_{−0.043}^{+0.053}</td>
<td>−0.180 ± 0.320</td>
<td>main-sequence star</td>
<td>photometry</td>
</tr>
</tbody>
</table>

**Notes.** From left to right: effective temperature, surface gravity, and metallicity. $^a$ Brown et al. (2011). $^b$ The estimate of Huber et al. (2014) incorporates priors (see their Sect. 2). This explains their improved uncertainties with respect to the Kepler Input Catalogue, especially on log $g$. 
object is a non-trivially shaped dust cloud rather than a solid sphere, this requires a numerical approach. Hence, we fit the phase-folded *Kepler* light curve using the 1-D dust cloud model of Brogi et al. (2012), adding $a_p/R_*$ as an extra free parameter. The outcome of the MCMC fitting shows a clear degeneracy between $a_p/R_*$ and $b$ (see Fig. 5.8). Nevertheless, there is a firm (3$\sigma$) constraint on the scaled semi-major axis of $2.8 \lesssim a_p/R_* \lesssim 4.9$, which corresponds to a mean-stellar-density range of $0.9$ g cm$^{-3} \lesssim \rho_* \lesssim 5.4$ g cm$^{-3}$.

To use this constraint to determine the evolutionary status of KIC 12557548, we compare it with the mean stellar densities predicted for the two scenarios by stellar models. Figure 5.9 shows Yonsei–Yale stellar evolutionary tracks (Yi et al. 2003; Demarque et al. 2004) and the associated curves of constant mean stellar density. It demonstrates that the stellar parameters found by Kawahara et al. (2013) correspond to subgiants with mean densities that are inconsistent with the constraints from the transit signal. In contrast, the values of $\rho_*$ corresponding to the mid-K dwarf parameters match well with the $a_p/R_*$ maximum in Fig. 5.8.

Based on the stellar density constraint, we reject the estimate of Kawahara et al. (2013) and conclude that KIC 12557548 is a K-type main-sequence star rather than a subgiant. It is unclear why Kawahara et al. (2013) arrive at a discrepant result despite their more advanced method of determining stellar parameters compared to the other efforts listed in Table 5.6.

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12 This model also contains implicit information about the stellar properties in the limb-darkening coefficient $u$. We tested running the model with $u = 0.76$ and $u = 0.79$, appropriate for the two possible stellar types (Claret 2000), and found the difference in outcome to be negligible. Fig. 5.8 shows the results for the $u = 0.79$ run.

13 The evolutionary tracks we show use a metallicity of $[\text{Fe/H}] = 0.05$ and an $\alpha$-enhancement of $[\alpha/\text{Fe}] = 0.0$. Varying the metallicity within the range suggested by Table 5.6 does not change the conclusions.
intriguing possibility is that gasses released by the evaporating planet contaminate the stellar spectrum, affecting the retrieval of the stellar parameters. If this is the case, it gives hope to probing the composition of the planet more directly by spectroscopically examining the gas component of the planet’s outflow.

5.B Complex-refractory-index treatment

Our dust-tail model uses a simple prescription for the complex refractory index of the dust material. This recipe, summarised by Eq. 5.7, consists of two free parameters: $n$, the refractory index (one value for all wavelengths), and $k_1$, the extinction coefficient at wavelengths below $\lambda_{\text{split}} = 8 \, \mu\text{m}$. The extinction coefficient at wavelengths of 8 $\mu\text{m}$ and higher is fixed at $k_2 = 1$. Our model uses the complex refractory index in Mie calculations to compute the optical efficiency factors of the dust grains. These, in turn, are needed to find the $\beta$ ratios and temperatures of the dust grains, and to generate the synthetic light curves. In this appendix, we demonstrate that our simple recipe for the complex refractory index can reproduce many of the dust temperatures found for real materials, and we explain how we assign values of $n$ and $k_1$ to the real materials as listed in Table 5.5.

Figure 5.10 shows the full wavelength-dependent complex refractory indices of the nine
Figure 5.10: Complex refractory indices as a function of wavelength for the dust species considered in this study. **Top:** The real part of the complex refractory index. The grey area indicates the range in wavelength used to determine the values of \( n \) listed in Table 5.5. **Middle right:** The imaginary part of the complex refractory index, also known as the extinction coefficient. The vertical dashed line at 8 \( \mu m \) marks \( \lambda_{\text{split}} \), which is the boundary between \( k_1 \) and \( k_2 \), the two constant values of extinction coefficient used in our recipe. The horizontal black line at unity marks the value we use for \( k_2 \). **Middle left:** The values of \( k_1 \) (listed in Table 5.5) that best reproduce the dust-temperature-vs.-grain-size profiles of the various materials. The symbols are offset horizontally for visibility only. No symbol is shown for SiO, since its temperatures cannot be reproduced by our complex-refractory-index recipe. **Bottom:** The spectrum of the Kurucz model atmosphere that we use for the host star KIC 12557548, together with thermal emission spectra of dust at different temperatures, all normalised to unity. Also shown is the *Kepler* response function (not normalised), for which the vertical axis should read “response”.

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materials considered in this study. The sources of these data are listed in Table 5.5. For materials with different complex refractory indices for different crystal axes, the data for only one axis is shown. While the number and positions of resonance features changes between the different crystal axes, the overall shape of the spectra does not change significantly. Also shown in Fig. 5.10 are the Kurucz (1993) model that we use for the stellar spectrum, two Planck curves for possible dust temperatures, and the Kepler response function (Koch et al. 2010). These can all be used to determine which parts of the $n(\lambda)$ and $k(\lambda)$ spectra are the most important for the Mie calculations.

From Fig. 5.10 it becomes clear that, at visible and near-infrared wavelengths, the values of $k(\lambda)$ vary significantly between materials. In the mid-infrared region, however, most materials surveyed here exhibit a range of narrow $k(\lambda)$ features, but have average $k(\lambda)$ values that fall within a much smaller range. This motivates our choice of a fixed $k_2 = 1$ for wavelengths beyond $\lambda_{\text{split}} = 8 \, \mu m$. The rise in $k(\lambda)$ in the ultraviolet seen for many materials does not affect their heating, since the stellar spectrum already drops off rapidly around $\lambda = 0.3 \, \mu m$. This justifies the use of a constant $k_1$ for all wavelengths lower than 8 $\mu m$.

Figure 5.11 compares equilibrium dust temperatures calculated using the full wavelength-dependent complex refractive indices and the simple recipe of constant $n$, $k_1$, and $k_2$. By varying $k_1$, a large range of the temperatures observed for the real dust species can be reproduced. Varying $n$ has a smaller effect of the temperature profiles, causing a bump in temperature at grain sizes of $s \sim 0.01 \, \mu m$ for higher values of $n$.

Of the materials we tested, the only one for which the simple recipe fails completely is silicon monoxide (SiO). The reason for this is that the $k(\lambda)$ profile of SiO changes abruptly around $\lambda = 0.8 \, \mu m$, causing the grains to efficiently absorb the stellar radiation, but not efficiently cool in the near infrared.

The very low temperatures of some silicates (enstatite, forsterite, and quartz) can be explained by their combination of low $k(\lambda)$ in the optical and near infrared, with the strong features in the mid infrared. This causes them to absorb stellar radiation very inefficiently, and only cool efficiently when they reach temperatures that are so low that the mid infrared features become relevant, as illustrated by the 500 K Planck curve in Fig. 5.10. Our complex-refractive-index recipe with $\lambda_{\text{split}} = 8 \, \mu m$ and $k_2 = 1$ manages to capture this behaviour well.

The complex refractive indices also affect other parts of the dust-tail model ($\beta$ ratios and the generation of light curves), but its effect on the final MCMC results through dust temperature is the most pronounced. This is apparent from the fact that $n$ has a much smaller effect on dust temperatures than $k_1$ (see Fig. 5.11) and it is not well constrained by the data (see Sect. 5.3.1).

In order to compare the real dust species to the model results, we wish to assign a single value of $n$ and $k_1$ to each of the materials. Figure 5.10 shows that $n(\lambda)$ remains almost constant in the wavelength range $0.4 \, \mu m < \lambda < 4 \, \mu m$ (grey area) for all tested materials except iron, for which $n(\lambda)$ gradually rises with wavelength. This wavelength regime includes the peak of the stellar spectrum, as well as the Kepler bandpass. To determine the value of $n$ for each of
Figure 5.11: Dust temperatures at the distance of the planet as a function of grain size calculated using different complex refractive indices: **Top:** Using the full wavelength-dependent complex refractive indices of real dust species, measured through laboratory experiments. **Middle:** Using our simple complex refractive index prescription, keeping $n$ fixed and varying $k_1$. **Bottom:** Using our simple complex refractive index prescription, keeping $k_1$ fixed and varying $n$. 
the dust species, we therefore take the average value of \( n(\lambda) \) over these wavelengths. These values are listed in Table 5.5.

The value of \( k(\lambda) \) of a given dust species can vary by orders of magnitude within the relevant wavelength regime. Therefore, rather than trying to estimate an effective or average value directly from the \( k(\lambda) \) profile, we take the value of \( k_1 \) for which our simple recipe yields the \( T_d(s) \) profile that best matches the one computed from the full data of that species. To do this, we set \( n \) at the value established before and go through \( \log_{10}(k_1) \) in steps of 0.25, visually checking for a good match between the two \( T_d(s) \) profiles, in particular at grain sizes of \( 0.01 \, \mu m < s < 10 \, \mu m \). The uncertainties in \( \log_{10}(k_1) \) that we quote reflect the values beyond which the profiles begin to deviate significantly.

The results are listed in Table 5.5 and shown in the middle left panel of Fig. 5.10. As mentioned before, our complex-refractive-index recipe fails to reproduce the \( T_d(s) \) profile of silicon monoxide. For the remaining materials, the match is good, except for iron and corundum, for which the temperatures predicted by the best-fitting \( n \) and \( k_1 \) values are slightly lower than the ones found using the full \( n(\lambda) \) and \( k(\lambda) \) data. This discrepancy means that these materials will have sublimation rates that are somewhat higher than suggested by their position in \( J_{1700 \, K} \)-\( k_1 \) space.
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Summary

In the past few decades, it has become clear that the Sun is not the only star accompanied by planets. Extrasolar planets (exoplanets) have been detected around more than a thousand stars and their ubiquity indicates that planet formation is more the rule than the exception. Surprisingly, many of the discovered exoplanetary systems are very different from the Solar System in the types of planets they harbour and the distances from the host stars at which those planets orbit. These realisations have raised some exciting questions:

- Why does planet formation have such diverse results?
- What processes determine the architecture of a planetary system?
- How special is the Earth as a planet and the Solar System as a planetary system?
- Are there other places in the Milky Way galaxy where life could have emerged?

Finding answers to such questions is one of the goals of modern astronomy.

In order to address these questions, it is necessary to obtain detailed information about extrasolar systems through astronomical observations. However, the vast distances between stars, as well as the small size of a planet with respect to its host star, present major obstacles to characterising exoplanets. The majority of what we currently know about exoplanets comes from the effects planets have on their host stars.

However, a planetary system consist of more than just planets. Also orbiting the star is a wide range of smaller bodies, such as asteroids, comets, and interplanetary dust grains. The study of these materials can provide information about the planetary systems that cannot be inferred from the planets themselves. Circumstellar dust is particularly useful in this respect. A swarm of dust grains may constitute a very small percentage of the mass of a system, but its collective cross-section is enormous. This makes it possible for modern telescopes to directly observe the radiation emitted by an extrasolar cloud of dust, and in some cases even to distinguish its spatial structure.

Circumstellar dust grains usually have a relatively short lifetime, because they are removed or destroyed by several mechanisms. For example, the pressure associated with a
star’s radiation can blow very small dust grains away, out of the system. This means that for any dust to be present, it must be replenished continuously from larger bodies that constitute a more stable mass reservoir, for instance through destructive collisions between asteroids or comets that produce dust as fragments. Because circumstellar dust originates in larger bodies, the two populations are related in location and composition, and observations of dust can be used to probe exoplanetary materials.

To make the step from dust observations to inferring properties of an exoplanetary system, one must understand in detail how dust grains are produced, how they behave after being released, and how they are destroyed or removed. Understanding the physics of circumstellar dust is the subject of this thesis. The thesis focusses specifically on dust grains that orbit extremely close to their host star, at only a few stellar radii. At these distances, the dust is heated to temperatures so high that the solid material it consists of turns into vapour, a process called sublimation.

Sublimation can add some interesting complications to the life of a circumstellar dust grain. Most importantly, as a dust grain sublimates, it gradually loses mass and becomes smaller. Generally, smaller dust grains have a larger cross-section in proportion to their mass. This means that as a dust grain becomes smaller, the forces on the grain due to stellar radiation pressure become more important with respect to gravity. Such changes influence the movement of the dust grain. We investigate two particular situations in which dust sublimation is relevant: hot exozodiacal dust and the dusty tails of evaporating exoplanets.

**Hot exozodiacal dust**

Chapters 2 and 3 are concerned with the phenomenon known as hot exozodiacal dust. This is dust located in the close vicinity of a star that can be detected through specialised infrared observations. Some 10% to 30% of all stars seem to have such a population of dust, but its origin is still unclear.

In Chapter 2, we investigate a possible mechanism that could explain the phenomenon. The dust could be created by collisions in a belt of asteroids or comets further out, and dragged inward by forces caused by the stellar radiation and the orbital motion of the dust grains. Once the dust is very close to the star, it starts sublimating, which decreases the size of the dust grains. This brings about changes in the radiation pressure forces, causing the inward migration to slow down and hence leading to a pile up of dust grains around the sublimation distance from the star. Although the pile-up mechanism seems promising to explain the location of hot exozodiacal dust, our detailed investigation of this process reveals that it is inefficient. We conclude that the proposed mechanism cannot explain the observed quantities of dust.

Chapter 3 is an in-depth study of the dust in the close vicinity of the star Fomalhaut. This star is known to have belts of asteroids and comets that produce dust through mutual collisions. Infrared observations show that some dust also exists very close to the star. In this chapter, we derive the properties and spatial distribution of the dust in the inner parts of the
Fomalhaut system from the observations. Using this information, we check several possible mechanisms that could explain the presence of the dust, finding that none can provide dust in sufficient quantities to explain the observations. The origin of hot exozodiacal dust remains a mystery.

**Dusty tails of evaporating exoplanets**

Chapters 4 and 5 are about small exoplanets that orbit so close to their stars that they evaporate due to the intense stellar irradiation. Such planets emit large amounts of dust grains, which end up in a comet-like tail behind the planet. As these objects pass in front of their host star, the cloud of dust causes the star to temporarily appear slightly dimmer. Using an accurate astronomical instrument, this effect can be followed in detail. From the precise way in which the star dims, it is then possible to derive the shape of the dust cloud. At the moment, three of these small evaporating exoplanets are known.

We describe why dust grains that are ejected from the planet trail behind the planet, forming a tail. Put simply, because the dust grains are pushed outward by the radiation pressure of the star, they travel on orbits that are a bit larger than that of the planet, and consequently they take marginally longer to circle the star. Hence, the dust grains lag behind the planet and form a tail. As they slowly drift away from the planet, the dust grains gradually sublimate. If the dust grains are made of a refractory material, they will take long to sublimate and yield a long tail. Conversely, volatile dust species disappear quickly and give short tails. Therefore, the length of the tail, which can be derived from the duration of the dimming of the star, can be used to learn about the composition of the dust.

In Chapter 4, we study this astrophysical problem using pen-and-paper-type techniques. These are useful to gain a better understanding of the situation and provide quick answers, but they also require making many simplifying assumptions about the dust grains and their behaviour, which may not be accurate in some circumstances. Chapter 5 introduces a numerical model, which uses more elaborate computer calculations to determine the dust composition from the dimming of the star. This technique is slower and more cumbersome, but it uses less assumptions than the pen-and-paper method, and provides a more detailed answer.

The results of the two different analysis methods are consistent. We find that the dust in the tail of one of the investigated evaporating planets could be made of the mineral corundum, an aluminium oxide. It is the same material that rubies and sapphires are made of. The study also demonstrates that many other dust compositions, such as iron, graphite, and magnesium-rich silicates, are much less likely. Most importantly, this work shows that it is possible to distinguish between different compositions for the dust in the tails of small evaporating planets from the way their host star dims as they pass in front of it. Since the dust originates in the evaporating planet, it can be used to probe the composition of this type of exoplanets.
In de afgelopen paar decennia is het duidelijk geworden dat de Zon niet de enige ster is die wordt vergezeld door planeten. Extrasolaire planeten (exoplaneten) zijn ontdekt rondom meer dan duizend sterren en het feit dat ze zo veelvoorkomend wijst erop dat de vorming van planeten meer de regel dan de uitzondering is. Verrassend genoeg zijn veel exoplanetaire systemen erg verschillend van het Zonnestelsel qua de types planeten die ze huisvesten en de afstanden tot de ster waarop die planeten ronddraaien. Deze realisaties hebben enkele spannende vragen opgeroepen:

- Waarom heeft planeetvorming zulke uiteenlopende resultaten opleveren?
- Welke processen bepalen de bouw van een planetenstelsel?
- Hoe speciaal is de Aarde als planeet en het Zonnestelsel als planetenstelsel?
- Zijn er andere plekken in de Melkweg waar leven zou kunnen zijn ontstaan?

Het beantwoorden van zulke vragen is één van de doelstellingen van de moderne sterrenkunde.

Om deze vragen aan te kaarten, is het nodig om uitvoerige informatie te verkrijgen over extrasolaire systemen door middel van sterrenkundige observaties. Echter, de gigantische afstanden tussen sterren, en de kleine afmeting van een planeet ten opzichte van zijn ster, zijn fikse obstakels voor het karakteriseren van exoplaneten. Het merendeel van wat we op het moment weten over exoplaneten komt uit de effecten die planeten hebben op de sterren waar ze omheen draaien.

Maar een planetenstelsel bestaat uit meer dan alleen planeten. Verder draaien er allerlei kleine lichamen rond de ster, zoals planetoiden, kometen, en interplanetaire stofkorrels. Het bestuderen van deze materialen kan informatie opleveren die niet van de planeten zelf kan worden afgeleid. Circumstellair stof is in dit opzicht erg leerzaam. Een zwerm stofkorrels maakt misschien maar een klein percentage uit van de massa van een systeem, maar heeft een enorme collectieve doorsnede. Dit maakt het mogelijk voor moderne telescopen om direct
Nederlandse samenvatting
de straling te observeren die wordt uitgezonden door een extrasolair stofwolk, en soms zelfs om de ruimtelijke structuur van de wolk te ontwaren.

Circumstellaire stofkorrels hebben normaal gesproken een relatief korte levensduur, omdat ze worden verwijderd of vernietigd door enkele mechanismen. De druk van de straling van de ster kan kleine stofkorreltjes bijvoorbeeld het systeem uit blazen. Dit betekent dat als er enig stof aanwezig is, dit continue moet worden bijgevoed vanuit grotere lichamen die een stabiel massareservoir vormen, bijvoorbeeld door middel van destructieve botsingen tussen planetoiden of kometen die stof produceren als brokstukken. Omdat circumstellaire stof zijn oorsprong heeft in grotere lichamen zijn de twee populaties verbonden in locatie en samenstelling en kunnen waarnemingen van stof gebruikt worden om exoplanetaire materialen te onderzoeken.

Om de stap te zetten van waarnemingen aan stof naar het achterhalen van de eigenschappen van een exoplanetair systeem, is het nodig om in detail te begrijpen hoe stofkorrels worden geproduceerd, hoe ze zich gedragen nadat ze zijn losgelaten, en hoe ze worden vernietigd of verwijderd. Het begrijpen van de fysica van circumstellaire stofkorrels is het onderwerp van dit proefschrift. Het proefschrift focust in het bijzonder op stofkorrels die zich extreem dichtbij hun ster bevinden, op afstanden van maar enkele ster-stralen. Op deze afstanden wordt het stof verwarmd tot temperaturen die zo hoog zijn dat het vaste materiaal waaruit het bestaat in gasvorm wordt omgezet, in een proces dat sublimatie heet.

Sublimatie kan enkele interessante complicaties teweeg brengen in het leven van een circumstellaire stofkorreltje. Het voornaamste effect is dat stofkorrels kleiner worden naarmate ze massa verliezen door sublimatie. Over het algemeen hebben kleinere stofkorreltjes een grotere doorsnede voor hun massa. Dit heeft tot gevolg dat naarmate een stofkorreltje kleiner wordt de krachten op het korreltje van stralingsdruk belangrijker worden in vergelijking met zwaartekracht. Dit soort veranderingen beïnvloeden de beweging van het stofkorreltje. We onderzoeken twee specifieke situatie waarin stofsublimatie relevant is: heet zodiakaal stof en de stofstraarten van verdampende exoplaneten.

Heet zodiakaal stof
Hoofdstukken 2 en 3 houden zich bezig met het fenomeen heet zodiakaal stof. Dit is stof dat zich in de nabijheid van een ster bevindt en dat gedetecteerd kan worden door middel van gespecialiseerde infrarood waarnemingen. Zo’n 10% tot 30% van alle sterren lijken een dergelijke populatie stof te hebben, maar de oorsprong ervan is nog steeds onduidelijk.

In Hoofdstuk 2 onderzoeken we een mogelijk mechanisme dat het fenomeen zou kunnen verklaren. Het stof zou geproduceerd kunnen worden door botsingen in een gordel van planetoiden of kometen op een grotere afstand van de ster, en naar binnen migreren door toedoen van krachten die worden veroorzaakt door de straling van de ster en de beweging van de stofkorreltjes. Wanneer het stof heel dicht bij de ster is, begint het te sublimeren, waardoor de stofkorreltjes kleiner worden. Dit veroorzaakt veranderingen in de stralingsdruk krachten, waardoor de migratie naar binnen toe langzamer wordt en er een opeenhoping van stof
teweeg wordt gebracht rondom de sublimatie afstand tot de ster. Hoewel het opeenhopingsmechanisme veelbelovend lijkt om de locatie van het hete exozodiakale stof te verklaren, blijkt uit onze nauwkeurige analyse van dit process dat het inefficient is. We concluderen dat het voorgestelde mechanisme de waargenomen hoeveelheden stof niet kan verklaren.

Hoofdstuk 3 is een grondige analyse van het stof in de nabijheid van de ster Fomalhaut. Het is bekend dat deze ster gordels planetoïden en kometen heeft die stof produceren door middel van onderlinge botsingen. Infrarood waarnemingen laten zien dat er zich ook stof erg dicht bij de ster bevindt. In dit hoofdstuk achterhalen we de eigenschappen en de ruimtelijke verdeling van het stof in de binnenste delen van het Fomalhaut systeem vanuit de waarnemingen. Met behulp van deze informatie testen we verscheidene mogelijke mechanismen die de aanwezigheid van het stof zouden kunnen verklaren, maar we komen tot de slotsom dat er geen voldoende stof kan leveren om de waarnemingen te verklaren. De oorsprong van heet exozodiakaal stof blijft een mysterie.

Stofstraarten van verdampende exoplaneten

Hoofdstukken 4 en 5 gaan over kleine exoplaneten die zo dicht bij hun ster ronddraaien dat ze verdampen als gevolg van de hevige bestraling door de ster. Dit soort planeten spuwen grote hoeveelheden stofkorrels uit, die in een komeetachtige staart achter de planeet terecht komen. Wanneer deze objecten voor hun ster langs komen, dan zorgt de stofwolk ervoor dat de ster tijdelijk een klein beetje minder helder lijkt. Met een zeer precies afgesteld sterrenkundig instrument kan dit effect in detail worden gevolgd. Van de precieze manier waarop de ster verduistert is het dan mogelijk om te vorm van de stofwolk af te leiden. Er zijn op dit moment drie van dit soort kleine verdampende exoplaneten bekend.

We beschrijven hoe het komt dat de stofkorrels die van de planeet af komen achter de planeet raken en een staart vormen. Simpel gezegd worden de stofkorrels naar buiten geduwd door de stralingsdruk van de ster, waardoor ze op omloopbanen komen die een beetje groter zijn dan die van de planeet, en bijgevolg kost het ze iets langer om hun baan om de ster af te leggen. Hierdoor blijven de stofkorrels achter op de planeet en vormen ze een staart. Terwijl ze weg van de planeet bewegen, sublimeren de stofkorrels geleidelijk. Als de stofkorrels gemaakt zijn van een vuurvast materiaal zal het langer duren voordat ze volledig gesublimeerd zijn en dit soort stofkorrels zullen langere staarten opleveren. Vice versa, vluchtige stofsoorten verdwijnen snel en zullen korte staarten geven. Daarom kan de lengte van de staart, die kan worden afgeleid van de duur van de verduistering van de ster, worden gebruikt om iets te leren over de samenstelling van het stof.

In Hoofdstuk 4 bestuderen we dit sterrenkundig probleem met behulp van pen-en-papier technieken. Die zijn handig om een beter begrip van de situatie te krijgen en geven snelle antwoorden, maar ze vereisen het maken van veel vereenvoudigende aannames over de stofkorrels en hun gedrag, die in sommige omstandigheden niet accuraat zouden kunnen zijn. Hoofdstuk 5 introduceert een numeriek model dat gebruik maakt van meer uitvoerige computerberekeningen om de stof samenstelling af te leiden van de verduistering van de ster.
Deze techniek is logger en langzamer, maar gebruikt minder aannames dan de pen-en-papier methode, en levert een gedetailleerder antwoord op.

De resultaten van de twee verschillende analyse-methodes zijn consistent. We bevinden dat het stof in de staart van één van de onderzochte verdampende planeten gemaakt zou kunnen zijn van het mineraal korund, een aluminiumoxide. Dit is het materiaal waarvan ook robijnen en saffieren gemaakt zijn. De studie toont verder aan dat veel andere stofsamenstellingen, zoals ijzer, grafit, en magnesiumrijke silicaten, een stuk minder waarschijnlijk zijn. Het belangrijkste is dat dit werk laat zien dat het mogelijk is om onderscheid te maken tussen verschillende samenstellingen voor het stof in de staarten van kleine verdampende planeten van de manier waarop hun sterren verduisteren als ze er voor langs passeren. Aangezien het stof afkomstig is van de verdampende planeet kan het gebruikt worden om de samenstelling van dit type exoplaneten te onderzoeken.
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